Exchange rate dynamics, balance sheet effects, and capital flows. A Minskyan model of emerging market boom-bust cycles

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Abstract

The paper provides a Minskyan open economy model of endogenous boom-bust cycles in emerging market economies, which explains the empirically observed procyclicality of exchange rates and the countercyclicality of the trade balance. It highlights the interaction of flexible exchange rate dynamics with balance sheets. Currency appreciation improves the net worth of firms with foreign currency debt, giving a boost to investment. Throughout the boom phase, the trade balance worsens. Pressures on the domestic exchange rate mount until the currency depreciates. Contractionary balance sheet effects then set in as domestic firms face a drop in their net worth. If capital inflows are driven by exogenous risk appetite, fluctuations can assume the form of shock-independent endogenous cycles. An increase in risk appetite raises the volatility of the cycle. Financial account regulation can reduce macroeconomic volatility, but the larger the risk appetite, the more financial account regulation is required to achieve this.

Keywords: Business cycles, emerging market economies, balance sheet effects, Minsky

JEL Codes: E11, E12, F36, F41

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1 Introduction

Business cycle research shows that macroeconomic fluctuations in output, exchange rates, and the current account are significantly stronger in emerging market economies (EMEs) compared to industrial economies (Agénor et al., 2000; Calderón and Fuentes, 2014; Uribe and Schmitt-Grohé, 2017, chap. 1). A common pattern is the coincidence of large capital inflows, exchange rate appreciation, and widening current account deficits during the boom, and capital outflows, currency depreciation, and current account reversals during the bust (Reinhart and Reinhart, 2009; Herr, 2013). The trade balance is thus strongly countercyclical, while nominal exchange rates - unlike in most industrial economies - behave procyclical (Cordella and Gupta, 2015).

A further characteristic of EMEs is the significance of foreign-currency borrowing (Eichengreen et al., 2007). Due to their subordinate position in the international currency hierarchy (Andrade and Prates, 2013; Kaltenbrunner, 2015), EMEs can borrow from abroad typically only in foreign currency, which creates currency mismatches on balance sheets. Currency mismatches expose economic units to exchange rate risk: a depreciation of the domestic currency raises the value of the unit's foreign currency debt, reducing its net worth. This, in turn, can lead to a drying up of financial sources or outright bankruptcy. Indeed, such balance sheet effects played a significant role in the East Asian crisis in the late 1990s (Kregel, 1998; Arestis and Glickman, 2002). A rich set of macroeconomic models with balance sheet effects has developed ever since (Krugman, 1999; Céspedes et al., 2004; Charpe et al., 2011, chap. 2; Kohler, 2017).[1]

Econometric studies confirm that depreciations are more likely to have negative effects on output and growth in countries with large external debt burdens (Galindo et al., 2003; Bebczuk et al., 2007; Blecker and Razmi, 2008; Kearns and Patel, 2016). The fact that the share of foreign currency-denominated liabilities on the balance sheets of non-financial corporations has increased sharply in many EMEs in the last decade (Feyen et al., 2015; Chui et al., 2018) suggests that balance sheet effects will remain an important feature of EM business cycles.

While both phenomena, procyclical exchange rates and balance sheet effects, have been studied in isolation, there is much less research on how they interact. The theoretical literature has not fully acknowledged that in flexible exchange rate regimes, foreign-currency debt not only has contractionary effects during depreciations, but also expansionary effects when the currency appreciates. The focus of models with balance sheet effects is typically on

[1] Ribeiro et al. (2017) consider non-financial channels through which currency depreciations may fail to be expansionary in a post-Keynesian framework.

currency crises, not business cycles. Business cycle models, in turn, have not yet explained how procyclical nominal exchange rates can be an endogenous driving force of business cycles in EMEs.

In this paper, we examine the role of procyclical exchange rates and corporate sector balance sheet effects in EM business cycle dynamics. We argue that a Minskyan framework is ideal for this purpose due to its focus on business investment as the most unstable component of aggregate demand, the role of financial factors in investment decisions, and idiosyncratic investor behaviour (Minsky, 2008 [1975], 2016 [1982]). In this framework, business cycles may arise endogenously through the interaction of the real side of the economy with the financial side. We supplement the Minskyan framework with a structuralist perspective, which highlights the role of the balance-of-payments for the business cycle in open developing economies (Ocampo et al., 2009; Ocampo, 2016).

The contribution of this article is threefold. First, it provides a simple Minskyan model of EM boom-bust cycles. Thereby, it formalises some mechanisms highlighted in the non-formal structuralist and Minskyan literature on boom-bust cycles in EMEs (Palma, 1998; Taylor, 1998; Arestis and Glickman, 2002; Cruz et al., 2006; Frenkel and Rapetti, 2009; Harvey, 2010; Kaltenbrunner and Painceira, 2015). While previous formal models in this tradition have highlighted interest rate dynamics and currency crises in fixed exchange rate regimes (Foley, 2003; Taylor, 2004, chap. 10; Gallardo et al., 2006), this paper examines the role of flexible exchange rates in business cycles. Second, it proposes an endogenous business cycle mechanism that not only captures the well-known countercyclicality of the trade balance but also explains the less researched procyclicality of nominal exchange rates in EMEs. The basic mechanism is as follows: Currency appreciations improve the balance sheets of foreign-currency indebted firms and induce an investment boom. At the beginning of the boom, debt-to-capital ratios decrease, which attracts capital inflows and leads to further exchange rate appreciation. The trade balance then gradually worsens over the boom. As capital inflows do not fully accommodate the growing trade deficit, excess demand for foreign currency emerges that puts pressure on the domestic exchange rate until it depreciates. Contractionary balance sheet effects then set in as domestic firms face a drop in their net worth and debt ratios rise. As the trade balance gradually improves during the downturn, the depreciation of the exchange rate comes to an end. Business net worth recovers and the cycle may repeat itself.

Third, the model combines the Minskyan notion of endogenous cycles with the structuralist argument that business cycles in EMEs are strongly influenced by external shocks. While the model generates shock-independent endogenous cycles, it shows that the volatility of these cycles is affected by exogenous capital inflows that are driven by risk appetite. We find that an exogenous increase in risk appetite increases the amplitude of cycles and that financial account regulation can reduce macroeconomic volatility. Interestingly, the higher the risk appetite, the higher the degree of regulation of the financial account that is needed to reduce volatility.

The article is structured as follows. The second section presents some stylized facts on the cyclical behaviour of exchange rates and real activity in EMEs and discusses the related literature. The third section develops a simple Minskyan model that explains the stylized facts highlighted in the second section. Its dynamic properties are first examined under the simplifying assumption of a constant external debt ratio, and then under the case of a flexible debt ratio that is driven by exogenous risk appetite. The section derives the conditions under which endogenous cycles emerge and considers financial account regulation as a stabilisation policy. The last section concludes.

2 Exchange rate dynamics and business cycles: stylized facts and related literature

After a series of EM crises in the 1990s and early 2000s that involved collapsing fixed exchange rate regimes (Frenkel and Rapetti, 2008), the majority of EMEs nowadays follows some form of exchange rate floating (Ghosh et al., 2015). Flexibility of nominal exchange rates raises the question how changes in the price of foreign currency are linked to business cycles. The applied policy literature suggests that exchange rate fluctuations may amplify macroeconomic volatility through balance sheet effects:

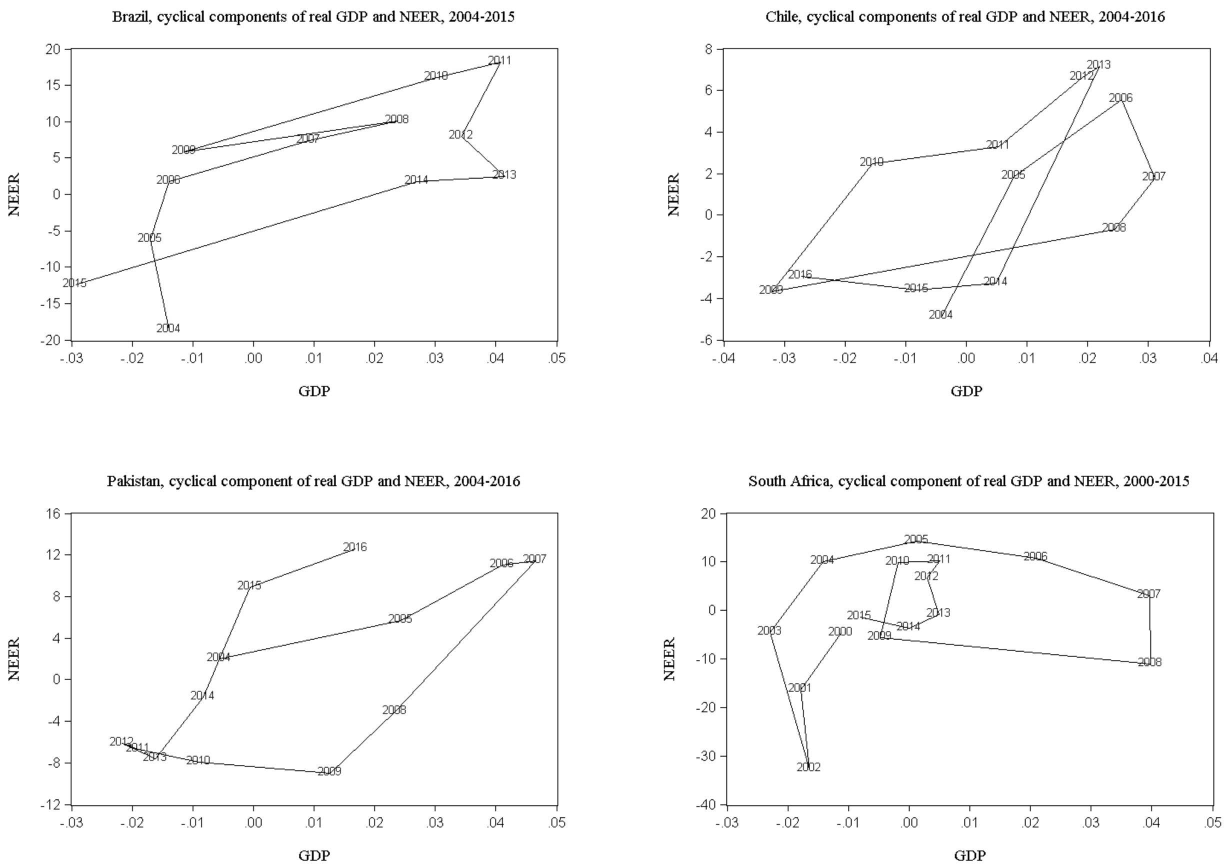
> 'In economies that have net external liabilities denominated in foreign currencies, exchange rate fluctuations [...] are pro-cyclical: real appreciation during the boom generates capital gains, whereas depreciation during crises generates capital losses. [...] The exchange rate fluctuations are themselves a result of some of the same forces that give rise to the economic fluctuations: capital inflows can fuel real exchange rate appreciation, at the same time that they lead to a private spending boom, while depreciation may have the opposite effects. In broader terms, in open developing economies the real exchange rate is an essential element in the dynamics of the business cycles' (Stiglitz et al., 2006, p.117).

