Accepted Manuscript

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PII: S0020-7403(17)33588-9
DOI: 10.1016/j.ijmecsci.2018.04.030
Reference: MS 4285

To appear in: *International Journal of Mechanical Sciences*

Received date: 18 December 2017
Revised date: 25 February 2018
Accepted date: 18 April 2018

Please cite this article as: Mehran Ghalami-Choobar , Gholamhossein Liaghat , Mojtaba Sadighi , Hamed Ahmadia Omid Razmkhah , Akbar Aboutorabi , STATIC ANALYSIS OF HIGHLY ANISOTROPIC LAMINATED BEAM USING UNIFIED ZIG-ZAG THEORY SUBJECTED TO MECHANICAL AND THERMAL LOADING, *International Journal of Mechanical Sciences* (2018), doi: 10.1016/j.ijmecsci.2018.04.030

Highlights

- Accurate prediction of stress filed plays an important role in highly anisotropic laminates.
- An Unified Zig-Zag Theory is developed with two primary variable under thermal environment.
- Comparison of the model predictions and that of literature indicates good agreement.
- The results show the interlaminar continuity(IC) causes more accurate stress in thin laminates

STATIC ANALYSIS OF HIGHLY ANISOTROPIC LAMINATED BEAM USING UNIFIED ZIG-ZAG THEORY SUBJECTED TO MECHANICAL AND THERMAL LOADING

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Abstract: In the present study, static behavior of short hybrid laminate beams was investigated using a unified zig-zag theory (ZZT) containing various beam theories as special cases. This theory satisfies transverse shear stresses continuity in the interface of layers via piece-wise continuous arbitrary shape functions. The principle of virtual work was employed to derive unified equilibrium equations and suitable boundary conditions. The present theory obviates the need for stress recovery for continuous transverse stresses. A general solution was presented to analyse high transversely anisotropic laminates under several kinds of transverse loads (general lateral, sinusoidal and point load) and non-linear thermal loads. The validity of this model is demonstrated by comparison of its predictions and good agreement with published results in literature. Numerical examples were given to investigate the impact of the transverse anisotropy on displacement, strain and stress fields through the thickness. The results show that the piece-wise continuous exponential and sinusoidal shape functions provide more accurate transverse stress distribution in comparison with other shape functions. In addition, the results show that the continuity of transverse shear stress through the thickness plays an important role in analysing transversely anisotropic laminated beams. A comparison of present ZZT and existing exact elasticity solutions shows that the current theory is simple and efficient.

Keywords: Zig-Zag theories; Hybrid laminate; interlaminar shear stress continuity, Equivalent Single Layer; Closed-form solution

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1. Introduction

In recent decades, the application of hybrid composite laminates and sandwich structures has found popularity in aerospace and automobile industries [1–3]due to their superior mechanical properties such as specific stiffness, strength and fatigue characteristics compared to the traditional structures[4,5]. As a result, the utilization of hybrid and Carbon reinforced composites is expected to increase in the coming years [6].

Accurate prediction of stress distribution through the body plays an important role in primary structure design[7]. Furthermore, thickening the multilayer composite to prevent damage in critical points exacerbates non-classical effects such as transverse shear and normal deformation [6]. For instance, Euler-Bernoulli Deformation Theory (EBDT) and Kirchhoff-Love plate/shell theory provide an inaccurate prediction of global deflection and local effects [8]. Although other Equivalent Single Layers (ESL) theories such as First-order Shear Deformation Theory(FSDT), High-order Shear Deformation Theory(HSDT) and Advanced High Order Theories [9,10] offer some improvements, they suffer from drawbacks of inaccurate prediction in some high transversely anisotropic laminates. On the other hand, in case of laminates having large numbers of layers, Layer-Wise (LW) models are computationally very expensive in spite of providing highly accurate results [11]. Therefore, developing novel theoretical models to capture non-classical effects is vital to produce the reliable design.

