Trade Liberalization and its Fiscal Implications in a North-South Trade Model

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February 9, 2007

Abstract

We study the fiscal implications of trade liberalization in a North-South trade model with nonhomothetic preferences. Combining a Ricardian trade model with a continuum of competitive goods and a public good, nonhomothetic preferences imply that both the global income distribution and the local income distribution matter for gauging the effects of different trade liberalization regimes on income taxes and public good provision. The fiscal implications of tariff reductions are typically more adverse for poorer countries than for richer countries. We also find that unilateral trade liberalization by richer countries is a more viable policy option to pursue than multilaterally reducing tariffs.

Keywords: Ricardian trade model; Asymmetric demand complementarities; Trade liberalization; Tax reform; Public goods; Income distribution

JEL classification: F11

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1 Introduction

It is widely accepted that trade liberalization enhances economic efficiency, accelerates growth, and very likely boosts income. However, freer trade may also lead to a loss of tax revenue as tariffs and other trade taxes are cut and an evaluation of the revenue implications of trade liberalization becomes important. For countries with significant fiscal imbalances, any loss of revenue would be an important consideration. It might endanger the provision of public goods, increase the social burden of tariff cuts and as such erode the willingness for future liberalization (Elborgh-Woytek *et al.,* 2006). This applies to many developing countries and emerging market economies alike. Despite significant progress in reducing trade barriers over the last decade, taxes on international trade still constitute a major source of revenue for these economies. Ebrill *et al.* (1999), for example, show that the share of trade taxes to GDP is inversely related to the level of development, with many low-income countries from Africa accounting for 5.5% of trade taxes to GDP on average in 1995, only marginally down from 6.7% in 1975.

There exists an extensive literature discussing the welfare effects of piecemeal tariff reforms precluding the revenue motive for deploying trade taxes, see, for example, Hatta (1977) and, for a survey, Woodland (1982), and more recently Anderson and Neary (2005). Formal treatments of the fiscal implications of trade liberalization when tariff revenue considerations are important are developed, for example, by Michael *et al.* (1991), Tsuneki (1995), Keen and Ligthart (2002), and Anderson and Neary (2006). Keen and Ligthart (2002), for example, develop a practical strategy whereby tariff cuts are offset by increases in consumption taxes to secure efficiency gains from trade liberalization while preserving public revenue for a small open economy. Since important information about the technology and preferences required for the implementation of such rules are often missing, Anderson and Neary (2006) propose sufficient conditions that guarantee welfare improving tariff reforms, making use of moments of the distribution of a cross-section of tariffs.[1]

What these contributions have in common is that they focus exclusively on efficiency effects of tariff changes for small open economies while ignoring distributional issues. By contrast, our paper analyzes various forms of trade liberalization under the assumption that revenue losses have to be compensated either by reducing the provision of the public good or by corresponding changes in income taxes in a framework that assigns a central role to income differences between and within countries.

Following Matsuyama (2000) and Stibora and de Vaal (2007) we use a Ricardian trade

[1] A related paper is Feehan (1988) who derives optimal tariff formulae to finance the provision of a public good in a Heckscher-Ohlin framework.

model with nonhomothetic preferences to analyze the consequences of trade liberalization in a North-South set-up.[2] They assume that tariff revenues are redistributed directly to households, which is an innocuous assumption in models with homothetic preferences, but not in models where the income elasticity deviates from one. Revenue from trade taxes are moreover rarely rebated directly to households but constitute part of the overall government budget. To capture this important aspect, we extend their framework by explicitly adding a government sector that provides public goods for the benefit of households. Public goods are produced with labor and are financed through revenues generated by income taxation and import tariffs. In addition to the public good, households consume a range of goods supplied by perfectly competitive markets. We assume that these goods are indivisible and that consumption for each good is satiated after one unit. We order goods according to priority in consumption, the lowest-indexed goods have the highest priority in consumption and the highest-indexed goods the lowest priority in consumption. This paves the way for nonhomothetic preferences since poor households are not able to consume the same consumption basket as rich households. All households purchase the lower-indexed, high-priority, goods, and when real income increases add higher-indexed, low-priority, goods to their consumption baskets, instead of buying more of the goods they already consume. The higher-indexed, low-priority, goods are therefore only affordable by households with sufficiently high income levels.[3] Of the two countries we consider, South has a comparative advantage in the production of lower-ranked, high priority, goods which all households consume, while the North has a comparative advantage in the production in higher-ranked, low-priority, goods, which household with higher income consume. This implies that the poor (rich) country produces goods with low (high) income elasticities in demand. This makes our framework appropriate for investigating North-South issues.

The assumption that the provision of public goods is financed by tariff revenue combined with nonhomothetic preferences generates new insights. Among other things, while North cannot lose from unilateral tariff reductions, South may lose from unilateral and may not gain from multilateral trade liberalization. This is the result of asymmetric demand responses. The fall in the price of lower-ranked goods increases real income and induces households to shift expenditures away from lower-ranked goods toward higher-ranked goods. The income effect makes higher-ranked goods complements to lower-ranked goods in demand. This

[2] According to Deaton and Muellbauer (1983) most household budget studies support the assumption of nonhomotheticity. Hunter and Markusen (1988) and Hunter (1991) report that as much as 29 percent of world trade may be caused by nonhomogeneous preferences.

[3] Assuming that goods are indivisible in consumption is a simple and tractable way to include nonhomothetic preferences in general equilibrium analyses, see e.g. Murphy *et al.* (1989), Krishna and Yavas (2005), and Bertola *et al.* (2006).

demand complementarity, however, is asymmetric in the sense, that when the price of higher-ranked goods falls, households do not increase their purchases of lower-ranked goods.

Nonhomotheticity of preferences also implies that tariff reductions by the rich, northern region have less adverse effect on northern public good provision than a similar tariff cut by the poor, southern region would have on southern public good provision. The reason is that the real income gains that follow for South when North cuts tariffs are spent on higher-index goods produced in North, increasing trade volumes and northern tariff revenues. By contrast, when South cuts its tariffs, such beneficial effect does not occur for South. The real income gains that arise in North are spent exclusively on northern goods and therefore do not yield a boost to South's import volume.

The structure of the paper is as follows. Section 2 details the model. Section 3 considers a situation where countries differ in their income level and conducts comparative static analysis to elaborate on the effects of demand complementarities. We divide this section into two subsections. The first subsection focuses on the impact on public good provision, keeping local income taxes constant. The second subsection keeps the level of the public good provision constant, examining the required change in taxation for countries undergoing trade liberalization. This section also provides a useful benchmark for the sections to follow. Section 4 extends the analysis by assuming heterogeneous income groups of households in each country. Section 5 concludes.

2 The model

We consider two countries, South and North, each having fixed labor supplies. Both countries produce a public good and a range of tradable competitive goods. The public good, being characterized by jointness in consumption and non-excludability, is financed in each region by levying tariffs on imported goods and by raising income taxes. The accruing tax revenues, which are collected exclusively by the country's government, are used to employ labor to provide the public good.

The competitive goods sector consists of a continuum of competitive industries, indexed by $z \in [0, \infty)$, each producing a homogeneous good also indexed by z. For good z, let $a(z)$ be the unit labor requirement in South and $a^*(z)$ the unit labor requirement in North. Here and in the sequel an asterisk denotes North. We define $a^*(z)/a(z)$ as the ratio of northern to southern labor productivity and follow standard practice ranking commodities in order of diminishing southern comparative advantage:[4]

$$A(z) \equiv \frac{a^*(z)}{a(z)} \qquad A'(z) < 0. \tag{1}$$

The function $A(z)$ is assumed to be continuous and strictly decreasing in z. Denoting percentage changes $da(\cdot)/a(\cdot) = \widehat{a}(\cdot)$, and likewise for all other variables, the elasticity of $A(z)$ at z is defined as $\zeta(z) \equiv [\widehat{a}(z)/\widehat{z} - \widehat{a}^*(z)/\widehat{z}] = -\hat{A}(z)/\widehat{z}$. $\zeta(z)$ which is positive and can be arbitrarily large.

Trade flows are distorted by tariffs. Let τ (τ^*) be one plus the ad valorem tariff imposed by South (North) when importing goods from North (South). All imported goods face the same tariff rate. Good z will be produced in South when the northern unit labor cost adjusted for the tariff exceeds the southern unit labor cost. We choose northern wages as numéraire ($w^* = 1$). The wage rate in South is w, which also constitutes South's factor terms of trade. It then follows that good z will be produced in South if

$$wa(z) \leq \tau a^*(z) \quad \text{or} \quad w \leq \tau A(z). \tag{2}$$

Similarly, any commodity z will be produced in North if

$$\tau^* wa(z) \geq a^*(z) \quad \text{or} \quad A(z) \leq \tau^* w. \tag{3}$$

The existence of import barriers imply that there are ranges of commodities that are not traded. Let $\widetilde{z}$ denote the borderline commodity between South's non-traded commodities and North's exports and let $\widetilde{z}^*$ be the borderline commodity between South's exports and North's non-traded commodities. By imposing equalities in (2) and (3), these commodities are seen to be

$$\widetilde{z} = A^{-1}(w/\tau)$$
$$\widetilde{z}^* = A^{-1}(\tau^* w),$$

where $\widetilde{z}^* < \widetilde{z}$ (the latter follows as A^{-1}, the inverse of $A(z)$, is monotonically decreasing, and $w/\tau < \tau^* w$). Equations (2) and (3) imply that South produces all $z \in [0, \widetilde{z}]$ and North all $z \in [\widetilde{z}^*, \infty)$. Commodities $z \in [0, \widetilde{z}^*]$ are exclusively produced in South and exported. South's inherent cost advantage in these goods is high enough to outweigh the trade taxes. Similarly, North's cost advantage in $z \in [\widetilde{z}, \infty)$ is so high that those higher-indexed goods are exclusively produced there and exported. Goods $z \in (\widetilde{z}^*, \widetilde{z})$ define an intermediate range of goods that both countries produce but do not trade. For these goods productivity differences are not high enough to outweigh the trade taxes for them to be traded. Even if traded, the local price of good z does not need to be identical across countries:

$$p(z) = \min\{wa(z), \tau a^*(z)\} \quad \text{and} \quad p^*(z) = \min\{a^*(z), \tau^* wa(z)\}.$$

[4]The continuum assumption is orginially due to Dornbusch, Fischer, and Samuelson (1977).

Turning to the demand side, we suppose there are N households in South and N^* in North. We follow Matsuyama (2000) and assume that the income distribution is nondegenerate and brought about by differences in skills reflected in differences in effective labor supply. We let $F(h)$ and $F^*(h^*)$ denote the distribution of effective (skill based) labor supply across households in South and North, respectively. The total labor supply thus equals $L = N \int_0^\infty h dF(h)$ in South and $L^* = N^* \int_0^\infty h^* dF^*(h^*)$ in North.

All households have identical preferences. The consumption set of a household includes the consumption of a public good G and a continuum of $z \in [0, \infty)$. Specifically, preferences for an individual southern household are

$$V_j = \ln G + \int_0^\infty b(z)x(z)dz. \tag{4}$$

An isomorphic utility function applies to northern households. The first expression on the right hand sight of (4) reflects the consumption of the public good, G. The specification implies that the marginal utility of the public good is positive but decreasing. The second part of (4) denotes the number of goods a households consumes of the competitive good z, where $b(z) > 0$ is the utility of consuming good z and $x(z) = \{0, 1\}$ denotes the consumption indicator. The household budget constraint is given by $\int_0^\infty p(z)x(z)dz \leq I$, where $x(z) = \{0, 1\}$ and where $I \equiv (1-t)wh$ denotes post-tax household income.[5] A household buys good z, $x(z) = 1$, if the utility from the last unit of income spent $\lambda \leq b(z)/p(z)$. The order in which each household purchases goods is assumed to be the same as the order of goods due to comparative advantage. Hence, we assume that households purchase lower-indexed, high-priority, goods first and with increasing income extend their consumption to higher-indexed, lower-priority, goods.[6] This requires that the order of utility per unit price is strictly decreasing in z, that is, we assume that

$$\frac{b(z)}{p(z)} = \frac{b(z)}{\min\{wa(z), \tau a^*(z)\}} \quad \text{and} \quad \frac{b(z)}{p^*(z)} = \frac{b(z)}{\min\{a^*(z), \tau^* wa(z)\}}$$

are strictly decreasing in z for given w, τ, and τ^*. This has the strong implication that, in contrast to standard analysis, an increase in real income is reflected in the consumption

[5] Alternatively, one could assume that the loss of tariff revenue is compensated by the strengthening of domestic consumption taxation, for example, in form of value-added tax.

[6] Even though one could label lower-indexed goods as 'necessities' and higher-ranked goods 'luxuries', in our model this classification is not appropriate. None of the goods satisfies the definition of necessary (luxury) goods usually found in textbooks. The reason is that the definition requires divisibility of goods and looks at inifinitesimal income changes while in our model goods come in discrete units. Income has to go up 'sufficiently' for a household to add a higher-indexed good to its consumption basket. Moreover, making a division between necessities and luxuries is artificial in our model, since our satiation assumption implies that each good becomes a necessity once it is consumed.

of an increased number of goods rather than in the consumption of higher quantities of a fixed number of goods. Combined with the ranking of factor intensity it follows that South has a comparative advantage in the production of lower-ranked goods which are consumed by poorer households, whereas North has a comparative advantage in the production of higher-ranked goods that are purchased by richer households.