This link between exchange rate dynamics and balance sheets has also been termed 'the risk-taking' or 'financial channel' of exchange rates (Shin 2016; Kearns and Patel, 2016). If the exchange rate of a global funding currency (typically the US-dollar) depreciates against the domestic currency, balance sheets of economic units with currency mismatches improve. This can lead to an investment boom. Conversely, an appreciation of the funding currency can depress domestic economic activity. The resulting procyclicality of exchange rates in EMEs has been well documented by recent empirical studies (Calderón and Fuentes, 2014; Cordella and Gupta, 2015). Notably, both nominal and real exchange rates are found to be procyclical (Cordella and Gupta, 2015), pointing to the importance of nominal exchange rates in driving the cyclical component of the real exchange rate.[2]

To gain insights into the nature of output and exchange rate cycles, we plot the cyclical component of the nominal effective exchange rate against the cyclical component of log real GDP for four selected EMEs. Figure 1 depicts the most recent one or two complete cycles that could be found in the data.

[2]Reinhart and Reinhart (2009, pp. 40-41) further show that real exchange rates appreciate during the boom phase of capital-inflow surges, while inflation tends to remain constant or fall. This is consistent with the well-known phenomenon of imperfect exchange-rate pass-through, which implies that nominal appreciations translate into real appreciations.

Figure 1: Phase diagrams for Brazil, Chile, Pakistan, and South Africa



Data sources: IFS (IMF), World Bank.
Notes: NEER: Nominal effective exchange rate (index, 2010 = 100). GDP: Natural logarithm of real gross domestic product. Cyclical components were extracted using the Hodrick-Prescott filter with a smoothing parameter of 100. An increase in the NEER indicates an appreciation of the domestic currency. The cyclical component of log real GDP is the percent deviation from trend.

All four countries exhibit clockwise cycles between output and the nominal effective exchange rate, with boom periods associated with currency appreciation and busts accompanied by depreciation. Converted into the more conventional definition of the exchange rate as domestic currency units per foreign unit, where an increase in the exchange rate indicates a depreciation, this implies counter-clockwise cycles in economic activity and the exchange rate. A counter-clockwise direction of cycles indicates that peaks and troughs in economic activity precede peaks and troughs in the exchange rate. This pattern points to predator-prey dynamics where economic activity takes on the role of the prey that is being squeezed by a rising exchange rate (i.e. depreciation), while the currency behaves like a predator that grows together with the prey. Stockhammer et al. (2019) argue that such an endogenous real-financial interaction mechanism lies at the heart of theories of financial-real cycles such as Hyman Minsky's Financial Instability Hypothesis (Minsky, 2008 [1975], 2016 [1982]; for a survey see Nikolaidi and Stockhammer, 2017).

A key aspect of the Minskyan approach is the claim that financial fragility increases during economic booms to the point where it spills over to the real economy and turns them into busts. The boom endogenously prepares its own bust, and it is finance that plays a decisive role in driving the cycle. In Minsky's view (2008 [1975]), corporate investment is the most unstable component of aggregate demand and is together with business debt at the heart of the financial-real cycle. This accords well with the empirical fact that investment is typically more than three times as volatile as output and about twice as volatile as consumption (Uribe and Schmitt-Grohé, 2017, chap. 1).

Open economy versions of the Minskyan approach often draw on the structuralist tradition (Taylor, 2004; Ocampo et al., 2009), which highlights the importance of the balance-of-payments for business cycle dynamics in EMEs. Due to their strong openness, capital flows and external debt play a much more important role in the interaction between the real and the financial side of the economy compared to advanced countries. Capital flows are largely detached from domestic fundamentals and follow their own dynamics, which are often driven by the risk appetite of investors. A rich non-formal literature analyses specific EM crisis episodes through such structuralist-Minskyan lenses, such as the 1994 Mexican peso crisis (Palma 1998; Cruz et al., 2006) and the 1998 Asian financial crisis (Kregel 1998; Palma 1998; Taylor, 1998; Arestis and Glickman, 2002), and identifies commonalities (Frenkel and Rapetti, 2009). While all these crises had different triggers, they featured fixed exchange rate regimes and were preceded by a boom in capital inflows and increasing external financial fragility in the private sector.

Foley (2003), Taylor (2004, chap. 10), and Gallardo et al. (2006) present formal models of some of the key mechanisms. In Foley (2003), cyclical dynamics are generated endogenously by the interaction of confidence-driven investment and a central bank that raises the interest rate during economic booms. The increase in interest rates will bring externally indebted firms into refinancing issues, thereby generating financial fragility. Taylor (2004, chap. 10) presents a model in which risk premia on interest rates are sensitive to the stock of foreign reserves. Economic booms come with current account deficits, which lead to a loss of reserves. This pushes up interest rates to the point where the economy contracts and the current account reverses. Gallardo et al. (2006) build a Minskyan currency crisis model to capture the Mexican peso crisis. The exchange rate depreciates when foreign reserves fall short of a critical level that foreign investors deem necessary to sustain an exchange rate peg.

Importantly, flexible nominal exchange rates do not play a role in these accounts, as exchange rates were fixed during the considered episodes.

The recent structuralist and Minskyan literature highlights the importance of exchange rate volatility for macroeconomic dynamics in EMEs (Herr, 2013; Kaltenbrunner and Painceira, 2015; Ocampo, 2016). Similar to the mechanism described in Stiglitz et al. (2006, p.117), Harvey (2010) develops a schematic depiction of a Minsky cycle in flexible exchange rate regimes, where exchange rates behave procyclical through their effects on foreign debt ratios, but does not develop a formal model. Botta (2017) presents a formal model in which the nominal exchange rate interacts with external debt in a cyclical manner. Capital inflows lead to nominal exchange rate appreciation. As the stock of external debt successively increases during the boom, foreign investors get anxious and curb the supply of foreign finance. Thus, the nominal exchange rate plays a key role for cyclical dynamics, but the real side of the economy is not explicitly modelled.

Other approaches that propose an explanation of the procyclicality of exchange rates in EMEs are the non-Minskyan models in Schneider and Tornell (2004), Korinek (2011) and Müller-Plantenberg (2015). Schneider and Tornell (2004) offer a microeconomic model in which bailout guarantees encourage currency mismatches in the non-tradables sector. Economic booms are self-reinforcing as they lead to a real appreciation through inflation, which improves balance sheets and allows for more borrowing. As the stock of debt rises, self-fulfilling crises may occur in which expectations of a real depreciation led to a collapse in investment that validates expectations. The boom-bust cycle is thus endogenous, but the model fails to capture the role of nominal exchange rates and the balance-of-payments highlighted in the recent structuralist literature (Ocampo, 2016). In Korinek (2011), a negative productivity shock in the tradable sector leads to a real depreciation, which amplifies the shock as domestic agents reduce consumption to cope with higher repayments of their foreign currency-denominated debts. The real exchange rate thereby exerts a procyclical effect. As in Schneider and Tornell (2004), the real exchange rate is reduced to a relative price and is not affected by nominal exchange rate dynamics. Furthermore, macroeconomic fluctuations are solely driven by exogenous shocks. Lastly, Müller-Plantenberg (2015) differs from the previous studies by considering how the balance-of-payments drives nominal exchange rate dynamics. In the model, a boom-bust cycle emerges exogenously through an increase and subsequent decline in domestic asset returns. During the boom, capital inflows lead to nominal appreciation and equity price inflation, and the current account worsens. These dynamics are reversed when asset returns fall back to their initial levels. The model thus captures the procyclicality of exchange rates. However, unlike the Minskyan approach taken in the present paper, Korinek (2011) and Müller-Plantenberg (2015) do not offer any economic mechanisms that would explain how cycles emerge endogenously.

3 A Minskyan open economy model with flexible exchange rates

Contributing to the Minsykan and structuralist approaches to endogenous cycles in EMEs, we build a Minskyan open economy model that accounts for a procyclical role of *flexible* nominal exchange rates through balance sheet effects. The model is analysed in two steps. First, under the simplifying assumption of a constant external debt ratio, damped oscillations in the exchange rate and capital accumulation arise. Second, under a flexible debt ratio that is determined by an exogenous debt target, the dynamics turn into shock-independent endogenous cycles.

3.1 The core model

The goods market of the model is kept simple and resembles other Minsky models (e.g. Foley, 2003; Charles, 2008). The economy consists of one sector that produces a homogenous good using capital and labour, which can be used for consumption and investment. For simplicity, there is no depreciation of the capital stock and no overhead labour. The technical coefficients of labour and capital are constant, so there is no substitution and no technical progress. Production is demand-driven, so there is Keynesian quantity adjustment to changes in demand. For the sake of simplicity, there is no fiscal policy and no inflation. We set the domestic and foreign price level to unity.[3] The economy is small and open, i.e. all foreign variables are exogenous. The exchange rate is flexible but adjusts only sluggishly due to restrictions on financial account transactions.

Equilibrium conditions[4]

Aggregate demand (Y^D) in the open economy is composed of consumption (C), investment (I), and net exports ($X - sM$), where s is the spot exchange rate defined as units of domestic currency per unit of foreign currency so that an increase in s corresponds to a currency

[3]As a result, the real exchange rate is fully determined by the nominal exchange rate. Although a simplification, this assumption reflects the empirical evidence discussed in section 2 suggesting that cyclical real exchange rate dynamics are strongly driven by the nominal exchange rate. For a post-Keynesian model that considers the effects of nominal exchange rate shocks on domestic inflation, see Bastian and Setterfield (2017).

[4]The superscript f denotes foreign variables. The rate of change of a variable x is denoted by $\frac{dx}{dt} = \dot{x}$. A list of symbols can be found in Appendix A1.

depreciation.[5] Equilibrium in the goods market requires that national income (Y) equals aggregate demand:

$$Y = Y^D \equiv C + I + X - sM. \tag{1}$$

The balance-of-payments (BoP), on the other hand, is given by

$$(X - sM - si^f D^f) + (s\dot{D}^f) = s\dot{Z}, \tag{2}$$

where the first term in brackets represents the current account, i.e. the trade surplus minus interest payments abroad, and the second term is the financial account, i.e. net capital inflows. The latter take the form of sales of foreign-currency denominated bonds (D^f) in this model (i^f is the interest rate those bonds). A surplus in the current (financial) account that is not fully matched by a deficit in the financial (current) account leads to an accumulation of foreign reserves ($s\dot{Z}$). Equilibrium in the balance of payments is given when reserve changes are zero:

$$(X - sM - si^f D^f) + (s\dot{D}^f) = s\dot{Z} = 0. \tag{3}$$

Budget constraints

The economy consists of workers who earn wages (W) and firms who make profits (R), so that $Y \equiv W + R$. Workers consume their entire income which exclusively consists of wages. Their budget constraint is thus always satisfied. Firms can finance their investment expenditures (I) through retained profits and via selling foreign currency-denominated bonds to foreigners (D^f) - a practice that has taken place on a large scale in emerging market economies in the last decade (Feyen et al. 2015; Chui et al., 2018). We furthermore make the simplifying assumption that if the spending and financing decisions of the firm sector do not add up, firms will be provided by an interest-free foreign-currency loan by the monetary authority (L).[6] The aggregate firm budget constraint reads:

$$0 = R - I - si^f D^f + s\dot{D}^f + s\dot{L}. \tag{4}$$

If only workers consume, and they consume their entire wage income, aggregate con-

[5]Notice the difference to the empirically widely used nominal effective exchange rate in Figure 1, where an increase indicates an appreciation.