Zig-Zag theories (ZZT) combine the low computational cost of ESL theories and the ability of LW theories to model laminates having layers with completely different material properties [9]. This theory incorporates piece-wise continuous displacement field shown in multilayer composites. The variation of slope through thickness due to transverse anisotropy is known as Zig-Zag (ZZ) effect (Fig. 1). Interlaminar continuity(IC)[12] for transverse stresses causes rapid changes in the slope of in-plane displacement field. ZZT includes three different and independent contributions: Lekhnitskii Multilayered Theory (LMT), Ambartsumian Multilayered Theory (AMT) and Reissner Multilayered Theory (RMT) [6,13]. Lekhnitskii apparently proposed a ZZT for the first time [13]. Ambartsumian developed a ZZT assuming a parabolic distribution for transverse shear stress [14]. Later on, Di Sciuva presented a displacement theory to enhance first-order shear deformation theory to take into account ZZ effects [15]. Tessler et al developed a refined zigzag theory RZT to overcome shortcomings of Di Sciuva formulation [16]. Similarly, Murakami presented an alternative method to improve FSDT by including a geometric zigzag function which is known as Murakami's ZZ function (MZZF)[13,17]. Carrera presented a unified description of several theories including ESL, LW and ZZ effects. The finite element matrices derived in a unified manner and vast numerical examples have been given [18,19]. MZZF employs an independent parabolic function for displacement and shear stress distribution via a mixed variational calculus method. Brischetto et al applied MZZF to sandwich panels[20]. Gherlone et al [1] studied the mixed formulation of MZZF in comparison with displacement-based MZZF, RZT and Timoshenko beam. They showed RZT is more accurate for arbitrary lay-ups by considering the ZZ effects. Groh and Weaver [6] investigated displacement-based and mixed formulation with Reddy shape function and MZZF. Furthermore, they proposed a unified general theory based on Hellinger–Reissner mixed formulation to capture non-classical effect due to highly heterogeneous multilayers[21,22].

Rodrigues et al. investigated bending, free vibration and buckling of laminated plates using MZZF in using local collocation method based on radial basis functions[23]. Kulkarni and Kapuria developed a quadrilateral element for static and free vibration analysis of composite and sandwich plats. They validated the finite formulation by comparing with the analytical Navier solution for simply supported plates[24,25]. Neves et al. employed Carrera's Unified Formulation (CUM) to study thin and thick functionally graded sandwich plates. Also, they considered thickness stretching effect in the model[26]. Sahoo and Singh presented an efficient element based on a new trigonometric ZZT for free vibration and buckling of laminated and sandwich plates[27]. Pandit et al proposed an improved high order zig-zag theory for static analysis of laminated sandwich plate with compressible soft core[28].

In addition to mentioned works in the linear elastic domain, a few attempts have employed ZZ to model delamination damage in multilayer composites[29]. Groh et al used the cohesive zone approach to predict the initiation and propagation of delamination in cross-ply composite beams and compare the predictions with those of experimental test[30]. Eijo et al. developed beam and plate/shell element to model delamination (mode II and III) in laminated composites using an isotropic damage model[31,32].

In this paper, a general solution of unified ZZT for arbitrary shape function is presented, where a piece-wise continuous arbitrary function is assumed as distribution of transverse shear stresses through the thickness. Interlaminar continuity of transverse stress and displacement can be applied via the assumed distribution. The unified governing equations and boundary conditions are derived using the principle of the virtual work, and the Navier-type and closed-form solutions are given for the proposed beam theory subjected to mechanical and thermal loads. By comparing present model results with those reported in literature, the validity of solutions is confirmed. Furthurmore, other numerical results from this work are given to investigate the impact of the transverse anisotropy on displacement and stress fields along the beam length and through the thickness.