Define

$$E(z) \equiv \int_0^z p(s)ds = \int_0^z \min\{wa(s), \tau a^*(s)\}ds \tag{5}$$

as the minimum level of income required to allow a southern household to consume good z. The highest-indexed commodity, $n(h)$, a southern household with net income $(1-t)wh$ is able to consume is determined by the requirement that

$$E[n(h)] = (1-t)wh, \tag{6}$$

where we assume that households take the import tariff and the income tax rate as given.[7] Similarly, North produces higher-ranked commodities and imports the lower-indexed goods from South paying a tariff inclusive price for every good z imported. The highest ranked good, $n^*(h^*)$, a northern household with net income $(1-t^*)h^*$ is able to purchase is given by

$$E^*[n^*(h^*)] = (1-t^*)h^*. \tag{7}$$

Southern households purchase the competitive good z only if their income is not lower than $E(z)$, or equivalently if their skill is such that $(1-t)wh$ exceeds $E(z)$. Likewise, northern households purchase good z if their skill is such that $(1-t^*)h^* > E^*(z)$. The fraction of southern households with income (skill) in excess of $E(z)$ is given by $1 - F[(E(z)/(1-t)w]$. Similarly, the fraction of northern households able to purchase good z is given by $1-F^*[E^*(z)/(1-t^*)]$. Aggregate demand for good z consists of all those households in both countries whose income is greater or equal than $E(z)$. Consequently, demand from each country is given by

$$\begin{aligned} Q(z) &= N[1-F\{E(z)/(1-t)w\}] \\ Q^*(z) &= N^*[1-F^*\{E^*(z)/(1-t^*)\}]. \end{aligned} \tag{8}$$

A further equilibrium condition reflects the clearing of factor markets. In South the public good sector demands $a_g G$ units of labor and since South produces goods in $[0, \widetilde{z}]$ and exports goods in $[0, \widetilde{z}^*]$, southern labor market equilibrium requires:[8]

$$L = N\int_0^\infty h dF(h) \quad = a_g G + \int_0^{\widetilde{z}} a(z)Q(z)dz + \int_0^{\widetilde{z}^*} a(z)Q^*(z)dz. \tag{9}$$

[7] We abstract from optimal tariff-tax rate competition between countries.

[8] We assume that public good provision does not entail scale economies so that the unit labor requirement of public good provision is constant: a_g in South and a_g^* in North.

The left hand side of (9) represents South's effective labor supply and the right hand side is the derived demand for its labor, consisting of the demand for public goods and competitive goods. Combining (9) and (8), and using (5), labor market equilibrium in South can be expressed as (see Stibora and de Vaal 2007)

$$\begin{aligned} wL = \ & wa_g G + N \textstyle\int_0^\infty \min\{(1-t)wh, E(\widetilde{z})\}\, dF(h) \\ & + \frac{N^*}{\tau^*} \textstyle\int_0^\infty \min\{(1-t^*)h^*, E^*(\widetilde{z}^*)\}\, dF^*(h^*), \end{aligned} \tag{10}$$

where

$$\begin{aligned} E(\widetilde{z}) &= \int_0^{\widetilde{z}} p(s)ds = \int_0^{\widetilde{z}} wa(s)ds \\ E^*(\widetilde{z}^*) &= \int_0^{\widetilde{z}^*} p^*(s)ds = \tau^* \int_0^{\widetilde{z}^*} wa(s)ds. \end{aligned} \tag{11}$$

Equation (10) shows that a southern household spends $\min\{(1-t)wh, E(\tilde{z})\}$ on southern goods while a northern household spends $(1/\tau^*) \min\{(1-t^*)h^*, E^*(\tilde{z}^*)\}$ on southern goods.

Similarly, $a_g^* G^*$ units of northern labor are used to produce the public good. Additionally, North produces goods in $[\widetilde{z}^*, \infty)$ of which $[\widetilde{z}, \infty)$ are possibly exported. Market clearing implies

$$L^* = \ a_g^* G^* + \textstyle\int_{\widetilde{z}^*}^\infty a^*(z) Q^*(z)dz + \int_{\widetilde{z}}^\infty a^*(z)Q(z)dz, \tag{12}$$

$$\begin{aligned} L^* = \ & a_g^* G^* + \frac{N}{\tau} \textstyle\int_0^\infty \max\{(1-t)wh - E(\widetilde{z}), 0\}\, dF(h) \\ & + N^* \textstyle\int_0^\infty \max\{(1-t^*)h^* - E^*(\widetilde{z}^*), 0\}\, dF^*(h^*). \end{aligned} \tag{13}$$

A southern household spends $(1/\tau) \max\{(1-t)wh - E(\widetilde{z}), 0\}$ on northern products, while a northern household spends $\max\{(1-t^*)h^* - E^*(\widetilde{z}^*), 0\}$ on northern goods.

Since both economies are linked by trade, Walras' law implies that equations (10) and (13) can be replaced by the equivalent statement that the world goods market is in equilibrium if the value of South's exports equals its value of imports. This yields

$$\begin{aligned} & \frac{N^*}{\tau^*} \textstyle\int_0^\infty \min\left\{\frac{(1-t^*)h^*}{w}, \tau^* \int_0^{\widetilde{z}^*} a(s)ds\right\} dF^*(h^*) = \\ & \frac{N}{\tau} \textstyle\int_0^\infty \max\left\{(1-t)h - \int_0^{\widetilde{z}} a(s)ds, 0\right\} dF(h), \end{aligned} \tag{14}$$

where we have used (11) to substitute for $E(\widetilde{z})$ and $E^*(\widetilde{z}^*)$. In the sequel, we will refer to (14) as the trade balance condition and note that it is a representation of the demand side only.

Finally, we have to specify government balance. The public good G is financed with an income tax of size t and by tariff revenues TR. Both the income tax and the tariff rate are set by the government, which also collects the taxes and hires labor to produce public goods. We assume that the collection of taxes is costly and introduce efficiency parameters to account for that. More specifically, we assume that of all income taxes (import tariffs) collected, a share Γ_t (Γ_{TR}) can be used to provide public goods. Higher values for Γ_t and Γ_{TR} imply more efficiency in generating government revenue. Both parameters are strictly between zero and one, implying that there is always some room for public goods provision, but also that collecting taxes always incur efficiency costs. Naturally, Γ_t and Γ_{TR} need not be the same.

Assuming that government's budgets are always balanced requires for South that

$$wa_gG = \Gamma_t Ltw + \Gamma_{TR}TR, \tag{15}$$

and for North that

$$a_g^*G^* = \Gamma_t^* L^* t^* + \Gamma_{TR}^* TR^*, \tag{16}$$

where $0 < \Gamma_t^*, \Gamma_{TR}^* < 1$ are the efficiency parameters for the northern government. Parametrizing the efficiency of alternative sources of government income also captures the notion that developing countries typically lack efficient alternative mechanisms to raise income than import taxation. Ebrill *et al.* (1999), for example, report that the share of trade taxes to total revenue is inversely related to the level of development, with many low-income countries earning half or more of their revenue from trade taxes.

Making use of (8), we express southern tariff revenue as

$$\begin{aligned} TR &= (\tau - 1)\int_{\widetilde{z}}^{\infty} a^*(z)Q(z)dz \\ &= \frac{(\tau-1)}{\tau}N\int_0^{\infty} \max\{(1-t)wh - E(\widetilde{z}), 0\}dF(h). \end{aligned}$$

Similarly, the expression for the northern tariff revenue is

$$TR^* = \frac{(\tau^*-1)}{\tau^*}N^*\int_0^{\infty} \min\{(1-t^*)h^*, E^*(\widetilde{z}^*)\}dF^*(h^*).$$

Regarding southern tariff revenue, TR, we note that it is only positive if there are southern households who import, that is if the income of (some of the) southern households exceeds $E(\widetilde{z})$. Otherwise $TR = 0$. By contrast, tariff revenue for the northern government, TR^*, are always positive, since northern households always import (some of) the lower-indexed goods produced by South.

Equations (2), (3), (6), (7), (14), (15) and (16) define a system of equations jointly determining the equilibrium values of $\widetilde{z}$, $\widetilde{z}^*$, w, n, n^*, G, and G^*.

3 Trade liberalization and its fiscal implications: homogeneous populations

Since we are primarily interested in studying the effects of trade liberalization and its fiscal implication between countries with significant income differences we assume throughout the paper that South has an absolute disadvantage in all industries, that is, $a^*(z) < a(z)$ for all z. This ensures that in equilibrium $w < 1$ and that northern households receive a higher relative wage rate than southern households with the same skill level. In this section we concentrate on the implications of income differences between countries and assume that the population in each country is homogeneous. That is, we momentarily ignore the potential impact income differences within countries might have. This will provide a useful benchmark for the analysis to follow. For this purpose let each household be endowed with one unit of effective labor, i.e., $h = h^* = 1$. We note that these assumptions also imply that households in both countries spend their last unit of income on the higher-indexed goods produced in North.

Under these circumstances the balanced trade condition (14) becomes

$$N^* \int_0^{\tilde{z}^*} a(s)ds = \frac{N}{\tau}[(1-t) - \int_0^{\tilde{z}} a(s)ds] \tag{17}$$

provided that

$$w < \overline{w} \equiv \left[1 + \frac{N^*\tau}{N}\right] \frac{(1-t^*)}{\tau^*} \left[(1-t) - \int_{\tilde{z}^*}^{\tilde{z}} a(s)ds\right]^{-1}. \tag{18}$$

Equation (17) characterizes the trade balance condition if w is sufficiently small in equilibrium, that is: $w < \overline{w}$.[9] The trade balance condition therefore is independent of w and the factor terms of trade only exerts an indirect impact through its effect on $\tilde{z}^*$ and $\tilde{z}$ since all households are rich enough to afford the higher-ranked northern goods. A lower relative wage in South would decrease the purchasing power of southern households, but the lower spending this generates only affects northern production. Similarly, the purchasing power of northern households increases, which enhances spending on northern goods. As there is balanced trade, these effects on northern production and labor cancel out and no wage adjustment is required to restore balanced trade.

Equation (17) indicates also that the trade balance condition depends on τ and t, but not on τ^* and t^*. Let us consider the asymmetric reliance on the tariff rate as a similar

[9] For $w > \bar{w}$, the trade balance condition becomes positively sloped in (w, z) space as it depends on w. This is the case when only southern households are rich enough to buy northern goods, which we will however not consider here.

argument applies to the tax rate. When all households are rich enough to spend their last unit of income on northern goods, a southern tariff reduction implies that prices of northern goods fall for southern consumers. The concomitant real income gains are spent on northern goods, increasing imports and the demand for northern labor. To preserve trade balance South has to increase its range of production. On the other hand, when northern tariffs fall the real income gains accrue to northern households who also expand their consumption basket toward the higher-ranked goods produced in North. There are no trade balance implications, explaining the absence of τ^* in the trade balance condition. As we will see, this asymmetry in how the tariff rates have an effect on the trade balance will help to explain the asymmetric outcomes of trade liberalization.[10]

For the sake of graphical presentation, we note that for given taxes and relative wage rate, (2) and (3) provide a relationship between $\widetilde{z}^*$ and $\widetilde{z}$ that must be satisfied. Since $A(z)$ is strictly decreasing in z, we can express $\widetilde{z}^*$ as a function of $\widetilde{z}$, specifically,

$$\widetilde{z}^* = A^{-1}\left[\tau A(\widetilde{z})\tau^*\right].$$

Inserting this in (17) the balanced trade condition becomes a function of $\widetilde{z}$ only:[11]

$$N^* \int_0^{A^{-1}[\tau A(\widetilde{z})\tau^*]} a(s)ds = \frac{N}{\tau}[(1-t) - \int_0^{\widetilde{z}} a(s)ds], \tag{19}$$

which is the relation that is plotted in $(w,\, z)$ space in the upper panels of Figure 1a and 1b together with the conditions of efficient production (2) and (3). The government balanced budget conditions replace (15) and (16) by

$$a_g G = \Gamma_t N t + \Gamma_{TR} \frac{(\tau-1)}{\tau} N \left[(1-t) - \int_0^{\widetilde{z}} a(s)ds\right] \tag{20}$$

for South and

$$a_g^* G^* = \Gamma_t^* N^* t^* + \Gamma_{TR}^* (\tau^* - 1) N^* \int_0^{\widetilde{z}^*} wa(s)ds, \tag{21}$$

for North. South's budget constraint does not directly depend on w, simply because both the cost and revenue side are linearly related to wages. Of course, w has an indirect impact through its effect on $\widetilde{z}$, $\widetilde{z}^*$ and n^*. We can depict both conditions in (G, z) space. In the lower panel of Figure 1a, (20) is plotted as the downward sloping line GG: the larger is the range of goods South produces, the smaller the imports and the concomitant tariff revenue,

[10] As shown in Stibora and de Vaal (2007), the asymmetry in the tariff rate does not appear when tariff revenues are redistributed back to households.

[11] Likewise, we could have expressed equation (19) as a function of $\widetilde{z}^*$only, with obviously no consequences for the analysis whatsoever. For a similar analysis see Obstfeld & Rogoff (1999).

reducing the provision of public good in South. In the lower panel of Figure 1b, (21) is shown as the positively sloped line G^*G^* for given w. The larger is the range of northern imports the higher the tariff revenues the government is able to collect and hence the provision of publicly provided good in the North.

Figures 1a and 1b illustrate the determination of $\widetilde{z}$, $\widetilde{z}^*$, w, G and G^* for the initial tariff distorted equilibrium. In the upper panel of both figures, the intersection of the TB and $A(z)\tau$ schedules determines $\tilde{z}$ and w, while the $A(z)/\tau^*$ schedule is used to read off the equilibrium $\widetilde{z}^*$. The lower panel uses (20) to read off the corresponding equilibrium value for G (Figure 1a) and similarly the equilibrium value for G^* from (21) (Figure 1b).