[6]The monetary authority might be thought of as a consolidated banking sector in this context. Note that foreign-currency bonds (D^f), domestic loans (L) and imports (M) are the only variables in this model that are denominated in foreign currency.

sumption is given by:

$$C = W = Y - R. \tag{5}$$

From (5) and (1), we get:

$$I - R = sM - X. \tag{6}$$

Thus, whenever investment exceeds profits, the economy runs a trade deficit. Substitution of (6) into (2) yields:

$$R - I - si^f D^f + s\dot{D}^f = s\dot{Z}. \tag{7}$$

Eq. (7) in conjunction with the budget constraint of the firm sector (1) implies that $s\dot{Z} = -s\dot{L}$. Thus, the BoP and the budget constraint of the firm sector coincide: whenever firms' financing needs are not fully covered by new foreign-currency bonds, the monetary authority must draw on its foreign reserves to fill the financing gap. In BoP equilibrium ($s\dot{Z} = 0$), there is no need for such external funding. As we focus on the short- to medium-run, we will assume that the domestic economy commands sufficient foreign reserves for this purpose, and that changes in the stock of reserves have negligible effects on the economy.

Aggregate demand and goods market equilibrium

We now scale all variables by the capital stock (K) and use lower case letters henceforth. From the assumption that workers consume all their income, we have the following consumption function:

$$c \equiv \frac{C}{K} = u - r = u - \pi u, \qquad u \equiv \frac{Y}{K}; r \equiv \frac{R}{K}; \pi \equiv \frac{R}{Y}. \tag{8}$$

We use a simplified investment function with the rate of profit and the external debt ratio (measured in domestic currency) as the only two arguments.

$$g^d = g_0 + g_r r - g_s s\lambda, \qquad \lambda \equiv \frac{D^f}{K}; g_r \in (0,1); g_s > 0. \tag{9}$$

The use of the profit rate in the investment function is common in many post-Keynesian models, as profits constitute an indicator for aggregate demand and a source of internal funds.[7] However, inclusion of an external debt in foreign currency ratio is not. The eco-

[7]The profit rate can be decomposed as follows: $r \equiv \frac{R}{Y}\frac{Y}{K} \equiv u\pi$. Since we assume the profit share (π) to be constant, the profit rate is monotonically related to the output-capital ratio (u), which is a measure

nomic rationale for this is the presence of foreign currency-denominated debt on the balance sheets of emerging market firms. Changes in the value of the domestic currency thus exert a strong impact on the net worth of firms, which in turn can affect investment demand. From a Minskyan perspective, the link between net worth and investment is due to 'borrower's risk', which is the subjective risk of illiquidity and bankruptcy of the entrepreneur due to the possibility of lower than expected cash flows despite fixed payment obligations (Minsky, 2008 [1975], pp.104-110). Note that price competitiveness effects of the exchange rate affect desired investment expenditures only via their impact on the equilibrium profit rate. This specification reflects the empirical finding in Kearns and Patel (2016) that in emerging markets, the depreciation of debt-weighted exchange rates depresses investment, which indicates that balance sheet effects outweigh price competitiveness effects.

Lastly, the net exports ratio is determined by the foreign output-capital ratio (u^f), the domestic output-capital ratio, and the exchange rate.[8]

$$b \equiv \frac{X - sM}{K} = b_{u^f} u^f - b_u u + b_s s, \qquad b_u \in (0,1); b_{u^f} > 0. \tag{10}$$

The foreign output-capital ratio captures export demand, whereas the domestic ratio measures import demand. Whether the effect of an increase in the exchange rate on the trade balance is positive depends on whether the Marshall-Lerner condition (MLC) holds ($b_s > 0$). As the MLC is often empirically not satisfied due to low exchange rate elasticities (Bahmani et al., 2013), we will assume that b_s has a low absolute magnitude and can assume positive or negative values.

Goods market equilibrium is established by quantity adjustment, rendering the output-capital ratio endogenous. Making use of (1) (normalised by the capital stock), (8), and (10), we obtain:

$$u^* = \frac{g + b_{u^f} u^f + b_s s}{\pi + b_u} = \eta(g + b_{u^f} u^f + b_s s), \qquad \text{where} \quad \eta \equiv \frac{1}{\pi + b_u} > 0. \tag{11}$$

Investment dynamics

As in Charles (2008), we introduce finite adjustment of the actual rate of capital accumulation

for capacity utilisation and thus aggregate demand. We abstract from interest payments in the investment function to keep the model simple and to focus on balance sheet effects. For a Minskyan model that examines the role of interest payments in EME cycles, see Foley (2003).

[8] Inclusion of the foreign output-capital ratio requires that the domestic and foreign capital stock grow at the same rate. Also note that the specification in (10) can be considered as a linear approximation to a more generic net exports function $NX(u^f, u, s) = X(u^f, s) - sM(u, s)$, which is inherently nonlinear due to the denomination of imports in foreign currency.

to the desired one, e.g. due to lags in the order and construction of new equipment:

$$\dot{g} = \gamma(g^d - g), \qquad \gamma > 0. \tag{12}$$

From (9), (11), and (12) we obtain the law of motion of the investment rate:

$$\dot{g} = \gamma[g(\theta g_r - 1) + s(\theta b_s g_r - g_s\lambda) + g_0 + \theta b_{u^f} u^f g_r], \quad \text{where} \quad \theta \equiv \pi\eta \equiv \frac{\pi}{\pi + b_u} \in (0,1). \tag{13}$$

Balance-of-payments equilibrium and exchange rate dynamics
Eq. (11) in conjunction with the normalised BoP equilibrium condition yields:

$$\underbrace{(\theta b_{u^f} u^f + s\theta b_s + \frac{s\dot{D}^f}{K})}_{\text{FX supply}} - \underbrace{(si^f\lambda + g\eta b_u)}_{\text{FX demand}} = 0. \tag{14}$$

The BoP can be interpreted as a market-clearing condition for the FX market, since it contains all sources of supply and demand for foreign currency (Federici and Gandolfo, 2012). The positive elements constitute sources of supply (foreign import demand for domestic goods and net capital inflows), whereas the negative elements represent sources of demand for foreign currency (interest payments on foreign debt and domestic import demand). Whenever equality (14) is violated, there is excess demand or supply in the FX market.

Recall that we assume that BoP-disequilibria lead to changes in reserves (see Eq. 7), which ensures that the BoP identity holds and that the firm budget constraint is satisfied. From the perspective of the BoP as an FX market-clearing condition, changes in reserves can be regarded as interventions in the FX market. In a completely unregulated FX market without FX intervention, the exchange rate would instantaneously adjust to clear the market. Interventions take away some of these adjustment pressure on the exchange rate. In line with the current practice of managed floating in many EMEs (Ghosh et al., 2015), we assume that BoP-disequilibria lead to gradual changes in the exchange rate, which only sluggishly responds to pressures in the FX market:

$$\dot{s} = \mu[g\eta b_u + s(i^f\lambda - \theta b_s - \frac{s\dot{D}^f}{K}) - \theta b_{u^f} u^f], \qquad \mu > 0. \tag{15}$$

The speed of adjustment of the exchange rate, μ, can be interpreted as the degree of deregulation of the financial account (Bhaduri, 2003).[9] Exchange rate dynamics are thus determined

[9] Montecino (2018) provides econometric evidence that capital controls can slow down the adjustment speed of the exchange rate towards its long-run level.

by the rate of capital accumulation (through its effect on the trade balance), as well as by the exchange rate itself (through its influence on net exports and the value of interest payments and capital flows). From the time derivative of the external debt-to-capital ratio $\frac{d\lambda}{dt} \equiv \dot{\lambda} \equiv \frac{\dot{D}^f}{K} - g\lambda$, we get $\frac{\dot{D}^f}{K} \equiv \dot{\lambda} + g\lambda$. Substituting this expression into (15) yields our law of motion for the exchange rate:

$$\dot{s} = \mu[g(\eta b_u - s\lambda) + s(i^f\lambda - \dot{\lambda} - \theta b_s) - \theta b_{u^f} u^f]. \tag{16}$$

3.2 Cyclical dynamics under a constant external debt ratio

To get a thorough understanding of the interaction mechanism between investment and the exchange rate, we first focus on a special case and suppose that the external debt ratio remains constant over time. A constant external debt ratio requires external debt to change proportionally to the rate of investment:

$$\frac{\dot{D}^f}{K} = \lambda g. \tag{17}$$

Under this assumption, we have $\dot{\lambda} = 0$, so the law of motion of the exchange rate (18) reduces to:

$$\dot{s} = \mu[g(\eta b_u - s\lambda) + s(i^f\lambda - \theta b_s) - \theta b_{u^f} u^f]. \tag{18}$$

The laws of motion of capital accumulation (13) and the exchange rate (16) then constitutes a nonlinear 2D dynamic system. The Jacobian matrix of the system is:

$$\mathbf{J}(g, s) = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s\lambda) \\ \mu(\eta b_u - s\lambda) & \mu[(i^f - g)\lambda - \theta b_s] \end{bmatrix}. \tag{19}$$

With respect to the fixed points of the system, we state the following proposition:

Proposition 1 *The dynamic system given by (13) and (16) has at most two fixed points.*

Proof. See Appendix A3.

In the following, we will evaluate the stability and dynamics of the system in the neighbourhood of a fixed point (g^*, s^*), where both g^* and s^* are positive. We are interested in the conditions under which the system gives rise to oscillations.

Oscillations

Element J_{11} constitutes a version of the Keynesian stability condition. We follow the approach of Kaleckian Minsky models[10] and assume that the stability condition is satisfied, so that $J_{11} < 0$. The sign of J_{12} is ambiguous. The first term captures the effect of a depreciation on the trade balance, which is mediated by the exchange rate elasticity of net exports (b_s) and the sensitivity of investment with respect to profitability (g_r). The second term captures the contractionary balance sheet effect of a currency depreciation on investment (g_s). Based on our previous discussion of the empirical evidence (Bahmani et al., 2013; Kearns and Patel, 2016), we suppose that the second effect outweighs the first: balance sheet effects are typically strong in EMEs, while exchange rate elasticities are low ($J_{12} < 0$).