2. Theoretical formulations

2.1 Transverse shear stress and displacement field

In the present study, a composite laminate is considered which is illustrated in Fig. 2. The x- and y-axes are assumed along the length (L) and the thickness (h), respectively. The unified transverse shear stress field of k_{th} layer at any point (x, z) for present ZZT theory is given by

$$\tau_{xz}^{(k)} = G\left\{A^{(k)} + m^{(k)}\left(\frac{d\varnothing}{dz} - 1\right)\right\}\bar{\gamma}_{xz}(x) \tag{1}$$

where G, $A^{(k)}$, $m^{(k)}$ and $\bar{\gamma}_{xz}$ are effective shear modulus, shear stress layer-wise constant, modification factor of layer k and transverse shear strain, respectively. A posteriori function $\varnothing(z)$ incorporates the through the thickness distribution of transverse shear stress. The effective shear modulus and modification factor can be defined as

$$m^{(k)} = e^{(k)}\left(g^{(k)} + \frac{1}{g^{(k)}} - 1\right) \tag{2}$$

where $e^{(k)}$ can be expressed by

$$e^{(k)} = \frac{\bar{Q}^{(k)}}{E} \tag{3}$$

$$E = 1/h\sum_{k=1}^{N} t^{(k)}\bar{Q}^{(k)} \tag{4}$$

and $g^{(k)}$ is given by

$$g^{(k)} = \frac{G}{G_{xz}^{(k)}} \tag{5}$$

$$G = \left[\frac{1}{h}\sum_{k=1}^{N}\frac{t^{(k)}}{G_{xz}^{(k)}}\right]^{-1} \tag{6}$$

where $\bar{Q}^{(k)}$, $G_{xz}^{(k)}$ and $t^{(k)}$ are reduced stiffness, transverse shear modulus, and thickness of k_{th} layer. It should be noted that Eq. (1) is the unified form of recent formula proposed by literature [6]. Groh and Weaver's formula is a piece-wise continues parabola distribution based on Reddy High order shear theory.

$$\tau_{xz}^{(k)} = G\left\{A^{(k)} - \frac{4}{t^2}e_{(k)}\left(g_k + \frac{1}{g_k} - 1\right)z^2\right\}\bar{\gamma}_{xz}(x) \tag{7}$$

It should be noted that transverse shear strain, $\bar{\gamma}_{xz}$, is measured on the mean-line of the beam.

$$\bar{\gamma}_{xz} = \mathrm{w}_{,\mathrm{x}} - \psi \tag{8}$$

where w and ψ are transverse displacement and section rotation on the mean-line of the beam. In both Eqs. (1) and (7), shear stress layer-wise constants, $A^{(k)}$, are determined by satisfying transverse stress continuity. The free surface and stress continuity conditions are given by

$$\tau_{xz}^{(1)}(z_0) = 0 \tag{9}$$

$$A^{(1)} = -m^{(1)}\left(\frac{d\varnothing}{dz} - 1\right) \tag{10}$$

And

$$\tau_{xz}^{(k)} = \tau_{xz}^{(k+1)} \tag{11}$$

$$A^{(k+1)} = A^{(k)} + \left(m^{(k)} - m^{(k+1)}\right)\left(\frac{d\varnothing}{dz} - 1\right) \tag{12}$$

Assuming linear strain-displacement relationship, unified displacement field $u_x^k(x,z)$ for present ZZT may be written as follows:

$$u_x^{(k)}(x,z) = -zw_{,x}(x) + g^{(k)}\left\{zA^{(k)} + m^{(k)}(\varnothing - z)\right\}\bar{\gamma}_{xz} + c^{(k)}\bar{\gamma}_{xz} \tag{13}$$

$$u_z(x,z) = w(x) \tag{14}$$

The function $\varnothing(z)$ represents shape functions defining the distribution of displacement and stress function through the thickness. It should be noted that the present ZZT is developed based on Reddy's high order theory which suffers from inconsistency in modeling clamped edges[33]. In this high order theories, enforcing $w_{,x} = 0$ at edges causes $\bar{\gamma}_{xz} = 0$, which does not match the 3D elasticity solution. The displacement field of various ZZTs may be obtained using shape functions given in Table 1. It can be noted the displacement field of HSDT is obtained by setting:

$$A^{(k)} = c^{(k)} = 0, \quad g^{(k)} = m^{(k)} = 1 \tag{15}$$

In Eq. (13), displacement layer-wise constants $c^{(k)}$ are determined by enforcing displacement continuity. The neutral axis location and continuity condition are given by:

$$u_x^{(k_0)} = 0 \tag{16}$$

Table 1

Shape function name	$\varnothing(z)$
EBDT	0
FSDT	z
Ambartsumyan[6, 11]	$\frac{z}{2}\left(\frac{h^2}{4}-\frac{z^2}{3}\right)$
Reddy[17]	$z\left(1-\frac{4z^2}{3}\right)$
Touratier[18, 19]	$\frac{h}{\pi}\sin(\frac{\pi z}{h})$
Karama[20]	$ze^{-2\left(\frac{z}{h}\right)^2}$