[please insert Figure 1a and b about here]

In the initial tariff ridden equilibrium, the highest-indexed good consumed by a household in South and North (n and n^*) is obtained from (6) and (7):

$$\int_0^{\widetilde{z}} wa(s)ds + \tau \int_{\widetilde{z}}^{n} a^*(s)ds = (1-t)w \tag{22}$$

$$\tau^* \int_0^{\widetilde{z}^*} wa(s)ds + \int_{\widetilde{z}^*}^{n^*} a^*(s)ds = (1-t^*). \tag{23}$$

If we allow tax rates to be different, the fact that $w < 1$ ensures that (22) and (23) satisfy $\widetilde{z}^* < \widetilde{z} < n < n^*$, unless the northern tax rate is considerably above South's. In this case the higher real wage rate for northern consumers is more than compensated by higher taxation, leaving northern consumers with a lower net real income. For the remainder of the analysis we exclude this possibility, making northern households richer than southern households in equilibrium. Thus southern households consume all the goods produced in South plus some northern goods ($\widetilde{z} < n$), while northern households consume all the goods southern households consume plus some ($n < n^*$). From the combination of the assumption on technology ($A(z)$ is strictly decreasing in z), the well-defined ordering of goods ($b(z)/p(z), b(z)/p^*(z)$ are strictly decreasing in z) and $a^*(z) < a(z)$ it follows that South specializes in goods whose demand is characterized by low income elasticities while North specializes in goods with high income elasticities.

3.1 Trade reform and the provision of public goods

Unilateral Tariff Reductions by South

Consider first the case in which South reduces uniformly its tariff rate on imports from North, that is $d\tau < 0$ while keeping τ^* and income taxes unchanged. This policy change affects the supply as well as the demand side. In terms of Figure 1, the $A(z)\tau$ schedule shifts downwards and the TB schedule shifts to the right on impact. The impact effect on the supply side is that lower tariffs imply that southern industries that compete closely with northern industries cannot compete anymore with imports from the North. Consequently, $\widetilde{z}$ falls, unless w falls equiproportionally. The impact effect on the demand side is that lower tariffs increase the value of southern imports. The lower price of northern goods in the South increases real income in two ways. First, real income gains are realized from lower price for all northern goods previously imported. Second, from importing previously not traded goods. Southern households spend these real income gains on higher-ranked goods produced by northern firms. To preserve trade balance the range of goods South produces has to increase, which also requires w to fall. The responds in the range of imports as a result of price changes is in stark contrast to the traditional literature where the revenue impact of a tariff change is determined by the elasticity of both demand and supply (Blinder 1981).

The general equilibrium effects of lower southern tariffs are obtained by total differentiation of (2), (3), (15), and (17). This yields

$$\frac{\widehat{\widetilde{z}}}{\widehat{\tau}} = -\frac{\tau\left[\zeta(\widetilde{z}^*)\frac{N}{w}\int_{\widetilde{z}}^{n} a^*(s)ds - N^*a(\widetilde{z}^*)\widetilde{z}^*\right]}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]} \gtrless 0 \tag{24}$$

$$\frac{\widehat{w}}{\widehat{\tau}} = \zeta(\widetilde{z}^*)\frac{\frac{N}{w}\left[\zeta(\widetilde{z})\tau\int_{\widetilde{z}}^{n} a^*(s)ds + wa(\widetilde{z})\widetilde{z}\right]}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]} > 0 \tag{25}$$

$$\frac{\widehat{\widetilde{z}}^*}{\widehat{\tau}} = -\frac{\frac{N}{w}\left[\zeta(\widetilde{z})\tau\int_{\widetilde{z}}^{n} a^*(s)ds + wa(\widetilde{z})\widetilde{z}\right]}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]} < 0. \tag{26}$$

We recall that $\zeta(z) \equiv [\hat{a}(z)/\hat{z} - \widehat{a}^*(z)/\hat{z}] = -\hat{A}(z)/\hat{z}$ denotes the elasticity of South's relative efficiency with respect to $z = \tilde{z}, \tilde{z}^*$, which can take any positive value. A large $\zeta(\widetilde{z}^*)$, for example, implies that South has a strong comparative advantage in lower-indexed goods which are exported, implying that changes in w hardly affect the range of goods exported. It also implies that comparative advantage is diminishing relatively quickly in z. In terms of the upper panels of Figure 1: a lower (higher) value of $\zeta(z)$ implies a flatter (steeper) slope of the $A(z)\tau$ and $A(z)/\tau^*$ curves at z.[12]

A unilateral reduction of South's tariff thus leads unambiguously to the deterioration of South's terms of trade and to an increase in the range of goods South exports. The effect on $\widetilde{z}$ is not clear and crucially depends on the value of $\zeta(\widetilde{z}^*)$. While lower southern tariffs by itself would imply a lower $\tilde{z}$, this effect is countered by a fall in w, which is larger the larger $\zeta(\widetilde{z}^*)$.

[12] Alternatively, the more equal unit labor requirements are and hence the more similar countries become with respect to the technology they use, the flatter the $A(z)$ schedule.

Moreover, the fall in South's factor terms of trade enhances the competitiveness of South's export industries. Consequently, the change in $\tilde{z}$ that is required to restore trade balance equilibrium also depends on how the change in w enhances the competitiveness of South's exporting industries. The larger South's comparative advantage in its export market, that is the larger is $\zeta(\widetilde{z}^*)$, the smaller the impact the lower w will have on South's export range and the larger the required change in $\widetilde{z}$. For sufficiently low $\zeta(\widetilde{z}^*)$, therefore, the overall effect of lower southern tariffs on $\tilde{z}$ might result in a reduction in the range of goods South produces. If the comparative advantage of South's exporting industries is weak, a fall in w will imply a large increase in South's range of exports and a fall of $\tilde{z}$ is consistent with restoring trade balance equilibrium. In terms of Figure 1, the TB schedule shifts further to the right leading northern firms to relocate to the South.

The tariff change and concomitant terms of trade effect also affect the range of goods households are able to consume. While the overall impact on the range of goods consumed is unambiguously positive ($Na^*(n)n\widehat{n}+N^*a^*(n^*)n^*\widehat{n}^* > 0$), these gains need not be evenly distributed. Inserting (24)-(26), into (22) and (23) we obtain for North

$$a^*(n^*)n^*\frac{\widehat{n}^*}{\widehat{\tau}} = -\tau^*\left[\int_0^{\widetilde{z}^*} wa(s)ds\right]\frac{\widehat{w}}{\widehat{\tau}} < 0,$$

and for South

$$\begin{aligned} a^*(n)n\widehat{n} &= \left[\int_{\widetilde{z}}^{n} a^*(s)ds\right](\widehat{w}-\widehat{\tau}) \\ &= \left[\int_{\widetilde{z}}^{n} a^*(s)ds\right]\zeta(\widetilde{z})\frac{\tau\left[\frac{N}{w}\zeta(\widetilde{z}^*)\int_{\widetilde{z}}^{n} a^*(s)ds - N^*a(\widetilde{z}^*)\widetilde{z}^*\right]}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}+\zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]}\widehat{\tau} \lesseqgtr 0. \end{aligned}$$

For North, the terms of trade improvement leads northern consumers unambiguously to expand their range of goods adding higher-ranked goods of lower priority to their consumption basket. By contrast, lower southern tariffs affect the range of goods southern households are able to consume in two opposing ways. The reduction in τ increases real income and induces southern households to expand their consumption set towards higher-ranked goods at impact. The subsequent deterioration of South's terms of trade, however, mitigates the benefit of lower tariffs. For sufficiently large $\zeta(\widetilde{z}^*)$, the adverse terms of trade change is so large that it more than offsets the primary effect of the tariff reduction and southern households shift expenditure from higher-ranked goods toward lower-ranked goods ($dn < 0$), and reducing its range of imports.

The impact of lower southern tariffs on public good provision shifts the GG schedule to the left and upwards in the lower panel of Figure 1a. Two effects play a role. First, the direct effect of lower τ reduces government revenue in the South as tariff revenues decline, holding constant the range of imports. This is the price effect of lower tariffs. Second, lower tariffs increase real income of southern households that is spend on goods produced by North, and thus may increase South's range of imports. This is the volume effect of lower tariffs in our analysis. The change in the terms of trade has no direct bearing on public good provision, as it reduces costs similarly, see (20). The effects become clearer by total differentiation of (15), while making use of (24) and (25). This yields

$$\begin{aligned}\frac{\widehat{G}}{\widehat{\tau}} = & \frac{\Gamma_{TR}}{wa_g G} N \int_{\widetilde{z}}^{n} a^*(s)ds \\ & + \frac{\Gamma_{TR}}{wa_g G} \frac{(\tau - 1)\, Na(\widetilde{z})\widetilde{z} \left[\zeta(\widetilde{z}^*) N^* \int_0^{\widetilde{z}^*} wa(s)ds - N^* a(\widetilde{z}^*)\widetilde{z}^* \right]}{[\zeta(\widetilde{z}^*) Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z}) N^* \tau a(\widetilde{z}^*)\widetilde{z}^*]} \lesseqgtr 0. \end{aligned} \tag{27}$$

The first term on the right-hand-side of (27) is the negative price effect of lower tariffs; the second term indicates by how much tariff revenues change due to the change in the range of goods imported. The second terms is positive (negative) if the range of imports decreases (increases) and therefore either adds to (counters) the negative effect of lower tariffs on the provision of public goods. Combining the results on dn and $d\widetilde{z}$, the range of imports changes according to

$$\frac{(dn - d\tilde{z})}{\widehat{\tau}} \gtrless 0 \quad \text{iff} \quad \zeta(\widetilde{z}^*) \gtrless \frac{a(\widetilde{z}^*)\widetilde{z}^*}{\int_0^{\widetilde{z}^*} a(s)ds}.$$

Consequently, for large enough $\zeta(\widetilde{z}^*)$ a unilateral reduction of southern tariffs lowers not only the range of goods South imports and but leads also to cuts in the provision of its public good G.

Similarly, the effect on the publicly provided good in North can be gauged from

$$\frac{\widehat{G}^*}{\widehat{\tau}} = \frac{\Gamma_{TR}^*(\tau^* - 1)N \left[\zeta(\widetilde{z})\tau \int_{\widetilde{z}}^{n} a^*(s)ds + wa(\widetilde{z})\widetilde{z} \right]}{\tau a_g^* G^* \left[\zeta(\widetilde{z}^*) Na(\widetilde{z})\tilde{z} + \zeta(\widetilde{z}) N^* \tau a(\widetilde{z}^*)\widetilde{z}^* \right]} \left[\zeta(\widetilde{z}^*) N^* \int_0^{\widetilde{z}^*} a(s)ds - N^* a(\widetilde{z}^*)\widetilde{z}^* \right] \lesseqgtr 0.$$

Of course, there is no direct effect of lower southern tariffs, holding constant the range of imports. Consequently, the effect on North's public good provision solely depends on how its tariff revenues change due to changes in its range of imported goods $\widetilde{z}^*$ via the induced change in w and the changes in import value via w. The former is represented by $N^* a(\widetilde{z}^*)\widetilde{z}^*$ in the bracketed term on the right-hand-side, while the latter is represented by $N^* \int_0^{\widetilde{z}^*} a(s)ds$. As before, which terms dominates depends on the magnitude of $\zeta(\widetilde{z}^*)$. In terms of Figure 1b, the G^*G^*-schedule shifts upwards as a result of the fall in w. This shift is larger the larger is $\zeta(\widetilde{z}^*)$.

Unilateral Tariff Reductions by North

Let us now consider the effect of unilateral tariff reductions by the North, that is $d\tau^* < 0$, *ceteris paribus*. This shifts the $A(z)/\tau^*$ upwards with the vertical TB condition unperturbed at impact. Hence w increases while $\widetilde{z}$ decreases.[13] Total differentiation of the equations involved yields

$$\frac{\widehat{\widetilde{z}^*}}{\widehat{\tau}^*} = -\frac{Na(\widetilde{z})\widetilde{z}}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]} < 0 \tag{28}$$

$$\frac{\widehat{w}}{\widehat{\tau}^*} = -\frac{\zeta(z)N^*\tau w a(\widetilde{z}^*)\widetilde{z}^*}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]} < 0 \tag{29}$$

$$\frac{\widehat{\widetilde{z}}}{\widehat{\tau}^*} = \frac{N^*\tau w a(\widetilde{z}^*)\widetilde{z}^*}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]} > 0. \tag{30}$$

A decline in τ^* reduces the price of southern goods, holding constant imports, and unambiguously increases the range of products North imports ($\widetilde{z}^*$ increases) as (some) northern firms lose their competitive edge. To prevent this from happening, North's factor terms of trade have to fall (w increases) thereby reducing the range of goods South produces ($\widetilde{z}$ falls). This leads to further adjustments through the trade balance condition yielding the general equilibrium effects represented in (28)-(30).

Given these changes, the overall impact on the range of goods consumed is unambiguously positive, with both countries benefitting. For northern households, we obtain

$$\begin{aligned} a^*(n^*)n^*\widehat{n}^* &= -\tau^* \int_0^{\widetilde{z}^*} wa(s)ds(\widehat{w} + \widehat{\tau}^*) \\ &= -\frac{\tau^*\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}\left[\int_0^{\widetilde{z}^*} wa(s)ds\right]}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]}\widehat{\tau}^* > 0, \end{aligned}$$

indicating that the initial gain from lower tariffs is powerful enough to dominate the subsequent terms of trade deterioration. Likewise, the range of goods a southern household is able to consume increases due to the improvement in South's terms of trade:

$$a^*(n)n\widehat{n} = -\frac{\zeta(z)N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\left[\int_{\widetilde{z}}^{n} a^*(s)ds\right]}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]}\widehat{\tau}^* > 0.$$

The impact of lower northern tariffs on public good provision in North is given by

$$\begin{aligned} \frac{\widehat{G}^*}{\widehat{\tau}^*} &= \frac{\Gamma^*_{TR}}{a^*_g G^*} N^*\tau^* \int_0^{\widetilde{z}^*} wa(s)ds \\ &\quad -\frac{\Gamma^*_{TR}}{a^*_g G^*}\frac{(\tau^*-1)\,N^*\tau a(\widetilde{z}^*)\widetilde{z}^*}{[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*]}\left[Na^*(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\int_0^{\widetilde{z}^*} wa(s)ds\right] \lesseqgtr 0. \end{aligned}$$

The price effect of lower τ^* on government revenue in North is given by the first term on the right-hand-side. As before, lower tariffs imply less public good provision on this account.