The sign of the element J_{21} is positive only if $\eta b_u > s^*\lambda$. The first term captures the fact that an increase in the rate of investment leads to a trade deficit, which in turn creates excess demand in the foreign exchange market. This effect is attenuated by the growth in the capital stock which raises the supply of foreign credit. We assume $J_{21} > 0$, which requires a relatively large propensity to import. This is realistic for developing countries where a high share of imported manufactured goods leads to high income elasticities of imports. If the steady state external debt ratio expressed in domestic currency becomes too large, J_{21} may become negative. Lastly, J_{22} determines the stability of the BoP. A destabilising element is interest payments on foreign currency debt. Capital accumulation and the exchange rate sensitivity of net exports are stabilising factors. Given that the Marshall-Lerner condition has been frequently rejected in empirical work (Bahmani et al., 2013), and that developing countries often have to cope with high debt service burdens (Frenkel, 2008), we will focus on the case where $J_{22} > 0$.[11]

We then have the following sign structure of the Jacobian matrix:

$$sgn[\mathbf{J}(g,s)] = \begin{bmatrix} - & - \\ + & + \end{bmatrix}.$$

Oscillations arise when the Jacobian matrix of the system exhibits complex eigenvalues. A sufficient condition for complex eigenvalues in a 2D system is $(J_{11} - J_{22})^2 + 4J_{12}J_{21} < 0$ (see Appendix A2). This requires

$$\underbrace{\{\gamma(\theta g_r - 1) - \mu[(i^f - g^*)\lambda - \theta b_s]\}^2}_{>0} + 4\mu\gamma \underbrace{(\eta b_u - s^*\lambda)}_{>0} \underbrace{(\theta b_s g_r - g_s\lambda)}_{<0} < 0,$$

[10]See Nikolaidi and Stockhammer (2017) for a survey of Minsky models that builds on the distinction between Kaleckian Minsky models with stable and Kaldorian Minsky models with unstable goods markets.

[11]The qualitative results do not hinge on this assumption.

which is satisfied when the import propensity is large ($b_u \gg 0$), which we consider to be likely, and when investment is very sensitive towards the external debt in foreign currency ($g_s \gg 0$), which is typical for developing countries where balance sheet effects have been shown to be strong.

Stability

The following proposition states the conditions under which the system is stable:

Proposition 2 *The dynamic system given by (13) and (16) is asymptotically stable if the following conditions are satisfied:*
(i) $i^f < g^* + \frac{\theta b_s}{\lambda} + \frac{\gamma(1-\theta g_r)}{\mu\lambda}$,
(ii) $i^f < g^* + \frac{\theta b_s}{\lambda} + \frac{(\eta b_u - s^*\lambda)(\theta b_s g_r - g_s\lambda)}{(\theta g_r - 1)\lambda}$.
Depending on whether $\frac{\gamma(1-\theta g_r)}{\mu\lambda} \gtrless \frac{(\eta b_u - s^*\lambda)(\theta b_s g_r - g_s\lambda)}{(\theta g_r - 1)\lambda}$, *either the first or the second inequality is binding.*

Proof. See Appendix A4.

For given structural parameters of the domestic economy, the system is thus stable if the exogenous foreign interest rate does not exceed a critical threshold. This is more likely to be the case, when the economy exhibits a high steady state growth rate or when the interaction mechanism between the exchange rate and the growth rate, i.e. $J_{12} < 0, J_{21} > 0$, is sufficiently strong. In the following, we will assume that the stability condition is satisfied.

Isoclines and dynamic trajectories

To examine the behaviour of the system graphically, consider the isoclines of the system:

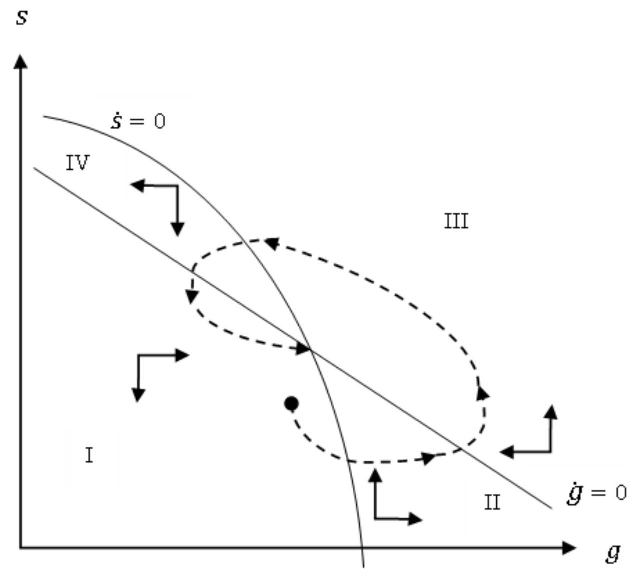
$$s^*_{|\dot{g}=0} = \frac{g(\theta g_r - 1) + g_0 + \theta b_{u^f} u^f g_r}{g_s\lambda - \theta b_s g_r}; \qquad \frac{\partial s^*_{|\dot{g}=0}}{\partial g} = -\frac{J_{11}}{J_{12}} < 0. \tag{20}$$

$$s^*_{|\dot{s}=0} = \frac{\theta b_{u^f} u^f - g\eta b_u}{(i^f - g)\lambda - \theta b_s}; \qquad \frac{\partial s^*_{|\dot{s}=0}}{\partial g} = -\frac{J_{21}}{J_{22}} < 0. \tag{21}$$

While the $\dot{g} = 0$ isocline is linear, the $\dot{s} = 0$ isocline is a rectangular hyperbola with two branches. Its vertical asymptote is $\bar{g} = \frac{i^f\lambda - \theta b_s}{\lambda}$. Given our assumption that $J_{22} = (i^f - g)\lambda - \theta b_s > 0$, we have $g < \bar{g}$. As we assume $|J_{12}J_{21}| > |J_{11}J_{22}|$ for stability, the $\bar{s} = 0$ isocline is steeper than the $\bar{g} = 0$ isocline.[12] The corresponding phase diagram is given in Figure 2.

[12]Since $|\frac{\partial s^*_{|\dot{s}=0}}{\partial g}| = |\frac{J_{21}}{J_{22}}|and|\frac{\partial s^*_{|\dot{g}=0}}{\partial g}| = |\frac{J_{11}}{J_{12}}|$, we have$|\frac{\partial s^*_{|\dot{s}=0}}{\partial g}| = |\frac{J_{21}}{J_{22}}| > |\frac{\partial s^*_{|\dot{g}=0}}{\partial g}| = |\frac{J_{11}}{J_{12}}|$.

Figure 2: Adjustment path after an appreciation shock



Note: The isoclines are based on $J_{11} < 0; J_{22} > 22; |J_{11}J_{22}| < |J_{12}J_{21}|$

Starting from an equilibrium position, consider a negative shock to the exchange rate (quadrant I). The currency appreciation feeds into investment demand via expansionary balance sheet effects, kicking off an investment boom. The resulting increase in the capital stock will trigger capital inflows, so that the debt ratio remains constant. The exchange rate keeps appreciating for a while, as the initial appreciation eases the burden of interest payments on foreign debt, which temporarily reduces demand for foreign currency. The investment boom boosts aggregate demand, which leads to a worsening of the trade balance. As the trade deficit grows faster than the incoming capital flows, excess demand for currency puts pressure on the domestic exchange rate. There is a short period in which investment keeps accelerating while the currency already depreciates (quadrant II), but this phase is quickly displaced by a sustained contractionary depreciation phase due to balance sheet effects (quadrant III). As a result, the downward trajectory of the trade balance eventually reverses until the pressure on the exchange rate is removed, and capital accumulation picks up again.

The economy experiences the empirically observed procyclicality of the exchange rate and countercyclicality of the trade balance, where a domestic boom coincides with currency appreciation and trade deficits, while busts are associated with depreciation and current account reversals (Reinhart and Reinhart, 2009; Cordella and Gupta, 2015). This trajectory matches the clockwise direction of the cycles observed in Figure 1 (recall that s is inversely defined compared to the NEER depicted in Figure 1).

3.3 Cyclical dynamics under flexible external debt ratios

3.3.1 Introducing a target external debt ratio

The previous result was established under the simplifying assumption of a constant external debt ratio. This assumption does not correspond well to the experience of large and unstable capital flows to EMEs highlighted in the structuralist literature (Ocampo, 2016; Ocampo et al., 2009). Structuralists argue that capital flows are often driven by external factors rather than domestic fundamentals. This notion is confirmed by a growing empirical literature showing that capital flows to EMEs are primarily determined by global factors such as liquidity, risk appetite, and funding cost in international financial centres (Nier et al., 2014; Rey, 2015). Recently, Feyen et al. (2015) showed that the surge in external bond issuance by EM firms in the decade after the crisis was predominantly driven by push factors such as expected volatility in the Standard & Poor's 500 (the VIX), as well as expansionary US monetary policy. They did not find evidence for a significant role of domestic factors and concluded that 'search-for-yield flows during loose global funding conditions do not strongly discriminate between EMDEs [emerging and developing economies] but are primarily driven by global factors' (Feyen et al. 2015, p.17).

To introduce externally driven capital flows into our model, we employ the notion of a target debt ratio – an approach that has recently been used in some closed economy Minsky models (Nikolaidi, 2014; Dafermos, 2018). While this literature explores the dynamics of endogenously changing domestic debt targets, we model the target external debt ratio as exogenous and independent of domestic fundamentals. The target ratio changes, for instance, in response to changes in risk appetite, liquidity or funding cost in global financial centres. Consequently, the actual debt ratio becomes a state variable that varies over time subject to capital inflows and domestic capital accumulation.

Consider a simple adjustment mechanism whereby capital flows increase whenever the actual debt ratio falls short of the target:

$$\frac{\dot{D}^f}{K} = \delta(\lambda^T - \lambda) \qquad \lambda^T, \delta > 0, \tag{22}$$

where λ^T is the target external debt-to-capital ratio and δ the adjustment speed of the actual ratio to the desired one. This translates into the following law of motion of the external debt-to-capital ratio:

$$\dot{\lambda} = \delta(\lambda^T - \lambda) - g\lambda. \tag{23}$$

Note that even if the target ratio was met ($\lambda^T = \lambda$), the actual debt ratio would still change, as long as the rate of investment and the actual debt ratio are different from zero. Under normal circumstances, the target is thus never meet and the actual debt ratio will be permanently changing over time.

Using (23) and (16), we obtain:

$$\dot{s} = \mu\{g\eta b_u + s[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] - \theta b_{u^f} u^f\}. \tag{24}$$

Eqs. (13), (23), and (24) constitute a 3D dynamic system that exhibits an intrinsically nonlinear structure due to valuation effects and normalisation of variables. Notice that this nonlinear structure emerges without having introduced nonlinearities in the behavioural functions. The Jacobian matrix of the system is given by:

$$\mathbf{J}(g,s,\lambda) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s\lambda) & -\gamma s g_s \\ \mu\eta b_u & \mu[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] & \mu s(i^f + \delta) \\ -\lambda & 0 & -(\delta + g) \end{bmatrix}. \tag{25}$$

With respect to the fixed points of the system, we state following:

Proposition 3 *The dynamic system given by (13), (23), and (24) has at most two fixed points.*

Proof. See Appendix A.5

We will focus on the fixed point where g^*, s^*, and λ^* are all positive.