Table 2

Mechanical properties	Value $\times10^6$ psi	Mechanical properties	Value $\times10^6$ psi	Mechanical properties	Value
E_1	25	G_{12}	5	ν_{12}	0.25
E_2	1	G_{13}	5	ν_{23}	0.25
E_3	1	G_{23}	2	ν_{13}	0.25

Table 3

Mechanical properties	Value (GPa)	Mechanical properties	Value (GPa)	Mechanical properties	Value
E_1	241.5	G_{12}	5.18	ν_{12}	0.24
E_2	18.89	G_{13}	5.18	ν_{23}	0.25
E_3	18.89	G_{23}	3.45	ν_{13}	0.24

Table 4

Beam	Theory	$W\left(L/2,h/2\right)$	$\sigma_x\left(L/2,h/2\right)$	$\tau_{xz}\left(L/2,0\right)$
[0/90/0]	Exact[6]	0.0116	0.7913	3.3176
	Parabolic[6]	0.0116	0.7997	3.3926
	Present-Sinusoidal	0.0116	0.8036	3.4187
	Present-Exponential	0.0116	0.8071	3.4410
[0/90 /0/90/0]	Exact[6]	0.0124	0.8672	3.3228
	Parabolic[6]	0.0124	0.8703	3.2688
	Present-Sinusoidal	0.0124	0.8725	3.3403
	Present-Exponential	0.0124	0.8747	3.4279

Table 5

l/h	Reference	$W\left(L/2,h/2\right)$	$U\left(L/2,h/2\right)$	$\tau_{xz}\left(L/2,0\right)$	$\sigma_x\left(L/2,h/2\right)$
100	Present Unified ZZT	0.5155	8020	45.10	6315.00
	Chakrabarti et al [38]	0.5140	8020	45.23	6315.00
	Pagano[39]	0.5153	8040	44.15	6315.00
50	Present Unified ZZT	0.5335	1009	22.52	1587.90
	Chakrabarti et al [38]	0.5270	1008	22.46	1587.07
	Liou and sun[40]	0.5270	-	-	-
	Pagano[39]	0.5283	1010	22.05	1587.00
20	Present Unified ZZT	0.6587	67.10	8.98	264.22
	Chakrabarti et al [38]	0.6176	66.86	8.90	263.19
	Lee and Liu[40]	0.6173	-	8.74	-
	Matsunaga[41]	0.6150	-	8.75	-
	Pagano[39]	0.6186	66.95	8.74	263.20
10	Present Unified ZZT	1.0050	9.5240	4.44	74.99
	Chakrabarti et al [38]	0.9329	9.3480	4.32	73.61
	Bambole and desai[42]	0.9321	-	4.25	73.81
	Liou and sun[40]	0.9315	-	-	-
	Pagano[39]	0.9357	9.3470	4.23	73.66