[13] This is most easily seen when the TB condition is expressed in terms of $\widetilde{z}^*$, instead of $\widetilde{z}$. In this case, the intersection of the TB condition with the $A(z)/\tau^*$ curve determines $\widetilde{z}^*$ and w, while $\widetilde{z}$ can be read off the $A(z)\tau$ curve.

The second term is the volume effect. It enters negatively, since North's range of imports increases unambiguously when it cuts tariffs.

The effect of lower northern tariffs on public good provision in South is unambiguously positive. Since the range of goods South imports expands — n goes up while $\tilde{z}$ goes down — tariff revenues increase. Formally,

$$\frac{\widehat{G}}{\widehat{\tau}^*} = -\frac{\Gamma_{TR}(\tau-1)Na(\widetilde{z})\widetilde{z}N^*\tau a(\widetilde{z}^*)\widetilde{z}^*}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}+\zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]a_g G} < 0.$$

When we compare the results of a unilateral tariff reduction by either of the two countries, we see that there are some striking differences. North gains from lower tariffs on southern imports in terms of an increase in the range of goods its households are able to purchase. South, on the other hand, may lose from liberalizing trade unilaterally with North. This is due to the asymmetric demand response. The income gains of lower southern tariffs on northern goods do not stimulate demand for southern goods because South specializes in goods with low income elasticity in demand. Lower τ leads to a loss of southern industries which compete directly with northern firms, thereby reducing demand for its labor. In order to preserve labor market equilibrium, South has to export a larger range of goods, requiring it to move into industries in which it has weak comparative advantage. This may lead to an adverse change in its terms of trade. The terms of trade deterioration could be so large (large $\zeta(\widetilde{z}^*)$) that the direct income gains from lower tariffs are more than compensated and consequently, the range of goods southern households are able to purchase falls. The concomitant fall in the range of goods South's imports, reduces tariff revenues and thus the provision of public goods.

North, the rich country, on the other hand, cannot lose from lower tariffs on imports from South, because it specializes in goods with high income elasticity in demand. Lower τ^* leads to a loss of northern industries directly competing with southern industries, reducing the demand for northern labor, and thus requiring a deterioration in its terms of trade to preserve labor market equilibrium. At the same time, the trade policy reduces the prices of lower-ranked goods, bringing about real income gains for northern consumers which are exclusively spent in North, increasing the demand for northern labor. As it turns out, the gain from higher real income outweighs the adverse terms of trade effect brought about by the loss of industries so that northern household are able to purchase a larger range of goods. Continuous trade liberalization leads to the emergence of new industries in the North as $dn^* > 0$. The real income gains of northern households provide the economic climate for new industries to be developed in North. While new industries appear in North, some older northern industries relocate to the South.

Multilateral Tariff Reductions

Let us consider the consequences of multilateral tariff reductions, that is $d\tau/\tau = d\tau^*/\tau^* \equiv d\tau_M/\tau_M < 0$. This is an interesting exercise in light of the collapse of the latest multilateral round of trade negotiations, the so-called Doha round. While the overall gains from reciprocal trade liberalization are unambiguously positive, that is, $Na^*(n)n\widehat{n}+N^*a^*(n^*)n^*\widehat{n}^* > 0$, they need not be evenly distributed. By adding the previous effects of unilateral reductions for τ and τ^*, we obtain

$$\frac{\widehat{w}}{\widehat{\tau}_M} = \frac{\left[\zeta(\widetilde{z}^*)\zeta(\widetilde{z})N^*\tau\int_0^{\widetilde{z}^*} a(s)ds + \zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} - \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]}$$

$$\frac{\widehat{\widetilde{z}}}{\widehat{\tau}_M} = -\frac{N^*\tau\left[\zeta(\widetilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds - 2a(\widetilde{z}^*)\widetilde{z}^*\right]}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]}$$

$$\frac{\widehat{\widetilde{z}}^*}{\widehat{\tau}_M} = -\frac{\left[\zeta(\widetilde{z})N^*\tau\int_0^{\widetilde{z}^*} a(s)ds + 2Na(\widetilde{z})\widetilde{z}\right]}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]} < 0$$

$$\frac{a^*(n)n\widehat{n}}{\widehat{\tau}_M} = \frac{\zeta(\widetilde{z})N^*\tau\left[\int_{\widetilde{z}}^n a^*(s)ds\right]\left[\zeta(\widetilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds - 2a(\widetilde{z}^*)\widetilde{z}^*\right]}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]}$$

$$\frac{a^*(n^*)n^*\widehat{n}^*}{\widehat{\tau}_M} = -\frac{\tau^*\zeta(\widetilde{z}^*)\left[\zeta(\widetilde{z})N^*\tau\int_0^{\widetilde{z}^*} wa(s)ds + 2Nwa(\widetilde{z})\widetilde{z}\right]\left[\int_0^{\widetilde{z}^*} a(s)ds\right]}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]} < 0.$$

While multilateral tariff reductions unambiguously increase the range of goods South exports, $d\widetilde{z}^* > 0$, and the range of goods northern households are able to consume, $dn^* > 0$, they exert an ambiguous effect on w, $\widetilde{z}$, and n. This is because our framework does not impose any restriction on $\zeta(\widetilde{z})$ and $\zeta(\widetilde{z}^*)$. South's terms of trade deteriorate if $\zeta(\widetilde{z}^*) > N^*\tau a(\widetilde{z}^*)\widetilde{z}^*/N^*\tau\int_0^{\widetilde{z}^*} a(s)ds$. If $\zeta(\widetilde{z}^*) > 2N^*\tau a(\widetilde{z}^*)\widetilde{z}^*/N^*\tau\int_0^{\widetilde{z}^*} a(s)ds$, the deterioration of the terms of trade is so large that southern households reduce the range of goods they are able to consume, $dn < 0$, thereby increasing the range of goods South produces, $d\widetilde{z} > 0$.[14] As a consequence, North would reap all benefits of reciprocal trade liberalization.

The effect on public good provision in both countries is given by

$$\frac{\widehat{G}}{\widehat{\tau}_M} = \frac{\Gamma_{TR}\left[N\int_{\widetilde{z}}^n a^*(s)ds\right]}{wa_gG} + \frac{\Gamma_{TR}}{wa_gG}\frac{(\tau-1)\,NN^*wa(\widetilde{z})\widetilde{z}}{\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right]}\left[\zeta(\widetilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds - 2a(\widetilde{z}^*)\widetilde{z}^*\right]$$

[14] Using l'Hopital, it is straightforward to see that for $\zeta(z^*) \to \infty$, South's terms of trade unambiguously deteriorate, while for $\zeta(z) \to \infty$, the terms of trade deteriorate if $\zeta(z^*)$ satiesfies the condition given in the text.

for South and

$$\begin{aligned}\frac{\widehat{G}^*}{\widehat{\tau}_M} &= \frac{\Gamma_t^* N^* t^*}{a_g^* G^*} + \frac{\Gamma_{TR}^*}{a_g^* G^*} \frac{(\tau^* - 1) N^* w \left[\zeta(\widetilde{z}^*) \int_0^{\widetilde{z}^*} a(s)ds - a(\widetilde{z}^*)\widetilde{z}^*\right]}{[\zeta(\widetilde{z}^*) N a(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z}) N^* \tau a(\widetilde{z}^*)\widetilde{z}^*]} \\ &\times \left\{ \frac{\tau N^* \zeta(\widetilde{z})}{\zeta(\widetilde{z}^*)} \left[\zeta(\widetilde{z}^*) \int_0^{\widetilde{z}^*} a(s)ds - a(\widetilde{z}^*)\widetilde{z}^*\right] + N a(\widetilde{z})\widetilde{z} \right\}\end{aligned}$$

for North. As before, the total impact on public good provision consists of a price effect (the first term in both expressions) and a volume effect (the second term). The latter enters ambiguously for both countries, as now both tariff rates are cut. For South, the volume effect contributes positively to public good provision provided its range of imported goods increases. For North the volume effect is clearly related to the terms of trade effect. If North's terms of trade improve, the volume effect contributes positively to its public good provision.

In general, multilateral tariff reductions are considered to be beneficial for all parties involved as it enhances economic efficiency. This perception is however not validated in our model, in particular for poor countries. Reciprocal tariff reductions may diminish the number of goods southern households are able to consume, while also the provision of public goods may come down. Only if South's comparative advantage in its export markets does not diminishes too quickly (i.e. $\zeta(\tilde{z}^*)$ is low) will n increase, while the impact on G is then also less detrimental. For households in the rich North, by contrast, multilateral tariff reductions are beneficial on account of the clear increase in n^*, while households may lose because of less public good provision.

Our analysis also shows that reciprocal tariff reductions are a better option for South than to unilaterally liberalize trade. The condition that settles whether or not South gains in terms of the highest good they are able to consume when cutting tariffs is less restrictive under the multilateral regime than under a regime where South cuts tariffs unilaterally. Households in North, by contrast, would rather have their government to cut tariffs unilaterally. Even though the effect on the range of goods they can consume is qualitatively the same (n^* goes up under both regimes), unilaterally cutting tariffs is less detrimental to public good provision in North. As such policy would also be highly welcomed by southern households, the interesting conclusion is therefore that unilateral trade liberalization by rich countries is to be preferred over any other trade liberalization scheme.

3.2 *Budget neutral trade liberalization*

In this section we are interested how trade liberalization affects income taxes assuming that the government wants to stabilize the provision of public goods. This is of interest because households are more often directly affected by changes in income taxes in contrast to changes in the provision of the public good. Formally, we fix the level of public expenditure at level $\overline{G}$ and $\overline{G}^*$ and derive the required change in income taxes when countries liberalize trade. Keeping the provision of public goods constant, we can interpret the induced changes in the highest good a household consumes, n and n^*, as changes in utility in the presence of public goods. This analysis is inspired by a recent IMF study of trade liberalization in countries undergoing IMF-supported programs that find a range of fiscal outcomes that let them to conclude that some programs could have targeted more trade reform if greater attention had been given to supporting fiscal policies and to revenue-neutral trade measures (see Ebrill *et al.* 1999). Since this is particular acute for many developing countries, where taxes on trade provide a substantial part of revenues, our analysis concentrates mainly on the South. (see appendix A.2 for details).

[please insert Figure 2 about here]

Figure 2 provides a diagrammatic derivation of the equilibrium values of w, t, and t^* by combining the reduced forms of the trade balance and government budget conditions. The right hand panel of Figure 2 depicts the trade balance condition, TB, and the budget constraint of South, SBC, while the left panel illustrates the budget constraint of North, NBC. The intersection of the TB schedule with the SBC schedule, point A, determines the equilibrium wage rate and the southern tax rate. Given the equilibrium wage rate, the equilibrium northern tax rate can be read off in the left panel, point A'.

The TB schedule is upward sloping since a higher income tax rate reduces income and thus curbs imports. To preserve balanced trade w has to increase to compensate for the loss in imports. The intuition for the slope of the SBC curve is more intricate. A higher income tax rate t directly raises tax revenues by the amount of $\Gamma_t wN$. At the same time, it reduces income and thus the range of imports and thereby lowering tariff revenues by $(\tau - 1)\Gamma_{TR} wN/\tau$, rendering the total effect on revenues ambiguous. For $[\tau\Gamma_t - (\tau - 1)\Gamma_{TR}] > 0$ a higher tax rate raises revenue and w has to fall to retain the public good provision. This is the case depicted in Figure 2. Given τ, this holds for $\Gamma_t \geqq \Gamma_{TR}$ but also for the policy relevant situation where the southern government efficiency of generating revenue from tariffs is mildly higher in comparison to tax revenue. By contrast, if the efficiency of taxing income falls seriously short of tariff collection, a higher income tax would reduce government revenue and w has to go up to compensate by increasing imports and tariff revenues. In this case, the SBC schedule is positively sloped. In our analysis we will rule out such perverse reactions of lowering tariffs on government revenues and assume that $[\tau\Gamma_t - (\tau - 1)\Gamma_{TR}] > 0$. [15]

The slope of the NBC schedule is also ambiguous. A higher northern income tax rate (t^*) increases directly government revenues. To retain a balanced budget, North's terms of trade have to improve (lower w). This shifts northern households' expenditure away from southern goods towards northern goods, reducing tariff revenues on previously traded goods. At the same time, lower w leads to the relocation of some northern firms to the South, increasing the range of goods North imports and hence tariff revenues. For sufficiently large $\zeta(\widetilde{z}^*)$, the former effect dominates and the NBC schedule slopes downward; otherwise NBC slopes upward.

Unilateral Tariff Reductions by South

For given w, lower τ creates a trade deficit. In Figure 2, the TB schedule shifts to the right. Holding imports constant, the immediate effect of a reduction in the southern tariff rate is also to lower revenues. However, lower tariffs lead to real income gains for southern households which are spent on imports and increase tariff revenues. The larger is $\zeta(\widetilde{z})$ the smaller these real income gains, yielding a government budget deficit. For given w, the income tax rate t has to increase. In terms of Figure 2, the SBC schedule shifts to the right, to SBC'.

For the ultimate effect on the factor terms of trade and tax rate, the extent of the rightward shifts of both schedules is important. This depends not only on the degree of comparative advantage southern firms have in their import market, i.e. on the value of $\zeta(\widetilde{z})$, but also on the relative efficiency of the southern government to raise revenue, i.e. Γ_t and Γ_{TR}.