Dynamic behaviour

We examine if this system can undergo a Hopf bifurcation giving rise to a stable limit cycle (Gandolfo, 1997, chap. 25). A limit cycle is a closed orbit of the variables of a dynamic system around a locally unstable fixed point. In the immediate neighbourhood of the fixed point, the system is unstable and gets pushed away from it. However, rather than exhibiting explosive behaviour, the system reaches a periodic cycle, as it is bounded due to its inherent nonlinearity. In contrast to the damped oscillations of the 2D model in the previous section, a limit cycle is periodic and displays persistent shock-independent oscillations.

Given the assumptions made in this paper, the Jacobian matrix evaluated at the positive fixed point has the following sign structure:

$$sgn[\mathbf{J}(g,s,\lambda)] = \begin{bmatrix} - & - & - \\ + & + & + \\ - & 0 & - \end{bmatrix}.$$

We now state the following:

Proposition 4 *If the following conditions simultaneously hold:*
(i) $|J_{11} + J_{33}| > |J_{22}|$
(ii) $|J_{11}J_{33} + J_{12}J_{21}| > |J_{22}(J_{11} + J_{33}) + J_{13}J_{31}|$
(iii) $|J_{33}(J_{11}J_{22} - J_{12}J_{21})| > |J_{31}(J_{12}J_{23} - J_{13}J_{22}|$,
then there exists a critical value of the adjustment speed of the exchange rate, μ_0*, at which the dynamic system in (13), (23), and (24) undergoes a Hopf bifurcation.*

Proof. See Appendix A6.

Economically these conditions require

- that the foreign exchange market is not excessively self-destabilising (i.e. the foreign interest rate must not be too high),
- that the cyclical interaction mechanism between the exchange rate and investment is strong, i.e. strong balance sheet effects coupled with a large propensity to import,
- a moderate steady state external debt ratio,
- that the adjustment speed of the exchange rate must exceed a critical value ($\mu_0 > \mu$), i.e. the financial account must be sufficiently open.

These conditions are similar to the conditions needed for damped oscillations in the 2D model. The existence of a self-stabilising debt ratio, however, allows the economy to cope with stronger instability in the foreign exchange market. Moreover, the adjustment speed of the exchange rate now assumes a key role for the dynamics of the model as it induces the Hopf bifurcation.

3.3.2 Endogenous cycles: a numerical illustration

In order to assess whether the limit cycle is stable, we resort to numerical simulation. Consider the following parameterisation (Table 1):

Table 1: Parameterisation of the 3D model

Parameter	i^f	λ^T	g_0	g_r	g_s	$b_{u^f}u^f$	b_u	b_s	γ	δ
Value	0.08	0.50	0.05	0.9	0.6	0.05	0.25	-0.002	1	1

The foreign interest rate is set at 8%, which roughly corresponds to the time average of the effective yield on the BofA Merrill Lynch Emerging Markets Corporate Plus Index[13] in the decade before the financial crisis (1998-2007). The target debt ratio is set at 50%. The remaining parameters are not directly observable. We choose a parameterisation that corresponds to the assumptions made in the text so far.[14] We suppose positive animal spirits (g_0), a moderate sensitivity of investment with respect to demand (g_r), and in line with the evidence in Kearns and Patel (2016) a strong sensitivity of investment with respect to the external debt ratio in foreign currency (g_s). Furthermore, we specify positive external demand for foreign goods ($b_{u^f} u^f$), a relatively large import propensity (b_u), and lastly a negative but low sensitivity of net exports with respect to the exchange rate. This is due to the empirical finding of Bahmani et al. (2013) that the MLC is not satisfied in the majority of empirical studies. The adjustment speeds of investment (γ) and external debt (δ) are set at unity for simplicity.

The system has two fixed points. For the chosen parameterisation, the first fixed point is economically meaningless, as all state variables are negative. We focus on the second equilibrium, which is given by:

$$(g^*, s^*, \lambda^*) = (0.055, 0.162, 0.474).$$

These steady state values imply an average capital stock growth rate of about 5.5%. This is well within the range of annual capital stock growth rates of the countries in Figure 1 (Brazil, Chile, Pakistan, South Africa) since the 1960s.[15] For this parameterisation, the conditions stated in Proposition 4 are satisfied. A positive critical value for which the Hopf bifurcation arises is given by $\mu_0 = 52.46$. Consider first an adjustment speed of the exchange below the critical value, $\mu = 50 < \mu_0$. The fixed point is locally stable and the system generates damped oscillations that converge towards the fixed point (Figures 3 and 4).

[13] See https://fred.stlouisfed.org/series/BAMLEMCBPIEY [downloaded 12/12/2017]. The BofA Merrill Lynch Emerging Markets Corporate Plus Index tracks the performance of US dollar and Euro denominated EM non-sovereign debt publicly issued within the major domestic and Eurobond markets.

[14] The purpose of the simulation is to investigate the stability of the limit cycle and to provide an illustration of the dynamics, not to match empirical data.

[15] Own calculations based on capital stock data at current purchasing power parities from Penn World Tables 9.

Figure 3: Damped oscillations of capital accumulation, the exchange rate, and the external debt-to-capital ratio

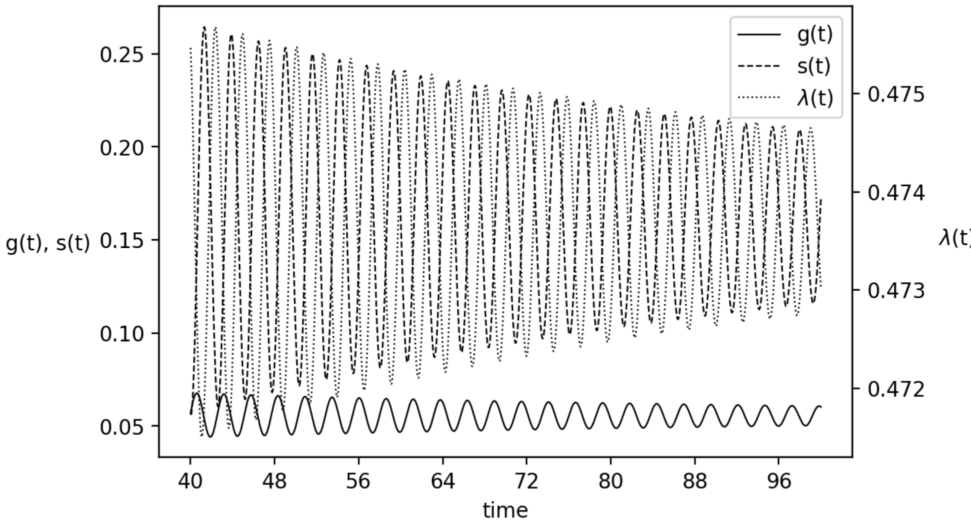
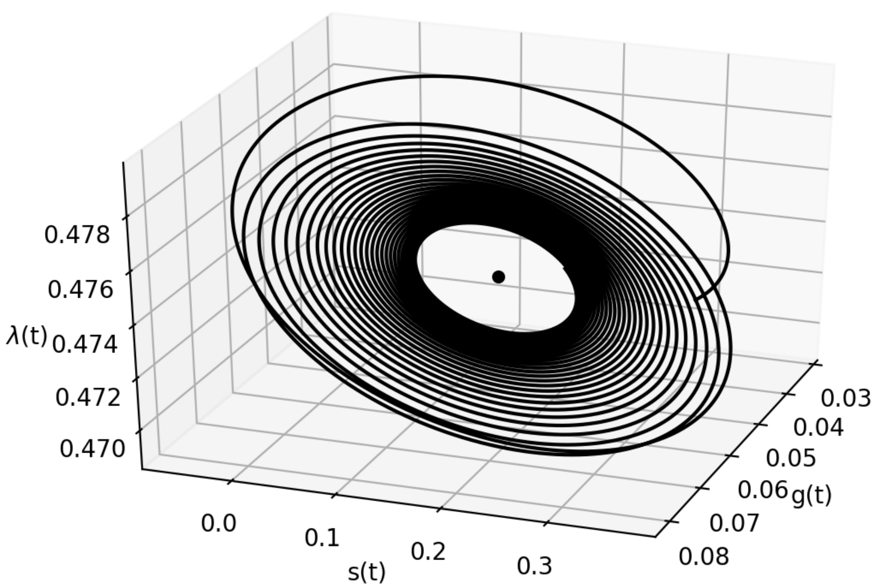


Figure 4: 3D phase plot of damped oscillations



Notes: The simulations are based on the parameters provided in Table 1, with $\mu = 50$, and the following initial values: $[g(0), s(0), \lambda(0)] = 0.055, 0.350, 0.474]$. The dot in Figure 4 marks the steady state towards which the system converges.

Now, consider a second parameterisation, where we leave all other parameters unchanged but set the adjustment speed of the exchange rate above its critical value: $\mu = 53 > \mu_0$. Now the fixed point loses its local stability and a limit cycle around the unstable equilibrium arises

(Figures 5 and 6). The simulations suggest that the Hopf bifurcation is supercritical and gives rise to a stable limit cycle in capital accumulation, the exchange rate, and the external debt-to-capital ratio. The middle panel in Figure 5 reveals that peaks and troughs in the investment rate precede those in the exchange rate, which implies a counter-clockwise cycle similar to the empirical phase plots depicted in Figure 1. It also corresponds to the results derived analytically in the simplified 2D model. However, in contrast to the 2D model, the external debt ratio now moves over time, as can be seen in the last panel.