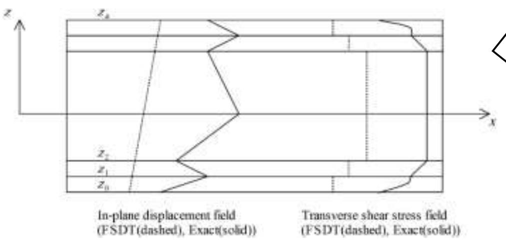
Table 6

Beam	Theory	$\frac{D^{\gamma 2}\pi^2}{DJL^2}$	$\frac{D^{\gamma\gamma}\pi^2}{JL^2}$	$U\left(L/2,h/2\right)$	$W\left(L/2,h/2\right)$	$.\sigma_x\left(L/2,h/2\right)$	$\tau_{xz}\left(L/2,0\right)$
Isotropic	Present	0.0210	0.0213	1.8151	11.8632	0.3071	0.0436
$E=E_1$	HSDT			1.8024	11.7805	0.3050	0.0433
$\upsilon=\upsilon_{12}$	FSDT			1.7961	11.7806	0.3039	0.0347
	EBDT			1.7961	11.5376	0.3039	0
$[0]_5$	Present	0.4947	0.5006	0.0963	0.8533	38311	0.0502
	HSDT			0.0827	0.7325	0.3288	0.0431
	FSDT			0.0764	0.7339	0.3039	0.0347
	EBDT			0.0764	0.4910	0.3039	-
[0/90 /0/90/0]	Present	0.6645	0.6718	0.1177	1.1474	0.4680	0.0420
	HSDT			0.1022	0.9379	0.4064	0.0603
	FSDT			0.0955	0.9332	0.3798	0.0456
	EBDT			0.0955	0.6135	0.3798	-

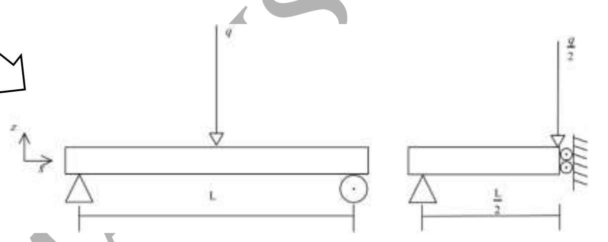
Table 7

Beam	Theory	$\frac{D^{\gamma 2}\pi^2}{DJL^2}$	$\frac{D^{\gamma\gamma}\pi^2}{JL^2}$	$U\left(L/2,h/2\right)$	$W\left(L/2,h/2\right)$	$\sigma_x\left(L/2,h/2\right)$	$\tau_{xz}\left(L/2,0\right)$
Isotropic	Present	0.1315	0.1331	0.7285	13.5551	0.3242	0.0995
	HSDT			0.6979	12.9854	0.3106	0.0953
	FSDT			0.6829	12.9878	0.3039	0.0763
	EBDT			0.6829	11.4774	0.3039	-
$[0]_5$	Present	3.0919	3.1287	0.0883	3.9501	0.9240	0.1870
	HSDT			0.0435	1.9452	0.4550	0.0921
	FSDT			0.0290	1.9988	0.3039	0.0763
	EBDT			0.0290	0.4884	0.3039	-
[0/90 /0/90/0]	Present	4.1534	4.1991	0.1018	5.4400	1.0648	0.1416
	HSDT			0.0519	2.5867	0.5430	0.1300
	FSDT			0.0363	2.5976	0.3798	0.1005
	EBDT			0.0363	0.6103	0.3798	-

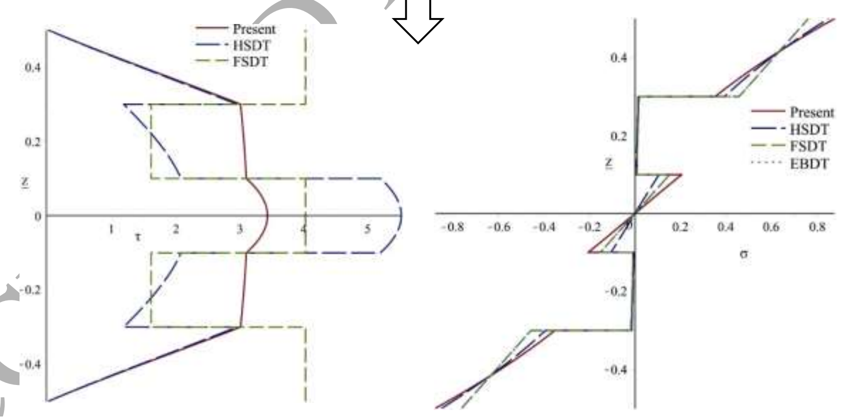
Graphical Abstract



Zig-Zag phenomenon in transversely anisotropic laminate



Boundary condition of the studied beam in thermal environment



Transverse shear stress and normal stress for different theories (Present, HSDT, FSDT, EBDT)