Suppose $\Gamma_t > \Gamma_{TR}$, only a small increase in the southern tax rate is necessary to compensate for the loss in tariff revenues. Since a higher income tax rate also extend the trade deficit South's terms of trade have to deteriorate to restore equilibrium. In terms of Figure 2, the shift in the TB schedule to the right is larger than that of the SBC schedule. The new equilibrium is indicated by point B and concomitantly by $B'(B'')$ for North. South experience an adverse reaction in its terms of trade which is accompanied by a higher tax rate. Formally, we derive

$$\frac{\widehat{w}}{\widehat{\tau}} = \frac{\zeta(\widetilde{z}^*)N\left\{\zeta(\widetilde{z})\left[\Gamma_t - \Gamma_{TR}\right]\tau\int_{\widetilde{z}}^{n} a^*(s)ds + \Gamma_t w a(\widetilde{z})\widetilde{z}\right\}}{wD_B} \tag{31}$$

[15] The is not a very restrictive assumption as the extent to which Γ_t can fall short of Γ_{TR} also depends on the initial tariff rate. The lower the initial tariffs, the less restrictive the condition will be, but even with initially high tariffs the condition will typically hold. For instance, if initial tariffs are 100%, Γ_t may still be half the value of Γ_{TR} for the condition to hold.

$$\frac{\widehat{t}}{\widehat{\tau}} = -\frac{\tau\Gamma_{TR}\left\{\int_{\widetilde{z}}^{n} a^*(s)ds\left[\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*a(\widetilde{z}^*)\widetilde{z}^*\right] - (\tau-1)a(\widetilde{z}^*)\widetilde{z}^*N^*a^*(\widetilde{z})\widetilde{z}\right\}}{twD_B}, \quad (32)$$

with $D_B = \{[\tau\Gamma_t - (\tau-1)\Gamma_{TR}]\,\zeta(\widetilde{z})N^*a(\widetilde{z}^*)\widetilde{z}^* + \zeta(\widetilde{z}^*)\Gamma_t Na(\widetilde{z})\widetilde{z}\} > 0$.

By contrast, if $\Gamma_t < \Gamma_{TR}$, i.e., South is less efficient in using tax revenue than tariff revenues, the degree of comparative advantage firms have in their export markets become important. Assuming that the efficiency gap is not too marked, that is, $[\tau\Gamma_t - (\tau-1)\Gamma_{TR}] > 0$, South's terms of trade might improve for sufficiently large $\zeta(\widetilde{z})$. In keeping the pre-liberalization level of public good provision, the loss of tariff revenues has to be compensated by a considerable increase in the tax rate, which turns the initial trade deficit into a surplus. South's terms of trade have to improve to preserve balanced trade. In terms of Figure 2, the shift in the SBC schedule is farther to the right than the one of the TB schedule. Provided that $\zeta(\widetilde{z}^*)$ is sufficiently large to make the NBC curve upward sloping, for North this implies that it can lower income taxation for a given provision of public goods.

From (31) and (32), equation (22) becomes

$$\begin{aligned}
a^*(n)n\frac{\widehat{n}}{\widehat{\tau}} &= \int_{\widetilde{z}}^{n} a^*(s)ds\left(\frac{\widehat{w}}{\widehat{\tau}} - 1\right) - \frac{tw}{\tau}\frac{\widehat{t}}{\widehat{\tau}} \\
&= \frac{(\Gamma_t - \Gamma_{TR})\,\zeta(\widetilde{z})\tau N^*\left[\int_{\widetilde{z}}^{n} a^*(s)ds\right]}{D_B}\left\{\zeta(\widetilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds - a(\widetilde{z}^*)\widetilde{z}^*\right\} \\
&\quad + \frac{\Gamma_{TR}N^*a^*(\widetilde{z})\widetilde{z}}{D_B}\left[\zeta(\widetilde{z}^*)\tau\int_0^{\widetilde{z}^*} a(s)ds - (\tau-1)\,a(\widetilde{z}^*)\widetilde{z}^*\right].
\end{aligned}$$

As is apparent from the above expression, the sign of $dn/d\tau$ does not only depend on the relative strength of each country's comparative advantage in its export market, $\zeta(\widetilde{z}^*)$ and $\zeta(\widetilde{z})$, but also on South's efficiency in collecting revenues.

For $\Gamma_t > \Gamma_{TR}$, t is raised and South's terms of trade deteriorate. The magnitude of the fall in w depends on South's degree of comparative advantage in its export market. For $\zeta(\widetilde{z}^*) > a(\widetilde{z}^*)\widetilde{z}^*/\int_0^{\widetilde{z}^*} a(s)ds$, southern firms have to move into industries in which they have a weak comparative advantage, which leads to a deterioration in the factor terms of trade. The range of goods South imports falls and the tax rate has to raise to replace the shortfall in tariff revenues, thereby reducing the range of goods southern households are able to consume, $dn < 0$. North, on the other hand, will reap all of the benefits of liberalization. Conversely, when $\Gamma_t < \Gamma_{TR}$, real income gains from liberalization are larger than the loss caused by the adverse movement in the terms of trade, increasing the range of goods of southern households. This is, however, countered by the increase in the tax rate, rendering the total expression ambiguous.

Unilateral Tariff Reductions by North

Lower northern tariffs generate a trade surplus for South and shifts the TB schedule to the left. Given that the SBC curve is downward sloping, South's terms of trade have to improve to restore equilibrium and the southern income tax rate falls. The better terms of trade for South reduce the range of domestic production, increases imports and the range of goods South exports (higher $\widetilde{z}^*$), allowing the southern government to reduce the tax rate. Formally

$$\begin{aligned}\frac{\widehat{w}}{\widehat{\tau}^*} &= -\frac{\zeta(\widetilde{z})\left[\tau\Gamma_t-(\tau-1)\Gamma_{TR}\right]N^*a(\widetilde{z}^*)\widetilde{z}^*}{D_B}<0\\ \frac{\widehat{t}}{\widehat{\tau}^*} &= \frac{\Gamma_{TR}(\tau-1)a^*(\widetilde{z})\widetilde{z}N^*\tau a(\widetilde{z}^*)\widetilde{z}^*}{twD_B}>0.\end{aligned}$$

As a result, southern household's welfare increases as they are able to purchase a larger range of goods, $dn>0$.

The northern government experiences a budget deficit on impact, provided that $\zeta(\widetilde{z}^*)$ is sufficiently large: the loss in northern revenue dominates the increase in tariff revenues caused by the migration of northern firms to the South and the NBC schedule shifts outward, rendering the effect on northern taxes and utility ambiguous. The effect on the range of goods northern households purchase and hence utility is ambiguous. On the one hand, they gain from lower tariffs but lose from a possible increase in income tax (higher t^*).[16]

Multilateral Tariff Reductions

Finally, when tariffs are cut multilaterally, goods that were previously not traded in both countries become tradeable. Additionally, and as before, imports of southern households increase due to real income gains on previously traded goods. The effect on South's trade balance is not clear and depends on the magnitude of $\zeta(\widetilde{z})$ and $\zeta(\widetilde{z}^*)$. For $\zeta(\widetilde{z}^*)>\zeta(\widetilde{z})$ sufficiently large, South's trade balance slips into deficit and both governments lose tariff revenues. In terms of Figure 2, the TB schedule and the SBC schedule shift to the right (with the TB schedule proportionally more), while the NBC schedule shifts the left. If $\Gamma_t>\Gamma_{TR}$, the income tax increases and South's terms of trade deteriorate to restore equilibrium:

$$\begin{aligned}\frac{\widehat{w}}{\widehat{\tau}_M} &= \frac{(\Gamma_t-\Gamma_{TR})}{D_B}\zeta(\widetilde{z}^*)\zeta(\widetilde{z})\frac{N}{w}\tau\int_{\widetilde{z}}^{n}a^*(s)ds+\frac{\Gamma_t}{D_B}\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}\\ &\quad -\zeta(\widetilde{z})\frac{N^*a(\widetilde{z}^*)\widetilde{z}^*}{D_B}\left[\tau\Gamma_t-\Gamma_{TR}(\tau-1)\right]\end{aligned}$$

[16] The tax rate falls if $\zeta(\widetilde{z}^*)$ is sufficiently small, making NBC'-curve the relevant curve in Figure 2. In that case, the northern government experiences a surplus, shifting the NBC'-curve inward.

$$
\begin{aligned}
\frac{\widehat{t}}{\widehat{\tau}_M} &= -\frac{\Gamma_{TR}}{D_B N t}\left[\zeta(\widetilde{z})N^* a(\widetilde{z}^*)\widetilde{z}^* + \zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}\right]\frac{N}{w}\tau\int_{\widetilde{z}}^{n} a^*(s)ds \\
&\quad + \frac{2(\tau-1)\Gamma_{TR}}{D_B t} a(\widetilde{z})\widetilde{z}N^* a(\widetilde{z}^*)\widetilde{z}^*.
\end{aligned}
$$

As a consequence, southern households experience a welfare loss, $dn < 0$.

4 Trade liberalization and its fiscal implications: heterogeneous populations

We now extend our analysis to allow the income distributions $F(h)$ and $F^*(h^*)$ to be nondegenerate. This is particularly interesting when the income of households with low skill levels is so low in equilibrium that they are not able to purchase higher-indexed goods produced in North. For the sake of concreteness, we therefore assume that there are two types of households in both countries, those with low skill levels (h_L, h_L^*) and those with high skill levels (h_H, h_H^*) which are equal in number. Hence, in South there are $N/2$ households who do not import, so that $n_L < \tilde{z}$, while in North there are $N^*/2$ households who only import, that is $n_L^* < \tilde{z}^*$. For simplification we keep the assumption that allows tax rates to differ between regions but identical within a region.

Since the population in North consists of households which spend their marginal income on southern goods, the trade balance condition will now depend on w. In terms of Figure 1, the $A(\widetilde{z})\tau$ schedule intersects the TB curve at the upward sloping part. Appendix B.1 derives the analytical results for positive tariffs.

4.1 Trade reform and the provision of public goods

Unilateral Tariff Reductions by South

Unilateral trade liberalization by South unambiguously causes South's factor terms of trade to deteriorate and its range of exports to increase, while having an ambiguous effect on the range of goods South produces. Qualitatively these effects are the same as for the homogeneous population case, while also the same condition applies to make South's range of production increase. Quantitatively there are some telling differences though. Introducing local income disparities, for instance, implies that a unilateral cut of southern tariffs has a much lower adverse effect on its terms of trade. Since half of South's population does not import, they are insulated from the effects of tariff changes, which mitigates the required change of w to restore labor market equilibrium. Moreover, in contrast to the homogeneous population case, the range of goods the poor southern population is able to purchase remains unaffected ($dn_L = 0$) while the range of goods the rich southern population is able to consume is ambiguously and is given by

$$a^*(n_H)n_H\widehat{n}_H = \left[\int_{\widetilde{z}}^{n_H} a^*(s)ds\right] \frac{\zeta(\widetilde{z})N^*\tau}{D_H} \left[\zeta(\widetilde{z}^*) \int_{\widetilde{z}}^{n_H} a^*(s)ds - a(\widetilde{z}^*)\widetilde{z}^*\right] \widehat{\tau} < 0,$$

where $D_H = \zeta(\widetilde{z})\zeta(\widetilde{z}^*)N^*\tau \int_0^{n_L^*} a(s)ds + \zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^* > 0$. For $\zeta(\widetilde{z}^*) > a(\widetilde{z}^*)\widetilde{z}^*/\int_{\widetilde{z}}^{n_H} a^*(s)ds$, rich southern households are worse off: the terms of trade change is so large that it more than offsets the primary effect of the tariff reduction. Northern households are better off because improved terms of trade induces both types of household to shift expenditure away from lower-ranked goods toward higher-ranked goods, that is $dn_L^* > 0$ and $dn_H^* > 0$. New industries emerge in North.

The effect on public good provision depends once more on a price effect and a trade volume effect. While the former affects G negatively, the latter effect is ambiguous and depends on the change of South's import range. If $\zeta(\widetilde{z}^*) < a(\widetilde{z}^*)\widetilde{z}^*/\int_0^{\widetilde{z}^*} a(s)ds)$ the import range increases (both n_H and $\tilde{z}$ go up) and the volume effect affects public good provision positively. Formally,

$$\begin{aligned}\frac{\widehat{G}}{\widehat{\tau}} &= \frac{\Gamma_{TR}N}{2wa_gG}\int_{\widetilde{z}}^{n_H} a^*(s)ds \\ &\quad + \frac{(\tau-1)\Gamma_{TR}Na(\widetilde{z})\widetilde{z}\left\{\left[\zeta(\tilde{z}^*)N^*\int_0^{\widetilde{z}^*} a(s)ds - N^*a(\widetilde{z}^*)\widetilde{z}^*\right]\right\}}{2D_h a_g G}.\end{aligned}$$

A unilateral tariff reduction in South also increases the range of goods North imports and more public goods can be produced on account of an increase in import volume. As North's terms of trade improves, however, the price effect on tariff revenues is negative, rendering the overall effect on G^* ambiguous. As with the homogeneous population case the overall effect is positive (negative) if $\zeta(\tilde{z}^*) < (>) a(\widetilde{z}^*)\widetilde{z}^*/\int_0^{\widetilde{z}^*} a(s)ds$.

Unilateral Tariff Reductions by North

This unambiguously increases the range of products North imports ($\widetilde{z}^*$ increases), and some of the previously non-traded goods industries are taken over by South. North's terms of trade deteriorate

$$\frac{\widehat{w}}{\widehat{\tau}^*} = -\frac{1}{\zeta(\widetilde{z})}\frac{\widehat{\widetilde{z}}}{\widehat{\tau}^*} = -\frac{\zeta(\widetilde{z})N^*\tau}{D_H}\left[\zeta(\widetilde{z}^*)\int_0^{n_L^*} a(s)ds + a(\widetilde{z}^*)\widetilde{z}^*\right] < 0. \tag{33}$$

In comparison to the homogeneous population case, (33) also includes a direct demand side effect of lower tariffs. Since the poor faction of North's population only import, lower northern tariffs directly increases demand for southern labor and North's terms of trade must deteriorate more to restore labor market equilibrium. In contrast to the homogeneous population case, the range of goods poor southern households consume is unaffected, $dn_L = 0$, while rich southern households are able to increase their consumption basket, $dn_H > 0$. Similarly, the range of goods both poor and rich northern households are able to consume goes up, i.e. $dn_L^* > 0$, $dn_H^* > 0$. Since the import range of North goes up, public good provision in North increases on account of the volume effect. The price effect is of course negative, leaving the total effect on G^* unclear. As with the homogeneous population case the effect of lower northern tariffs on South's public good provision is unambiguously positive as South's import range goes up.