Figure 5: Limit cycle dynamics of capital accumulation, the exchange rate, and the external debt-to-capital ratio

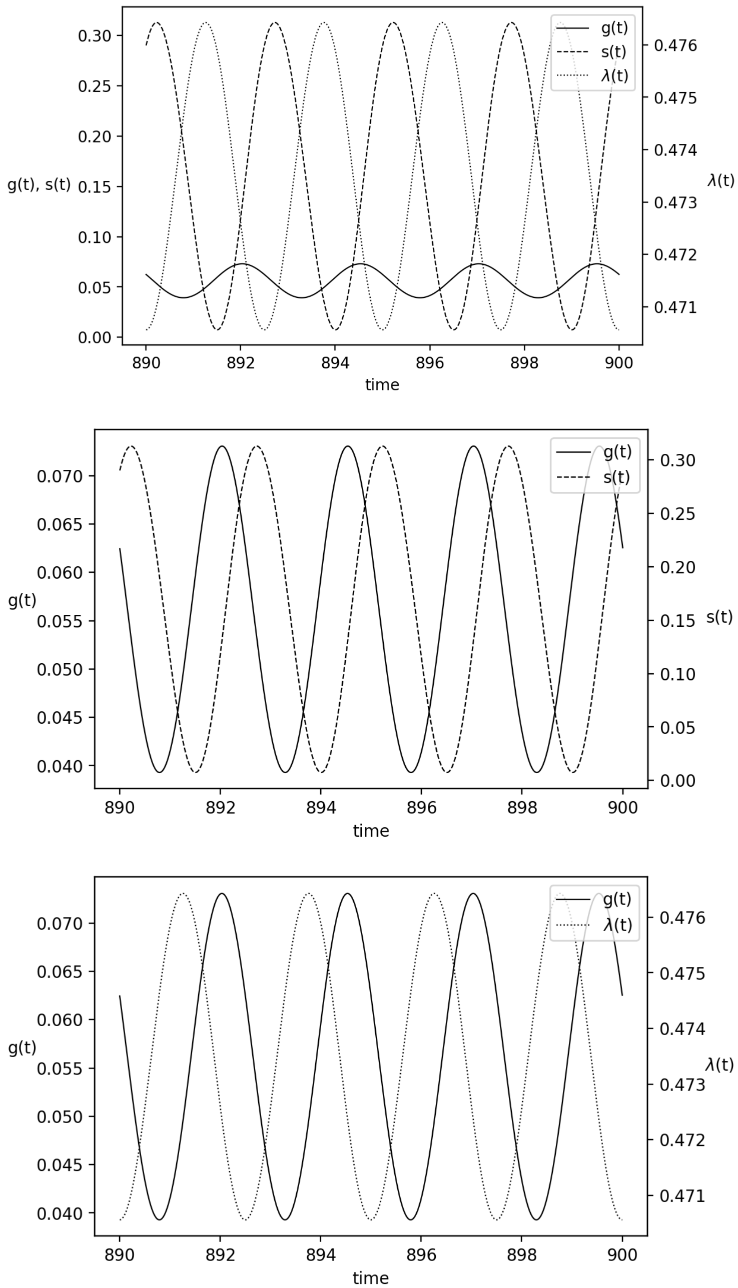
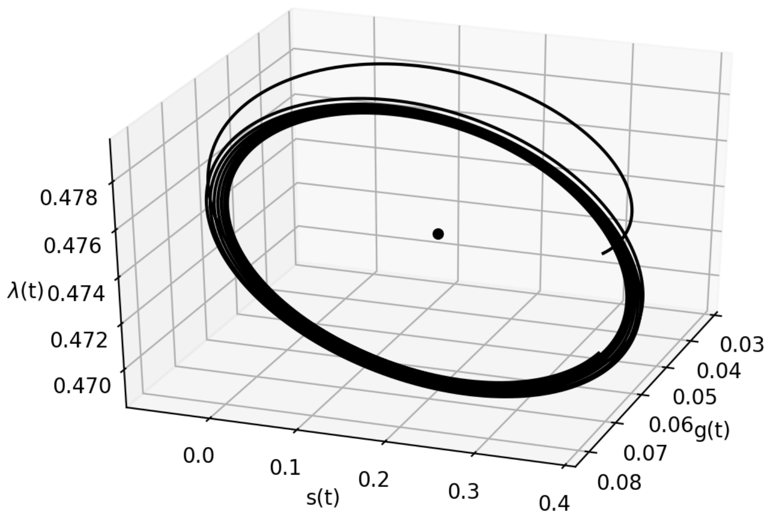
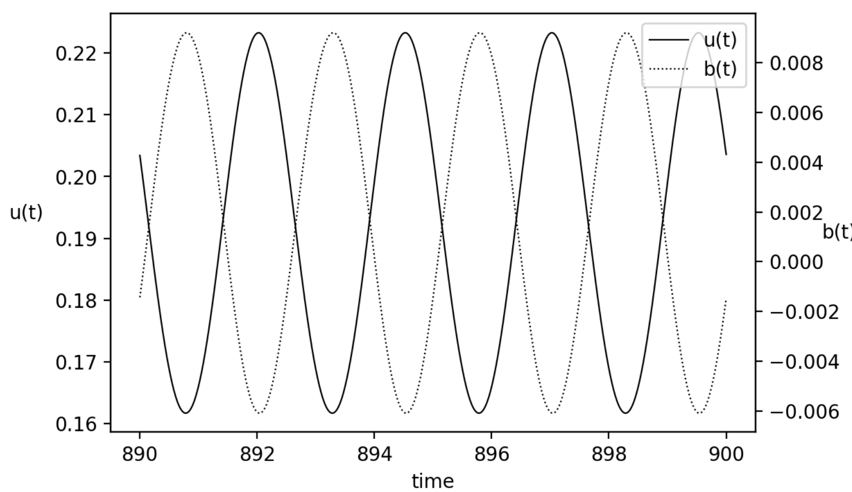


Figure 6: Phase plot of 3D limit cycle dynamics



Notes: The simulations are based on the parameters provided in Table 1, with $\mu = 53$, and the following initial values: $[g(0), s(0), \lambda(0)] = [0.055, 0.350, 0.474]$. The dot in Figure 6 marks the locally unstable steady state.

Figure 7: Dynamics of the output-capital ratio and trade balance



Notes: The simulation is based on the parameters provided in Table 1, with $\mu = 53$, and the following initial values: $[g(0), s(0), \lambda(0)] = [0.055, 0.350, 0.474]$. The equilibrium output-capital ratio is given by (11). The equilibrium trade balance is given by (10) in conjunction with (11).

Figure 7 further plots the time paths of the output-capital ratio and the trade balance, which reveals a strong negative correlation. As the economy moves from the peak of the boom to the trough, the trade balance reverses from a deficit to a surplus. Thus, the model captures the strong countercyclicality of the trade balance, as well as the phenomenon of current account reversals observed during EME cycles (Reinhart and Reinhart, 2009).

To understand what happens over the cycle, consider again an appreciation shock to the exchange rate that pushes the economy off the locally unstable equilibrium. Through the improvement of balance sheets, the appreciation induces a boost to capital accumulation. As a result, the actual external debt ratio starts declining. This process induces further capital inflows, which fuel currency appreciation. However, as the growing trade deficit over the boom is not fully accommodated by exogenously determined capital inflows, there is downward pressure on the domestic currency. At some point, the depreciation becomes contractionary due to its effects on corporate balance sheets. The initial phase of the contractionary depreciation is accompanied by rising external debt ratios as the capital stock shrinks, which slows down the inflow of capital. However, as the trade balance gradually improves during the bust, excess demand for foreign currency is removed, and the exchange rate starts to appreciate again. The cycle then repeats itself.

3.3.3 Shocks to risk appetite and policy intervention

What happens if there is a shock to the external debt target, for instance because of an increase in risk appetite? Table 2 presents the comparative dynamic effects of a change in the target debt ratio based on the parameterisation reported in Table 1. An increase in the debt target raises the steady state investment rate and leads to a more appreciated exchange rate. This corresponds to the empirical finding that surges in capital flows to EMEs are typically associated with higher domestic growth rates and exchange rate appreciation (Reinhart and Reinhart, 2009). An increase in the target also leads to a higher steady state debt ratio, as one would expect.

Table 2: Comparative dynamics of a change in the target debt ratio

	$\frac{\partial g^*}{\partial \lambda^T}$	$\frac{\partial s^*}{\partial \lambda^T}$	$\frac{\partial \lambda^*}{\partial \lambda^T}$	$\frac{\partial \mu_0}{\partial \lambda^T}$
Sign	$+$	$-$	$+$	$-$

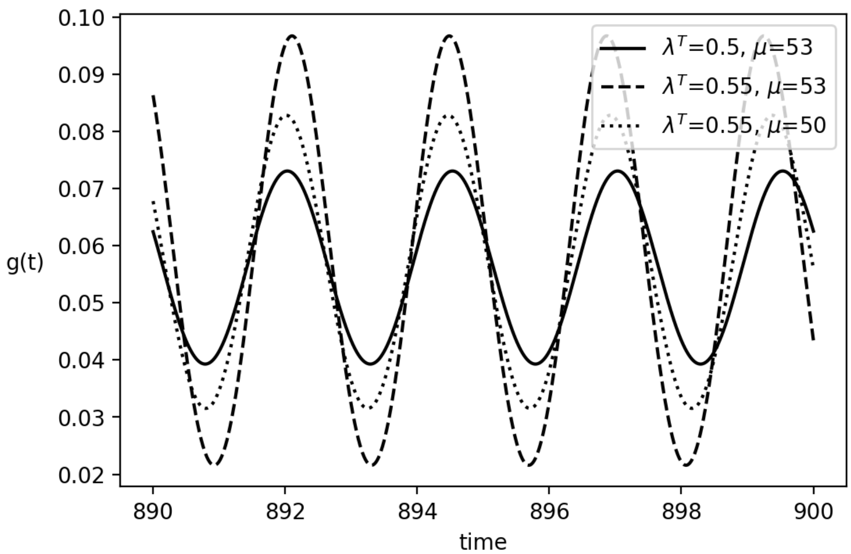
Notes: The partial effects are based on the parameterisation in Table 1.

Interestingly, a rise in the target external debt ratio lowers the critical adjustment speed of the exchange rate for which the limit cycle occurs. This implies that if there is a positive shock to risk appetite, stricter regulations on capital flows are needed to prevent limit cycle dynamics.

On the other hand, for a given target leverage ratio, a sufficiently strong regulation of the financial account may prevent the occurrence of volatile limit cycle dynamics altogether.

Figure 8 explores the effects of an increase in the target ratio on business cycle dynamics. An increase in the debt target ratio by 10% (dashed line) significantly increases the amplitude of the cycle compared to the baseline (solid line). Thus, although the cycle is driven by an endogenous mechanism, exogenous capital-flow surges increase the volatility of the domestic macroeconomy. The model thereby combines the notion of endogenous cycles with the phenomenon of exogenous capital flow shocks that make the domestic economy more volatile.

Figure 8: Amplitude of limit cycle for different target debt ratios and policy intervention



Notes: The simulations are based on the parameters provided in Table 1 and on the parameters stated in the legend. The following initial values were used: $[g(0), s(0), \lambda(0)] = [0.055, 0.350, 0.474]$. The solid line is the baseline parameterisation. The dashed line represents an increase in the target debt ratio compared to the baseline. The dotted line depicts a reduction in the adjustment speed of the exchange rate for the scenario with the increased debt target.

Lastly, Figure 8 also shows that policy intervention can reduce the volatility of the cycle. The dotted line displays the case where the target debt ratio is still increased compared to the baseline, but where the adjustment speed of the exchange rate is reduced. Empirical research has shown that such a reduction in the adjustment speed of the exchange rate can be achieved by restrictions on financial account transactions (Montecino, 2018). Our model shows that such capital account regulation can have stabilising effects on the macroeconomy – a view that has been taken by structuralists (e.g. Ocampo and Palma, 2008) and has recently also gained cautious support from mainstream institutions such as the IMF (Ostry et al., 2011).