Multilateral Tariff Reductions

Reciprocal trade reductions increases South's range of exports (higher $\widetilde{z}^*$) and renders the effect on w, $\widetilde{z}$, and n_H ambiguous:

$$\begin{aligned}
\frac{\widehat{\widetilde{z}}}{\widehat{\tau}_M} &= -\tfrac{N^*\tau}{D_H}\left[\zeta(\widetilde{z}^*)\left(\int_0^{\widetilde{z}^*} a(s)ds - \int_0^{n_L^*} a(s)ds\right) - 2a(\widetilde{z}^*)\widetilde{z}^*\right] \\
\frac{\widehat{w}}{\widehat{\tau}_M} &= \tfrac{1}{D_H}\left[\zeta(\widetilde{z})\zeta(\widetilde{z}^*)N^*\tau\int_0^{\widetilde{z}^*} a(s)ds + \zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z} - \zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*\right] \\
a^*(n_H)n_H\frac{\widehat{n}_H}{\widehat{\tau}_M} &= \tfrac{\zeta(\widetilde{z})N^*\tau}{D_H}\left[\int_{\widetilde{z}}^{n} a^*(s)ds\right]\left[\zeta(\widetilde{z}^*)\left(\int_0^{\widetilde{z}^*} a(s)ds - \int_0^{n_L^*} a(s)ds\right) - 2a(\widetilde{z}^*)\widetilde{z}^*\right],
\end{aligned}$$

but leaves poor southern households unaffected, $dn_L = 0$. While the terms of trade effect is comparable to the homogeneous population case the effect on $\widetilde{z}$ and n_H include $\zeta(\widetilde{z}^*)\int_0^{n_L^*} a(s)ds$ as an additional term. This reflects the fact that North's poor population only purchase southern goods in equilibrium, implying that any effect on their real income has additional balance of payments consequences. The range of goods northern households consume expands, i.e., $dn_L^* = dn_H^* > 0$, and new industries emerge in North. The impact on public good provision is ambiguous for both countries. Holding $\zeta(\widetilde{z})$ constant, both South and North experiences a deterioration in the provision of their public goods if $\zeta(\widetilde{z}^*)$ is sufficiently large.

4.2 Budget neutral trade liberalization

Assuming that poor northern households spend their last unit of income on southern goods implies that the trade balance depends additionally on the northern income tax rate t^*. We use Figure 3a and 3b for a diagrammatic analysis of the comparative statics exercise and delegate a formal treatment to appendix B.2. The figures are similar to Figure 2 but differ by depicting the trade balance's dependency on both tax rates explicitly, for example $TB(t_A)$ in Figure 3b. As before, the slope of the SBC schedule is positive, provided that $C \equiv \tau\Gamma_t(h_L + h_H) - (\tau - 1)\Gamma_{TR}h_H > 0$ ruling out perverse reactions of tax increases on government revenue. To reduce clutter on notation, we further assume that $\Gamma_t^* = \Gamma_{TR}^* = \Gamma^*$ so that $C^* \equiv \tau^*\Gamma^*(h_L^* + h_H^*) - (\tau^* - 1)\Gamma^* h_L^* > 0$ regarding the NBC schedule.

Unilateral Tariff Reductions by South

As in the homogeneous case, the effects on w and t are ambiguous. On impact, a drop in τ, leads to twin deficits provided that $\zeta(\widetilde{z})$ is sufficiently large.

[please insert Figure 3a about here]

In terms of Figure 3a, the $TB(t_A^*)$ schedule and the SBC schedule shift to the right on impact, respectively to $TB'(t_A^*)$ and SBC'. To restore southern government budgets t has to increase. As in the homogeneous case, if $\Gamma_t > \Gamma_{TR}$, only a small increase in the income tax rate would be sufficient, which has to be accompanied by a fall in w to restore trade balance:

$$\begin{aligned} \frac{\widehat{w}}{\widehat{\tau}} &= -\frac{C^* t^* t N^2 N^*}{D_{BH} 8(\tau^* - 1)(\tau - 1)\zeta(\widetilde{z})\Gamma_{TR}^*\Gamma_{TR}} \\ &\quad \times \left\{ \left[\Gamma_t(h_L + h_H) - \Gamma_{TR} h_H\right] \zeta(\widetilde{z})\tau \int_{\widetilde{z}}^{n_H} a^*(s)ds + \Gamma_t(h_L + h_H) w a(\widetilde{z})\widetilde{z} \right\}, \end{aligned}$$

where D_{BH} is a complex expression specified in appendix B.2 that is most likely negative.[17] The deterioration in South's factor terms of trade reduces tariff revenues for the northern government on previously imported goods by the rich faction of the population. The tariff revenues paid by the poor northern households also fall but this is exactly compensated by higher imports brought about by the improved terms of trade. But the fall in w also causes some northern firms to migrate to the South, increasing imports and hence tariff revenues for North. The extent of this relocation of firms depends on the value of $\zeta(\widetilde{z}^*)$. For sufficiently large $\zeta(\widetilde{z}^*)$, the increase in trade is small so that northern tariff revenues fall – the downward sloping NBC schedule applies in Figure 3a – and an increase in t^* is required to keep the level of public expenditure unchanged:

$$\frac{\widehat{t^*}}{\widehat{\tau}} = -C_0 \left[\zeta(\widetilde{z}^*) \int_0^{\widetilde{z}^*} a(s)ds - a(\widetilde{z}^*)\widetilde{z}^* \right] \frac{\widehat{w}}{\widehat{\tau}},$$

where $C_0 > 0$ is a collection of parameters. In comparison to the homogeneous case, the higher income tax reduces the spending power of the poor northern consumers who spent their last unit of income on southern goods, who react by reducing imports. In terms of Figure 3a, an increase in t^* from t_A^* to t_B^* shifts the $TB'(t_A^*)$ schedule further downwards to

[17] A sufficient condition for D_{BH} to be negative is that the gap in the northern skill levels is not too marked.

$TB^{'}(t_B^*)$, accentuating the deterioration in South's terms of trade (lower w) and the increase in t. All southern households are worse off ($dn_L < 0$, $dn_H < 0$). If southern firm's comparative advantage in their export market is rather weak, i.e., $\zeta(\widetilde{z}^*) < a(\widetilde{z}^*)\widetilde{z}^*/\int_0^{\widetilde{z}^*} a(s)ds$, the increase in trade is substantial so that northern tariff revenues raise and t^* falls. In this case, the $TB^{'}(t_A^*)$ schedule is shifting upwards, mitigating the fall in w and the increase in t. Consequently, southern households lose less.

By contrast, if $\Gamma_t < \Gamma_{TR}$, the increase in the southern income tax is substantial and the factor terms of trade might improve (higher w) for sufficiently large $\zeta(\widetilde{z})$. Poor southern households are unambiguously worse off due to the higher tax rate, $dn_L < 0$. The welfare change of rich southern households is ambiguous, though. The effect on t^* depends on the degree of comparative advantage southern firms have in their export markets, $\zeta(\widetilde{z}^*)$.

Unilateral Tariff Reductions by North

With lower τ^*, South experiences a trade surplus. Poor northern households expand their consumption basket with southern goods, while rich northern households substitute previously non-traded northern goods for southern imports. In terms of Figure 3b, the TB schedule shifts to $TB^{'}(t_A)$, *ceteris paribus*, improving South's factor terms of trade and raising the northern tax rate.

[please insert Figure 3b1 and 3b2 about here]

The impact on the northern government budget constraint is ambiguous though. The tariff revenues from poor northern households fall while those from rich northern household may either fall or increase, depending on the value of $\zeta(\widetilde{z}^*)$. The smaller is $\zeta(\widetilde{z}^*)$, the smaller the difference in the unit labor requirements in South's export markets, the larger the additional imports that come from the rich northern households. Provided that $\zeta(\widetilde{z}^*)$ is sufficiently small tariff revenues increase, generating a budget surplus on impact. The NBC schedule is upward sloping and shifts to $NCB^{'}$, as indicated Figure 3b1. By this effect, w would increase while the northern tax rate would fall. Taken together, this renders the impact effect on the northern tax rate ambiguous. However, for a sufficiently small $\zeta(\widetilde{z}^*)$, it turns out that the upward shift of the NBC schedule is larger than that of the TB schedule so that the northern tax rate falls on impact. If, by contrast, $\zeta(\widetilde{z}^*)$ is large, lower northern tariffs are bound to decrease government revenues. In this case, the NBC schedule is negatively sloped as in Figure 3b2.[18] The combined impact effect results in a higher relative wage rate while rendering the effect on the northern income tax rate unclear.

[18]For $\zeta(\widetilde{z}^*) > a(\widetilde{z}^*)\widetilde{z}^*/\int_0^{\widetilde{z}^*} a(s)ds$, both the TB and the NBC schedule slope negatively, with NBC schedule being steeper, see Appendix B.2.

Both impact effects point at an improved terms of trade for South, allowing the southern government to reduce income taxes. However, the induced changes in t also entail consequences for the position of the TB schedule. Since t falls to t_B, the TB curve shifts downwards. If $\zeta(\widetilde{z}^*)$ is small, South's factor terms of trade ultimately improve and its income tax rate falls, point C in Figure 3b1. Southern households are better off. The ultimate impact on northern taxes is then also positive. If $\zeta(\widetilde{z}^*)$ is large, however, the effects become ambiguous.

5 Conclusion

In this paper we look at the effects of trade liberalization and its fiscal implications in a North-South setting when preferences are nonhomothetic. We use a two country Ricardian trade model with a continuum of goods which we augment by introducing government sectors that turn revenues from income taxation and import tariffs into a public good. Additionally, households consume a range of indivisible goods supplied by competitive markets. We order these goods according to priority in consumption. We assume that the consumption of each good is satisfied after one unit. The lowest-indexed goods have the highest priority in consumption, whereas the highest-indexed goods have the lowest priority in consumption. All households consume the lower-indexed goods, and when real income increases, they add higher-indexed goods to their consumption baskets, instead of buying more of the goods they already consume. The higher-indexed goods are therefore only affordable by households with sufficiently high income levels. South, the poorer country, has a comparative advantage in the production of lower-ranked goods which all households consume, while North has a comparative advantage in the production of low-priority goods, which household with higher income consume. This implies that the poor (rich) country produces goods with low (high) income elasticities in demand.

The assumption of nonhomotheticity in consumption implies that goods have non-unitary income elasticities and that poor and rich households consume goods in different proportions. In contrast to the standard trade model exposition, the effects of policy changes are therefore not invariant to the income level of the incipient country. Otherwise symmetrical policy interventions may work out asymmetrically.

Take, for example, the case of a unilateral tariff reduction by either North or South. While North gains from lowering its tariffs on southern imports, a similar tariff reduction by South may imply a loss for southern households. The reason is that the income gains of lower southern tariffs on northern goods do not stimulate demand for southern goods because South specializes in goods with low income elasticities in demand.

The dependence of effects on the incipient country's income level also apply to the implications of trade liberalization for the provision of public goods. The direct revenue effect of lowering import tariffs in either country is the same - lower tariffs imply lower tariff revenues for either country - but the volume effects are not. A tariff reduction by North unambiguously increases North's import range and thus raises revenues, while a similar tariff reduction by South has an ambiguous effect on South's import range and concomitant tariff revenues. Consequently, the implications for public good provision of similar tariff reductions are more adverse for poorer countries. Regarding the fiscal implications of tariff reductions there are less clear differences between poor and rich countries. This becomes clear when considering the implications of government budget neutral tariff reductions, investigating required changes in income taxation that secure public good provision at pre-liberalization levels. For both countries, cutting import tariffs typically implies that income taxes should go up. This will be particularly the case when the strength of northern and southern firms' comparative advantage in their respective import markets is strong. If it is weak, import ranges hardly increase upon trade liberalization and income taxes must go up to balance the government budget.

Allowing for nondegenerate income distributions in both countries leaves most of the results qualitatively the same, but quantitatively there are some telling differences. For instance, a unilateral cut of southern tariffs has a much lower adverse effect on its terms of trade. Since half of South's population does not import, they are insulated from the effects of tariff changes, mitigating the required change of its factor terms of trade to restore labor market equilibrium. As a consequence, the effect of trade liberalization on the provision of public goods is mitigated. For North, the effects now also include a direct demand side effect since a faction of the population consumes only southern imports. Lower northern tariffs directly increase demand for southern labor and North's terms of trade must deteriorate by more to restore labor market equilibrium.

Nonhomotheticity in preferences also leads to asymmetric fiscal implications as a result of multilateral tariff reductions. While multilateral tariff reductions are generally considered to be beneficial for all parties involved, this perception is not validated in our model, in particular not for poor countries. Reciprocal tariff reductions may reduce both the number of goods southern households are able to consume as well as the provision of its public good. Only if South's comparative advantage in its export markets is strong will the number of goods consumed increase, making the negative impact on the provision of the public good negligible. For households in the rich North, multilateral tariff reductions unambiguously increase the consumption baskets, while losses incur because of lower public good provision. It also appears that reciprocal tariff reductions are a better option for South than to unilaterally liberalize trade, while households in North would favor their government to cut tariffs unilaterally. Since the latter option would also be favorable to South, unilateral trade liberalization by rich countries is the more viable policy option than any other trade liberalization scheme.

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A Derivations for Homogeneous Population

A.1 Unilateral and multilateral trade liberalization

Total differentiation of (17), (20), and (21), and making use of (22), (23), (2), and (3), yields:

$$\left[-\frac{\tau N^* a(\tilde{z}^*)\tilde{z}^*}{\zeta(\tilde{z}^*)} - \frac{Na(\tilde{z})\tilde{z}}{\zeta(\tilde{z})}\right]\widehat{w} = \left[\frac{\tau N^* a(\tilde{z}^*)\tilde{z}^*}{\zeta(\tilde{z}^*)}\right]\widehat{\tau}^* + \left[-\frac{Na(\tilde{z})\tilde{z}}{\zeta(\tilde{z})} - \tau N^*\int_0^{\tilde{z}^*} a(s)ds\right]\widehat{\tau}$$

$$-\frac{Nwa(\tilde{z})\tilde{z}}{\zeta(\tilde{z})}\widehat{w} + \frac{\tau w a_g G}{\Gamma_{TR}(\tau-1)}\widehat{G} = \left[\left(\frac{1}{\tau-1}\right)N\tau\int_{\tilde{z}}^{n} a^*(s)ds - \frac{Nwa(\tilde{z})\tilde{z}}{\zeta(\tilde{z})}\right]\widehat{\tau}$$

$$\frac{N^*\tau^* w}{\zeta(\tilde{z}^*)}\left[\zeta(\tilde{z}^*)\int_0^{\tilde{z}^*} a(s)ds - a(\tilde{z}^*)\tilde{z}^*\right]\widehat{w} - \frac{a_g^* G^* \tau^*}{\Gamma_{TR}^*(\tau^*-1)}\widehat{G}^* = \left[\begin{array}{c}\frac{N^*\tau^* w a(\tilde{z}^*)\tilde{z}^*}{\zeta(\tilde{z}^*)} \\ -\frac{\tau^*}{\tau^*-1}N^*\tau^*\int_0^{\tilde{z}^*} wa(s)ds\end{array}\right]\widehat{\tau}^*$$

In matrix form:

$$\begin{bmatrix} w_1 & 0 & 0 \\ w_2 & G_2 & 0 \\ w_3 & 0 & G_3^* \end{bmatrix}\begin{bmatrix} \widehat{w} \\ \widehat{G} \\ \widehat{G}^* \end{bmatrix} = \begin{bmatrix} \tau_1 & \tau_1^* \\ \tau_2 & 0 \\ 0 & \tau_3^* \end{bmatrix}\begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix}$$

where

$$w_1 = -\frac{\zeta(\tilde{z}^*)Na(\tilde{z})\tilde{z} + \zeta(\tilde{z})N^*\tau a(\tilde{z}^*)\tilde{z}^*}{\zeta(\tilde{z})\zeta(\tilde{z}^*)} \qquad w_2 = -\frac{Nwa(\tilde{z})\tilde{z}}{\zeta(\tilde{z})}$$

$$w_3 = \frac{N^*\tau^* w}{\zeta(\tilde{z}^*)}\left[\zeta(\tilde{z}^*)\int_0^{\tilde{z}^*} a(s)ds - a(\tilde{z}^*)\tilde{z}^*\right] \qquad G_2 = \frac{\tau}{\tau-1}\frac{wa_g G}{\Gamma_{TR}}$$

$$G_3^* = -\frac{\tau^*}{\tau^*-1}\frac{a_g^* G^*}{\Gamma_{TR}^*} \qquad \tau_1 = -\left[N^*\tau\int_0^{\tilde{z}^*} a(s)ds + \frac{Na(\tilde{z})\tilde{z}}{\zeta(\tilde{z})}\right]$$

$$\tau_2 = N\left[\frac{\tau}{\tau-1}\int_{\tilde{z}}^{n} a^*(s)ds - \frac{wa(\tilde{z})\tilde{z}}{\zeta(\tilde{z})}\right] \qquad \tau_1^* = \frac{\tau N^* a(\tilde{z}^*)\tilde{z}^*}{\zeta(\tilde{z}^*)}$$

$$\tau_3^* = \frac{N^*\tau^* w}{\zeta(\tilde{z}^*)}\left[a(\tilde{z}^*)\tilde{z}^* - \frac{\tau^*}{\tau^*-1}\zeta(\tilde{z}^*)\int_0^{\tilde{z}^*} a(s)ds\right]$$

Taking the inverse yields

$$\begin{bmatrix} \widehat{w} \\ \widehat{G} \\ \widehat{G}^* \end{bmatrix} = \frac{1}{D_1}\begin{bmatrix} G_2G_3^* & 0 & 0 \\ -w_2G_3^* & w_1G_3^* & 0 \\ -w_3G_2 & 0 & w_1G_2 \end{bmatrix}\begin{bmatrix} \tau_1 & \tau_1^* \\ \tau_2 & 0 \\ 0 & \tau_3^* \end{bmatrix}\begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix},$$

with $D_1 = w_1G_2G_3^* > 0$. Consequently,

$$\frac{\widehat{w}}{\widehat{\tau}} = \frac{\tau_1}{w_1} \qquad \frac{\widehat{w}}{\widehat{\tau}^*} = \frac{\tau_1^*}{w_1} \qquad \frac{\widehat{w}}{\widehat{\tau}_M} = \frac{\tau_1+\tau_1^*}{w_1}$$

$$\frac{\widehat{G}}{\widehat{\tau}} = \frac{w_1\tau_2 - w_2\tau_1}{w_1G_2} \qquad \frac{\widehat{G}}{\widehat{\tau}^*} = -\frac{w_2\tau_1^*}{w_1G_2} \qquad \frac{\widehat{G}}{\widehat{\tau}_M} = \frac{w_1\tau_2 - w_2(\tau_1+\tau_1^*)}{w_1G_2}$$

$$\frac{\widehat{G}^*}{\widehat{\tau}} = -\frac{w_3\tau_1}{w_1G_3^*} \qquad \frac{\widehat{G}^*}{\widehat{\tau}^*} = \frac{w_1\tau_3^* - w_3\tau_1^*}{w_1G_3^*} \qquad \frac{\widehat{G}^*}{\widehat{\tau}_M} = \frac{w_1\tau_3^* - w_3(\tau_1+\tau_1^*)}{w_1G_3^*}$$

and the expressions as given in the main text readily follow, recognizing that in equilibrium also $\widehat{\tilde{z}} = -\left[\widehat{w} - \widehat{\tau}\right]/\zeta(\tilde{z})$ and $\widehat{\tilde{z}}^* = -\left[\widehat{w} + \widehat{\tau}^*\right]/\zeta(\tilde{z}^*)$.

A.2 Budget neutral trade liberalization

Requiring both South and North to keep a balanced government budget upon trade liberalization, changes the matrix into:

$$\begin{bmatrix} w_1 & -t_1 & 0 \\ w_2 & -t_2 & 0 \\ w_3 & 0 & -t_3^* \end{bmatrix} \begin{bmatrix} \widehat{w} \\ \widehat{t} \\ \widehat{t^*} \end{bmatrix} = \begin{bmatrix} \tau_1 & \tau_1^* \\ \tau_2 & 0 \\ 0 & \tau_3^* \end{bmatrix} \begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix}$$

where all short-hand notations are as before and where:

$$t_1 = -Nt; \quad t_2 = \frac{Ntw\left[\tau\Gamma_t - (\tau-1)\Gamma_{TR}\right]}{(\tau-1)\Gamma_{TR}}; \quad t_3^* = -\frac{\tau^*\Gamma_t^*}{(\tau^*-1)\,\Gamma_{TR}^*}N^*t^*.$$

Taking the inverse yields

$$\begin{bmatrix} \widehat{w} \\ \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix} = \frac{1}{D_2} \begin{bmatrix} t_2t_3^* & -t_1t_3^* & 0 \\ w_2t_3^* & -w_1t_3^* & 0 \\ t_2w_3 & -t_1w_3 & w_2t_1 - w_1t_2 \end{bmatrix} \begin{bmatrix} \tau_1 & \tau_1^* \\ \tau_2 & 0 \\ 0 & \tau_3^* \end{bmatrix} \begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix},$$

with

$$\begin{aligned} D_2 &= \left[t_2w_1 - w_2t_1\right]t_3^* = w\tau\tau^*NN^*tt^*\Gamma_t^* \\ &\times \frac{\left[\tau\Gamma_t - (\tau-1)\Gamma_{TR}\right]\zeta(\widetilde{z})N^*a(\widetilde{z}^*)\widetilde{z}^* + \Gamma_t\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}}{(\tau-1)\,(\tau^*-1)\,\Gamma_{TR}\Gamma_{TR}^*\zeta(\widetilde{z})\zeta(\widetilde{z}^*)} \end{aligned}$$

positive since $\left[\tau\Gamma_t - (\tau-1)\Gamma_{TR}\right] > 0$ by assumption. Defining $D_B \equiv \left[\tau\Gamma_t - (\tau-1)\Gamma_{TR}\right] \times \zeta(\widetilde{z})N^*a(\widetilde{z}^*)\widetilde{z}^* + \zeta(\widetilde{z}^*)\Gamma_tNa(\widetilde{z})\widetilde{z}$, the expressions in the main text then readily follow. Moreover, we get

$$\begin{aligned} \frac{\widehat{\widetilde{z}}}{\widehat{\tau}} &= -\frac{\left[\zeta(\widetilde{z})\zeta(\widetilde{z}^*)N\left[\Gamma_t - \Gamma_{TR}\right]\tau\int_{\widetilde{z}}^n a^*(s)ds - \left[\tau\Gamma_t - (\tau-1)\Gamma_{TR}\right]w\zeta(\widetilde{z})N^*a(\widetilde{z}^*)\widetilde{z}^*\right]}{\zeta(\widetilde{z})wD_B} \\ \frac{\widehat{\widetilde{z}}^*}{\widehat{\tau}} &= -\frac{N\left\{\zeta(\widetilde{z})\left[\Gamma_t - \Gamma_{TR}\right]\tau\int_{\widetilde{z}}^n a^*(s)ds + \Gamma_twa(\widetilde{z})\widetilde{z}\right\}}{wD_B} \\ \frac{\widehat{t^*}}{\widehat{\tau}} &= -\frac{(\tau^*-1)N\Gamma_{TR}^*}{D_B\Gamma_t^*t^*}\left(\left[\Gamma_t - \Gamma_{TR}\right]\zeta(\widetilde{z})\tau\int_{\widetilde{z}}^n a^*(s)ds + \Gamma_twa(\widetilde{z})\widetilde{z}\right)\left[\zeta(\widetilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds - a(\widetilde{z}^*)\widetilde{z}^*\right] \end{aligned}$$

$$
\begin{aligned}
\frac{\widehat{z}}{\widehat{\tau}^{*}} &= = \frac{\left[\tau\Gamma_{t}-(\tau-1)\Gamma_{TR}\right]N^{*}a(\widetilde{z}^{*})\widetilde{z}^{*}}{D_{B}} \\
\frac{\widehat{z}^{*}}{\widehat{\tau}^{*}} &= -\frac{\Gamma_{t}Na(\widetilde{z})\widetilde{z}}{D_{B}} \\
\frac{\widehat{t}^{*}}{\widehat{\tau}^{*}} &= -\frac{\Gamma_{TR}^{*}}{D_{B}\Gamma_{t}^{*}t^{*}}\left\{\begin{array}{c}\left[D_{B}+(\tau^{*}-1)\Gamma_{t}\zeta(\widetilde{z}^{*})Na(\widetilde{z})\widetilde{z}\right]\int_{0}^{\widetilde{z}^{*}}wa(s)ds \\ -Na(\widetilde{z})\widetilde{z}\Gamma_{t}\left(\tau^{*}-1\right)wa(\widetilde{z}^{*})\widetilde{z}^{*}\end{array}\right\}
\end{aligned}
$$

The effects on n and n^* not given in the main text follow from $a^*(n)n\widehat{n} = \int_{\widetilde{z}}^{n} a^*(s)ds(\widehat{w}-\widehat{\tau}) - \dfrac{tw}{\tau}\widehat{t}$ and $a^*(n^*)\widehat{n}^* = -\tau^* \int_0^{\widetilde{z}^*} wa(s)ds\,[\widehat{w}+\widehat{\tau}] - t^*\widehat{t}^*$:

$$
\begin{aligned}
a^{*}(n^{*})n^{*}\frac{\widehat{n}^{*}}{\widehat{\tau}} &= \left[\frac{N\left\{\zeta(\widetilde{z})\left[\Gamma_{t}-\Gamma_{TR}\right]\tau\int_{\widetilde{z}}^{n}a^{*}(s)ds+\Gamma_{t}wa(\widetilde{z})\widetilde{z}\right\}}{\Gamma_{t}^{*}D_{B}}\right]\times \\
&\quad \left\{\zeta(\widetilde{z}^{*})\left[(\tau^{*}-1)\,\Gamma_{TR}^{*}-\tau^{*}\Gamma_{t}^{*}\right]\int_{0}^{\widetilde{z}^{*}}a(s)ds-(\tau^{*}-1)\,\Gamma_{TR}^{*}a(\widetilde{z}^{*})\widetilde{z}^{*}\right\} \\
a^{*}(n)n\frac{\widehat{n}}{\widehat{\tau}^{*}} &= -\frac{N^{*}a(\widetilde{z}^{*})\widetilde{z}^{*}}{D_{B}}\left\{\begin{array}{c}\left[\tau\Gamma_{t}-(\tau-1)\Gamma_{TR}\right]\zeta(\widetilde{z})\int_{\widetilde{z}}^{n}a^{*}(s)ds \\ +(\tau-1)\Gamma_{TR}a^{*}(\widetilde{z})\widetilde{z}\end{array}\right\} \\
a^{*}(n^{*})n^{*}\frac{\widehat{n}^{*}}{\widehat{\tau}^{*}} &= \frac{\Gamma_{t}Nwa(\widetilde{z})\widetilde{z}}{\Gamma_{t}^{*}D_{B}}\left\{\left(\Gamma_{TR}^{*}-\Gamma_{t}^{*}\right)\tau^{*}\zeta(\widetilde{z}^{*})\int_{0}^{\widetilde{z}^{*}}a(s)ds-\Gamma_{TR}^{*}\left(\tau^{*}-1\right)a(\widetilde{z}^{*})\widetilde{z}^{*}\right\} \\
&\quad +\frac{\zeta(\widetilde{z})\left[\tau\Gamma_{t}-(\tau-1)\Gamma_{TR}\right]N^{*}a(\widetilde{z}^{*})\widetilde{z}^{*}\Gamma_{TR}^{*}}{\Gamma_{t}^{*}D_{B}}\int_{0}^{\widetilde{z}^{*}}wa(s)ds
\end{aligned}
$$