4 Conclusion

This paper has been concerned with boom-bust cycles in emerging market economies. It has provided a Minskyan approach to business cycles in EMEs with structuralist features, incorporating the stylized facts of procyclical exchange rates, a countercyclical trade balance, and significant levels of foreign currency-denominated corporate debt. The model shows how nominal exchange rates that are driven by disequilibria in the balance-of-payments can interact with foreign-currency debt on balance sheets in a procyclical way, thereby generating business cycle dynamics. This is more likely to occur when balance sheet effects from exchange rate dynamics on investment spending are strong and dominate competitiveness effects, and when the import propensity is large. Although business cycles are endogenous, they are affected by exogenous capital flows which increase the amplitude of domestic fluctuations. The model shows that by imposing stricter regulations on the financial account, the volatility of business cycles can be reduced. The higher the risk appetite that drives capital flows, the more financial account regulation is needed to reduce volatility.

The model captures the key role of exchange rate and balance-of-payments dynamics in emerging market business cycles that has been highlighted in the structuralist and Minskyan literature (Ocampo et al., 2009; Frenkel and Rapetti, 2009; Harvey, 2010; Ocampo, 2016). Unlike previous formal models (Foley, 2003; Taylor, 2004, chap. 10; Gallardo et al., 2006) the approach presented in this paper shifts the focus away from interest rate issues and currency crises towards exchange rate dynamics and balance sheet effects. We consider this an important step forward, given that the majority of emerging markets economies presently follow some form of exchange rate floating. Despite its macroeconomic focus, the model also highlights structural sources of volatility. A high share of foreign currency-denominated debt and a large import propensity are identified as key factors for business cycle dynamics in EMEs. The former renders private spending sensitive to the procyclical exchange rate, while the latter generates strong cyclical behaviour of the trade balance, which feeds back into exchange rate dynamics. Finally, our approach combines the Minskyan notion of endogenous cycles with the structuralist emphasis on external shocks. Capital-flow surges amplify fluctuations, but the business cycle ultimately emanates endogenously from the interaction of foreign currency debt on domestic balance sheets with a procyclical exchange rate.

This combined approach allows us to identify three areas for policy interventions: first, on the external front, capital controls may curb macroeconomic fluctuations that stem from capital inflow shocks (Ocampo and Palma, 2008; Ostry et al., 2011). Second, a more active exchange rate policy can smooth exchange rate fluctuations and reduce their procyclical effects. Such managed floating has gained growing theoretical support among structuralist and post-Keynesian authors (Frenkel, 2006; Ocampo, 2016; de Paula et al., 2017). Third, our approach also suggests structural policies that reduce foreign-currency indebtedness and encourage domestic lending, for instance strengthening the domestic banking system through prudential regulation and public banks (Herr and Priewe, 2005).

Although we place a strong emphasis on the interaction of exchange rate dynamics and balance sheet effects, we do no claim that this is the only channel that can drive business cycles in EMEs. Firstly, interest rates and their effect on capital flows can play an important role too, especially in inflation-targeting regimes. Monetary policy that raises interest rates during a boom may attract more capital inflows which can further fuel the boom. Similarly, market interest rates may be endogenous to economic activity due to risk premia on external debt (Kohler, 2017). Secondly, exchange rate dynamics are presently modelled in a simplified way. The presence of heterogenous agents in the foreign exchange market may have important ramifications for macroeconomic stability (Proaño, 2011). Future research could introduce these aspects into the present framework.

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Appendix

A1 List of symbols

Roman Letters

$b \equiv \frac{X-sM}{K}$	Net exports rate
b_u	Sensitivity of net exports w.r.t. output-capital ratio
b_{u^f}	Sensitivity of net exports w.r.t. foreign output-capital ratio
b_s	Sensitivity of net exports w.r.t. exchange rate
C	Consumption
$c \equiv \frac{C}{K}$	Consumption rate
D^f	Foreign currency-denominated corporate bonds
$g \equiv \frac{I}{K}$	Investment rate
g^d	Desired investment rate
g_0	Animal spirits
g_r	Sensitivity of capital accumulation w.r.t. profit rate
I	Investment
i^f	Interest rate on foreign currency-denominated corporate bonds
L	Domestic loans (denominated in foreign currency)
M	Imports (denominated in foreign currency)
R	Profits
$r \equiv \frac{R}{K}$	Profit rate
s	Spot exchange rate (units of domestic currency per unit of foreign currency)
t	Time
u	Output-capital ratio
u^f	Foreign output-capital ratio
W	Wage bill
Y	National income
Y^D	Aggregate demand
Z	Foreign reserves

Greek Letters

γ	Adjustment speed of investment rate
δ	Adjustment speed of external debt-to-capital ratio
$\eta \equiv \frac{1}{\pi+b_u}$	Composite parameter
$\theta \equiv \frac{\pi}{\pi+b_u}$	Composite parameter
$\lambda \equiv \frac{D^f}{K}$	External debt-to-capital ratio
λ^T	Target external debt-to-capital ratio
$\pi \equiv \frac{R}{Y}$	Profit share
μ	Adjustment speed of exchange rate (Hopf bifurcation parameter)
μ_0	Critical value of Hopf bifurcation parameter

A2 Mathematical condition for oscillations in 2D systems of differential equations

Consider the characteristic equation of a 2D Jacobian matrix:

$$\lambda^2 - \lambda Tr(J) + Det(J) = 0.$$

The characteristic roots of this equation are given by:

$$\lambda_{\pm} = \frac{Tr(J) \pm \sqrt{Tr(J)^2 - 4Det(J)}}{2}.$$

Oscillatory behaviour occurs when the characteristic roots are a pair of complex conjugates, which requires the discriminant $\Delta = Tr(J)^2 - 4Det(J)$ to become negative. This condition can be simplified as follows:

$$\begin{aligned}\Delta &= Tr(J)^2 - 4Det(J)\\ &= (J_{11} + J_{22})^2 - 4(J_{11}J_{22} - J_{12}J_{21}) < 0\\ &= (J_{11} - J_{22})^2 + 4J_{12}J_{21} < 0.\end{aligned}$$

A3 Proof of Proposition 1: Fixed points of the 2D system

First, we determine the fixed points of s by setting equations (13) and (18) equal to zero and solving for g:

$$g^*_{|\dot{g}=0} = \frac{s(\theta b_s g_r - g_s \lambda) + g_0 + \theta b_{u^f} u^f g_r}{1 - \theta g_r},$$

$$g^*_{|\dot{s}=0} = \frac{s(i^f\lambda - \theta b_s) - \theta b_{u^f} u^f}{s\lambda - \eta b_u}.$$

Setting these two equations equal and solving for s yields:

$$\begin{aligned} 0 &= s^2\lambda(\theta b_s g_r - g_s\lambda) + s[(g_0 + \theta b_{u^f} u^f g_r)\lambda - (\theta b_s g_r - g_s\lambda)\eta b_u + (i^f\lambda - \theta b_s)(\theta g_r - 1)] \\ &+ (1 - \theta g_r)\theta b_{u^f} u^f - (g_0 + \theta b_{u^f} u^f g_r)\eta b_u. \end{aligned}$$

This is an inverted U-shaped parabola. Let us define:

$$\begin{aligned} \alpha_s &= (\theta b_s g_r - g_s\lambda) < 0, \\ \beta_s &= \underbrace{(g_0 + \theta b_{u^f} u^f g_r)\lambda}_{>0} - \underbrace{(\theta b_s g_r - g_s\lambda)\eta b_u}_{<0} + \underbrace{(i^f\lambda - \theta b_s)(\theta g_r - 1)}_{<0}, \\ \varsigma_s &= \underbrace{(1 - \theta g_r)\theta b_{u^f} u^f}_{>0} - \underbrace{(g_0 + \theta b_{u^f} u^f g_r)\eta b_u}_{>0}. \end{aligned}$$

Its roots, which are the two fixed points of s, are given by:

$$s^*_{1,2} = \frac{-\beta_s \pm \sqrt{\beta_s^2 - 4\alpha_s\varsigma_s}}{2\alpha_s}. \tag{A.1}$$

Second, we obtain the two fixed points of g by taking (22) and (23),

$$s^*_{|\dot{g}=0} = \frac{g(\theta g_r - 1) + g_0 + \theta b_{u^f} u^f g_r}{g_s\lambda - \theta b_s g_r}, \tag{20}$$

$$s^*_{|\dot{s}=0} = \frac{\theta b_{u^f} u^f - g\eta b_u}{(i^f - g)\lambda - \theta b_s}, \tag{21}$$

setting them equal and solving for g:

$$\begin{aligned} 0 &= g^2\lambda(1 - \theta g_r) + g[(\theta g_r - 1)(i^f\lambda - \theta b_s) - \lambda(g_0 + \theta b_{u^f} u^f g_r) + \eta b_u(g_s\lambda - \theta b_s g_r)] + \\ &(g_0 + \theta b_{u^f} u^f g_r)(i^f\lambda - \theta b_s) - \theta b_{u^f} u^f(g_s\lambda - \theta b_s g_r). \end{aligned}$$

This is a U-shaped parabola. We define:

$$\alpha_g = \lambda(1-\theta g_r) > 0,$$
$$\beta_g = \underbrace{(\theta g_r - 1)(i^f\lambda - \theta b_s)}_{<0} - \underbrace{\lambda(g_0 + \theta b_{u^f} u^f g_r)}_{>0} + \underbrace{\eta b_u(g_s\lambda - \theta b_s g_r)}_{>0},$$
$$\varsigma_g = \underbrace{(g_0 + \theta b_{u^f} u^f g_r)(i^f\lambda - \theta b_s)}_{>0} - \underbrace{\theta b_{u^f} u^f (g_s\lambda - \theta b_s g_r)}_{>0}.$$

The fixed points of g are then given by:

$$g^*_{1,2} = \frac{-\beta_g \pm \sqrt{\beta_g^2 - 4\alpha_g\varsigma_g}}{2\alpha_g}, \tag{A.2}$$

where $\beta_g^2 - 4\alpha_g\varsigma_g = \beta_s^2 - 4\alpha_s\varsigma_s$, i.e. the discriminants of (A.1) and (A.2) are identical. If the discriminant is positive, there will be two fixed points. ■

A4 Proof of Proposition 2: Asymptotic stability of the 2D system

The trace and determinant of the Jacobian (19) evaluated at the positive fixed point are given by:

$$Tr(J) = \gamma(\theta g_r - 1) + \mu[(i^f - g^*)\lambda - \theta b_s],$$
$$Det(J) = \gamma\mu\{(\theta g_r - 1)[(i^f - g^*)\lambda - \theta b_s] - (\theta b_s g_r - g_s\lambda)(\eta b_u - s^*\lambda)\}.$$

Stability requires $Tr(J) < 0$ and $Det(J) > 0$. Some algebra shows that:

$$Tr(J) < 0 \Leftrightarrow i^f < g^* + \frac{\theta b_s}{\lambda} + \frac{\gamma\overbrace{(1-\theta g_r)}^{>0}}{\mu\lambda},$$
$$Det(J) > 0 \Leftrightarrow i^f < g^* + \frac{\overbrace{(\theta b_s g_r - g_s\lambda)(\eta b_u - s^*\lambda)}^{<0}}{\underbrace{(\theta g_r - 1)\lambda}_{<0}}.$$

■

A5 Proof of Proposition 3: Number of fixed points of the 3D system

The 3D system is reproduced here for convenience:

$$\begin{aligned}
\dot{g} &= \gamma[g(\theta g_r - 1) + s(\theta b_s g_r - g_s\lambda) + g_0 + \theta b_{u^f} u^f g_r] \\
\dot{\lambda} &= \delta(\lambda^T - \lambda) - g\lambda \\
\dot{s} &= \mu\{g\eta b_u + s[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] - \theta b_{u^f} u^f\}.
\end{aligned}$$

First, we set (13) equal to zero and solve for g:

$$g = \frac{s(\theta b_s g_r - g_s\lambda) + g_0 + \theta b_{u^f} u^f g_r}{1 - \theta g_r}. \tag{A.5}$$

In order to reduce clutter, we introduce the following composite parameters:

$$\begin{aligned}
\Phi_0 &= \frac{g_0 + \theta b_{u^f} u^f g_r}{1 - \theta g_r} > 0, \\
\Phi_1 &= \frac{\theta b_s g_r}{1 - \theta g_r} > 0, \\
\Phi_2 &= \frac{g_s}{1 - \theta g_r} > 0,
\end{aligned}$$

which allows us to re-write (A.5) as:

$$g = \Phi_0 + \Phi_1 s - \Phi_2 s\lambda. \tag{A.6}$$

Substituting (A.6) into (23), setting it equal to zero and solving for s yields:

$$s = \frac{\delta\lambda^T - \lambda(\delta + \Phi_0)}{\lambda(\Phi_1 + \Phi_2\lambda)}. \tag{A.7}$$

Likewise, substituting (A.6) into (24), setting it equal to zero and solving for s yields:

$$s = \frac{\eta\Phi_0 b_u - \theta b_{u^f} u^f}{\lambda(\Phi_2\eta b_u + \delta - i^f) - \Phi_1\eta b_u - \delta\lambda^T - \theta b_s}. \tag{A.8}$$

Let us introduce further composite parameters:

$$\begin{aligned}
\rho_0 &= \eta\Phi_0 b_u - \theta b_{u^f} u^f, \\
\rho_1 &= \Phi_2\eta b_u + \delta - i^f, \\
\rho_2 &= -(\Phi_1\eta b_u + \delta\lambda^T + \theta b_s) < 0.
\end{aligned}$$

We can then rewrite (A.8) as:

$$s = \frac{\rho_0}{\lambda\rho_1 + \rho_2}. \tag{A.9}$$

Setting (A.7) and (A.9) equal and solving for λ yields the following second-order polynomial:

$$\lambda^2[\rho_0\Phi_2 + \rho_1(\delta + \Phi_0)] + \lambda[\Phi_2\rho_0 + \rho_2(\delta + \Phi_0) - \rho_1\delta\lambda^T] - \rho_2\delta\lambda^T = 0, \tag{A.10}$$

with roots:

$$\lambda^*_{1,2} = \frac{-[\Phi_2\rho_0 + \rho_2(\delta + \Phi_0) - \rho_1\delta\lambda^T] \pm \sqrt{[\Phi_2\rho_0 + \rho_2(\delta + \Phi_0) - \rho_1\delta\lambda^T]^2 + 4[\rho_0\Phi_2 + \rho_1(\delta + \Phi_0)]\rho_2\delta\lambda^T}}{2[\rho_0\Phi_2 + \rho_1(\delta + \Phi_0)]}. \tag{26}$$

If the discriminant is positive there are exactly two real roots, which constitute the fixed points of λ. Thus, there are at most two fixed points in the 3D system. ■

A6 Proof of Proposition 4: Hopf bifurcation in the 3D system

The Jacobian matrix of the 3D system (25) evaluated at the positive fixed points is given by:

$$\mathbf{J}(g, s, \lambda) = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix} = \begin{bmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s gr - g_s\lambda^*) & -\gamma s^* g_s \\ \mu\eta b_u & \mu[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] & \mu s^*(i^f + \delta) \\ -\lambda & 0 & -(\delta + g^*) \end{bmatrix}. \tag{27}$$

The Jacobian has the characteristic equation

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{28}$$

where

$$\begin{aligned} a_1 &= -Tr(J), \\ a_2 &= Det(J_1) + Det(J_2) + Det(J_3), \\ a_3 &= -Det(J), \end{aligned}$$

and where J_i is the 2x2 minor obtained by deleting row and column i from the Jacobian. The Hopf bifurcation emerges for $a_1, a_2, a_3 > 0$ and $a_1 a_2 - a_3 = 0$, where the Jacobian matrix exhibits a non-zero real root and a pair of pure imaginary eigenvalues (Gandolfo, 1997, pp. 475-479). We thus have the following four conditions:

$$Tr(J) = -a_1 < 0, \tag{HBF.1}$$

$$Det(J_1) + Det(J_2) + Det(J_3) = a_2 > 0, \tag{HBF.2}$$

$$Det(J) = -a_3 < 0, \tag{HBF.3}$$

$$-Tr(J)[\sum_{i=1}^{3} Det(J_i)] + Det(J) = a_1a_2 - a_3 = 0. \tag{HBF.4}$$

(HBF.1). The first condition is:

$$-a_1 = J_{11} + J_{22} + J_{33} = \underbrace{\gamma(\theta g_r - 1)}_{<0} + \underbrace{\mu[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s]}_{>0} - \underbrace{(\delta + g^*)}_{>0},$$

which becomes negative if $|J_{11} + J_{33}| > |J_{22}|$.

(HBF.2). The second condition is:

$$\begin{aligned}
a_2 &= \begin{vmatrix} J_{22} & J_{23} \\ J_{32} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{13} \\ J_{31} & J_{33} \end{vmatrix} + \begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{vmatrix} \\
&= \underbrace{\begin{vmatrix} \mu[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] & \mu s^*(i^f + \delta) \\ 0 & -(\delta + g^*) \end{vmatrix}}_{<0} + \begin{vmatrix} \gamma(\theta g_r - 1) & -\gamma s^* g_s \\ -\lambda^* & -(\delta + g^*) \end{vmatrix} \\
&+ \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s\lambda^*) \\ \mu\eta b_u & \mu[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] \end{vmatrix}}_{>0} \\
&= \underbrace{J_{22}J_{33}}_{<0} + \underbrace{J_{11}J_{33}}_{>0} - \underbrace{J_{13}J_{31}}_{>0} + \underbrace{J_{11}J_{22}}_{<0} - \underbrace{J_{12}J_{21}}_{<0},
\end{aligned}$$

which becomes positive if $|J_{11}J_{33} + J_{12}J_{21}| > |J_{22}(J_{11} + J_{33}) + J_{13}J_{31}|$.

(HBF.3). The third condition is given by:

$$
\begin{aligned}
-a_3 &= J_{31}\begin{vmatrix} J_{12} & J_{13} \\ J_{22} & J_{23} \end{vmatrix} + J_{33}\begin{vmatrix} J_{11} & J_{12} \\ J_{21} & J_2 \end{vmatrix} \\
&= -\lambda^* \underbrace{\begin{vmatrix} \gamma(\theta b_s g_r - g_s\lambda^*) & -\gamma s^* g_s \\ \mu[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] & \mu s^*(i^f + \delta) \end{vmatrix}}_{<0} - (\delta + g^*) \underbrace{\begin{vmatrix} \gamma(\theta g_r - 1) & \gamma(\theta b_s g_r - g_s\lambda^*) \\ \mu\eta b_u & \mu[i^f\lambda - \delta(\lambda^T - \lambda) - \theta b_s] \end{vmatrix}}_{>0} \\
&= \underbrace{J_{31}}_{<0}\underbrace{(J_{12}J_{23} - J_{13}J_{22})}_{<0} + \underbrace{J_{33}}_{<0}\underbrace{(J_{11}J_{22} - J_{12}J_{21})}_{>0},
\end{aligned}
$$

which becomes negative if $|J_{33}(J_{11}J_{22} - J_{12}J_{21})| > |J_{31}(J_{12}J_{23} - J_{13}J_{22})|$.

(HBF.4). Lastly, we have $a_1a_2 - a_3$, which must become zero for a Hopf bifurcation. We will use μ as the Hopf bifurcation parameter as it has the convenient property that it does not affect the steady state values. Let us write the three parameters of the characteristic equation as functions of μ:

$$
\begin{aligned}
a_1 &= \mu\kappa_1 + \kappa_2, \\
a_2 &= \mu\kappa_3 + \kappa_4, \\
a_3 &= \mu\kappa_5.
\end{aligned}
$$

We can then rewrite (HBF.4) as:

$$a_1a_2 - a_3 = f(\mu) = \kappa_1\kappa_3\mu^2 + (\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5)\mu + \kappa_2\kappa_4 = 0.$$

This expression must become zero for a Hopf bifurcation to occur. The coefficients κ_1 to κ_5

are given by:

$$\kappa_1 = -[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] < 0,$$
$$\kappa_2 = \gamma(1-\theta g_r) + (\delta + g^*) > 0,$$
$$\kappa_3 = \underbrace{[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s]}_{>0}\underbrace{[\gamma(\theta g_r - 1) - (\delta + g^*)]}_{<0} - \underbrace{\eta b_u \gamma(\theta b_s g_r - g_s\lambda^*)}_{<0},$$
$$\kappa_4 = \gamma(1-\theta g_r)(\delta + g^*) - \lambda^*\gamma s^* g_s,$$
$$\kappa_5 = \lambda^* \underbrace{\{\gamma(\theta b_s g_r - g_s\lambda^*)s^*(i^f+\delta) + [i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s]\gamma s^* g_s\}}_{<0}$$
$$+ (\delta + g^*)\underbrace{\{\gamma(\theta g_r - 1)[i^f\lambda^* - \delta(\lambda^T - \lambda^*) - \theta b_s] - \eta b_u\gamma(\theta b_s g_r - g_s\lambda^*)\}}_{>0}.$$

κ_3 is likely to be positive given that we assume strong interaction effects and weak independent dynamics. κ_4 and κ_5 can be positive or negative. We then have:

$$f(\mu) = \underbrace{\kappa_1\kappa_3}_{<0}\mu^2 + (\kappa_1\kappa_4 + \underbrace{\kappa_2\kappa_3}_{<0} - \kappa_5)\mu + \kappa_2\kappa_4 = 0. \quad \text{(A.11)}$$

This parabola is opened downward. If its discriminant is positive, the equation has two real roots. This requires the expression $\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5$ to be sufficiently large in absolute value. At these two roots, a Hopf bifurcation occurs:

$$\mu_\pm = \frac{-(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5) \pm \sqrt{(\kappa_1\kappa_4 + \kappa_2\kappa_3 - \kappa_5)^2 - 4\kappa_1\kappa_2\kappa_3\kappa_4}}{2\kappa_1\kappa_3}.$$

■