For multilateral tariff reductions we calculate:

$$
\begin{aligned}
\frac{\widehat{t}^{*}}{\widehat{\tau}_{M}} &= -\tfrac{(\tau^{*}-1)N\Gamma_{TR}^{*}}{D_{B}\Gamma_{t}^{*}t^{*}}\left(\left[\Gamma_{t}-\Gamma_{TR}\right]\zeta(\widetilde{z})\tau\int_{\widetilde{z}}^{n}a^{*}(s)ds+\Gamma_{t}wa(\widetilde{z})\widetilde{z}\right)\left[\zeta(\widetilde{z}^{*})\int_{0}^{\widetilde{z}^{*}}a(s)ds-a(\widetilde{z}^{*})\widetilde{z}^{*}\right] \\
&\quad -\tfrac{\Gamma_{TR}^{*}}{D_{B}\Gamma_{t}^{*}t^{*}}\left\{\begin{array}{c}\left[D_{B}+(\tau^{*}-1)\Gamma_{t}\zeta(\widetilde{z}^{*})Na(\widetilde{z})\widetilde{z}\right]\int_{0}^{\widetilde{z}^{*}}wa(s)ds \\ -Na(\widetilde{z})\widetilde{z}\Gamma_{t}\left(\tau^{*}-1\right)wa(\widetilde{z}^{*})\widetilde{z}^{*}\end{array}\right\}.
\end{aligned}
$$

B Derivations for Heterogeneous Population

B.1 Unilateral and multilateral trade liberalization

In case of heterogeneous population, the budget constraints vary with the assumed skill level of households. For southern households the budget constraints become:

$$
\begin{aligned}
\int_{0}^{\widetilde{z}}wa(s)ds+\tau\int_{\widetilde{z}}^{n_{H}}a^{*}(s)ds &= (1-t)wh_{H} \qquad \text{(high skilled households)} \\
\int_{0}^{n_{L}}a(s)ds &= (1-t)h_{L} \qquad \text{(low skilled households)}
\end{aligned}
$$

while for northern households they are:

$$\begin{aligned}\tau^* \int_0^{\widetilde{z}^*} wa(s)ds + \int_{\widetilde{z}^*}^{n_H^*} a^*(s)ds &= (1-t^*)h_H^* \qquad \text{(high skilled households)} \\ \tau^* \int_0^{n_L^*} wa(s)ds &= (1-t^*)h_L^* \qquad \text{(low skilled households)}\end{aligned}$$

Consequently, South's balanced trade condition becomes

$$N^* \left[\int_0^{n_L^*} wa(s)ds + \frac{1}{\tau^*}\left((1-t^*)h_H^* - \int_{\widetilde{z}^*}^{n_H^*} a^*(s)ds\right)\right] = N \int_{\widetilde{z}}^{n_H} a^*(s)ds$$

as compared to (17) in the homogenous population setting. The government's budget constraint for South and North become, respectively

$$\begin{aligned}wa_g G &= \frac{N}{2}\Gamma_t t(h_L + h_H)w + \frac{N}{2}(\tau - 1)\Gamma_{TR}\int_{\widetilde{z}}^{n_H} a^*(s)ds \\ a_g^* G^* &= \frac{N^*}{2}\Gamma_t^* t^*(h_L^* + h_H^*) + \frac{N^*}{2}(\tau^* - 1)\Gamma_{TR}^*\left[\frac{(1-t^*)h_L^*}{\tau^*} + \int_0^{\widetilde{z}^*} wa(s)ds\right]\end{aligned}$$

Total differentiation of the equilibrium equations, making use of the household budget constraints for heterogeneous population and the conditions for efficient production (2) and (3), yields, in matrix notation:

$$\begin{bmatrix} w_1 & 0 & 0 \\ w_2 & G_2 & 0 \\ w_3 & 0 & G_3^* \end{bmatrix}\begin{bmatrix} \widehat{w} \\ \widehat{G} \\ \widehat{G}^* \end{bmatrix} = \begin{bmatrix} \tau_1 & \tau_1^* \\ \tau_2 & 0 \\ 0 & \tau_3^* \end{bmatrix}\begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix}$$

where:

$$w_1 = -\frac{1}{2}\frac{D_H}{\tau\zeta(\tilde{z}^*)\zeta(\tilde{z})} \qquad w_2 = -\frac{N}{2}\frac{wa(\widetilde{z})\widetilde{z}}{\zeta(\tilde{z})}$$

$$w_3 = \frac{\tau^* w}{\zeta(\tilde{z}^*)}\frac{N^*}{2}\left[a(\widetilde{z}^*)\widetilde{z}^* - \zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds\right] \qquad G_2 = \frac{\tau w a_g G}{\Gamma_{TR}(\tau - 1)}$$

$$G_3^* = \frac{a_g^* G^* \tau^*}{\Gamma_{TR}^*(\tau^* - 1)} \qquad \tau_1 = -\frac{N}{2}\left[\frac{1}{w}\int_{\widetilde{z}}^{n_H} a^*(s)ds + \frac{1}{\tau}\frac{a(\widetilde{z})\widetilde{z}}{\zeta(\tilde{z})}\right]$$

$$\tau_2 = \frac{N}{2\zeta(\tilde{z})}\left[\frac{\tau}{\tau - 1}\zeta(\tilde{z})\int_{\widetilde{z}}^{n_H} a^*(s)ds - wa(\widetilde{z})\widetilde{z}\right] \qquad \tau_1^* = \frac{N^*}{2}\left[\int_0^{n_L^*} a(s)ds + \frac{a(\widetilde{z}^*)\widetilde{z}^*}{\zeta(\tilde{z}^*)}\right]$$

$$\tau_3^* = \frac{N^*}{2}\frac{\tau^*}{\tau^* - 1}\left[\int_0^{n_L^*} wa(s)ds + \tau^*\int_0^{\widetilde{z}^*} wa(s)ds - \frac{(\tau^* - 1)wa(\widetilde{z}^*)\widetilde{z}^*}{\zeta(\tilde{z}^*)}\right]$$

and

$$D_H \equiv \zeta(\tilde{z}^*)\zeta(\tilde{z})N^*\tau\int_0^{n_L^*} a(s)ds + \zeta(\tilde{z}^*)Na(\widetilde{z})\widetilde{z} + \zeta(\tilde{z})\tau N^* a(\widetilde{z}^*)\widetilde{z}^* > 0.$$

Taking the inverse yields

$$\begin{bmatrix} \widehat{w} \\ \widehat{G} \\ \widehat{G}^* \end{bmatrix} = \frac{1}{D_3} \begin{bmatrix} G_2G_3^* & 0 & 0 \\ -w_2G_3^* & w_1G_3^* & 0 \\ -w_3G_2 & 0 & w_1G_2 \end{bmatrix} \begin{bmatrix} \tau_1 & \tau_1^* \\ \tau_2 & 0 \\ 0 & \tau_3^* \end{bmatrix} \begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix},$$

with

$$D_3 = w_1G_2G_3^* = -\frac{1}{2}\frac{1}{\tau-1}\frac{\tau^*}{\tau^*-1}\frac{wa_gG}{\Gamma_{TR}}\frac{a_g^*G^*}{\Gamma_{TR}^*}\frac{D_H}{\zeta(\tilde{z}^*)\zeta(\tilde{z})} < 0.$$

Consequently,

$$\begin{array}{lll} \dfrac{\widehat{w}}{\widehat{\tau}} = \dfrac{\tau_1}{w_1} & \dfrac{\widehat{w}}{\widehat{\tau}^*} = \dfrac{\tau_1^*}{w_1} & \dfrac{\widehat{w}}{\widehat{\tau}_M} = \dfrac{\tau_1+\tau_1^*}{w_1} \\ \dfrac{\widehat{G}}{\widehat{\tau}} = \dfrac{w_1\tau_2 - w_2\tau_1}{w_1G_2} & \dfrac{\widehat{G}}{\widehat{\tau}^*} = -\dfrac{w_2\tau_1^*}{w_1G_2} & \dfrac{\widehat{G}}{\widehat{\tau}_M} = \dfrac{w_1\tau_2 - w_2(\tau_1+\tau_1^*)}{w_1G_2} \\ \dfrac{\widehat{G}^*}{\widehat{\tau}} = -\dfrac{w_3\tau_1}{w_1G_3^*} & \dfrac{\widehat{G}^*}{\widehat{\tau}^*} = \dfrac{w_1\tau_3^* - w_3\tau_1^*}{w_1G_3^*} & \dfrac{\widehat{G}^*}{\widehat{\tau}_M} = \dfrac{w_1\tau_3^* - w_3(\tau_1+\tau_1^*)}{w_1G_3^*} \end{array}$$

as in the homogeneous population case. The expressions for unilateral tariff changes as given in the main text then readily follow. Furthermore, we derive:

$$\begin{aligned} \frac{\widehat{G}}{\widehat{\tau}^*} &= -\frac{\Gamma_{TR}(\tau-1)Na(\widetilde{z})\widetilde{z}N^*\left[\zeta(\tilde{z}^*)\displaystyle\int_0^{n_L^*} a(s)ds + a(\widetilde{z}^*)\widetilde{z}^*\right]}{2D_Ha_gG} < 0 \\ \frac{\widehat{G}^*}{\widehat{\tau}} &= -\frac{\Gamma_{TR}^*(\tau^*-1)N^*N\left[\zeta(\tilde{z})\tau\int_{\tilde{z}}^{n_H} a^*(s)ds + wa(\widetilde{z})\widetilde{z}\right]\left[a(\widetilde{z}^*)\widetilde{z}^* - \zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*} a(s)ds\right]}{2D_ha_g^*G^*} \\ \frac{\widehat{G}^*}{\widehat{\tau}^*} &= \frac{\Gamma_{TR}^*\left\{\begin{array}{c} \dfrac{N^*}{2}wD_h\left[\int_0^{n_L^*} a(s)ds\right] - Na(\widetilde{z})\widetilde{z}\left[(\tau^*-1)a(\widetilde{z}^*)\widetilde{z}^*\right] \\ +\left[D_H + (\tau^*-1)\zeta(\tilde{z}^*)Na(\widetilde{z})\widetilde{z}\right]\int_0^{\widetilde{z}^*} a(s)ds \end{array}\right\}}{D_Ha_g^*G^*} \end{aligned}$$

Regarding multilateral tariff reductions, we calculate

$$\frac{\widehat{\widetilde{z}}^*}{\widehat{\tau}_M} = -\frac{1}{D_H}\left[\zeta(\tilde{z})\tau N^*\left(\int_0^{\widetilde{z}^*} a(s)ds + \int_0^{n_L^*} a(s)ds\right) + 2Na(\widetilde{z})\widetilde{z}\right]$$

$$\begin{aligned} \frac{\widehat{G}}{\widehat{\tau}_M} &= \frac{\Gamma_{TR}}{wa_gG}\frac{N}{2}\int_{\tilde{z}}^{n_H} a^*(s)ds \\ &\quad + \frac{(\tau-1)\Gamma_{TR}Na(\widetilde{z})\widetilde{z}N^*}{2D_ha_gG}\left\{\zeta(\tilde{z}^*)\left(\int_0^{\widetilde{z}^*} a(s)ds - \int_0^{n_L^*} a(s)ds\right) - 2a(\widetilde{z}^*)\widetilde{z}^*\right\} \end{aligned}$$

$$\frac{\widehat{G}^*}{\widehat{\tau}_M} = \frac{w\Gamma_{TR}^*}{2D_h a_g^* G^*} \left\{ \begin{array}{c} D_H N^* \left[\int_0^{n_L^*} a(s)ds + \tau^* \int_0^{\widetilde{z}^*} a(s)ds - \frac{(\tau^*-1)a(\widetilde{z}^*)\widetilde{z}^*}{\zeta(\widetilde{z}^*)} \right] \\ -(\tau^*-1)N^* \left[a(\widetilde{z}^*)\widetilde{z}^* - \zeta(\widetilde{z}^*) \int_0^{\widetilde{z}^*} a(s)ds \right] \times \\ \left[N^* \tau \zeta(\widetilde{z}) \int_0^{\widetilde{z}^*} a(s)ds - N^* \frac{\zeta(\widetilde{z})\tau a(\widetilde{z}^*)\widetilde{z}^*}{\zeta(\widetilde{z}^*)} + Na(\widetilde{z})\widetilde{z} \right] \end{array} \right\}$$

B.2 Budget neutral trade liberalization

Requiring both South and North to keep balanced government budgets, changes the matrix of total differentiation into:

$$\begin{bmatrix} w_1 & -t_1^* & -t_1 \\ w_3 & -t_3^* & 0 \\ w_2 & 0 & -t_2 \end{bmatrix} \begin{bmatrix} \widehat{w} \\ \widehat{t^*} \\ \widehat{t} \end{bmatrix} = \begin{bmatrix} \tau_1 & \tau_1^* \\ 0 & \tau_3^* \\ \tau_2 & 0 \end{bmatrix} \begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix}$$

where all short-hand notations are as before and where additionally:

$$t_1 = -\frac{N}{2}\frac{th_H}{\tau}, \qquad t_2 = \frac{Nwt}{2}\left[\frac{\Gamma_t \tau}{\Gamma_{TR}\tau - 1}(h_L + h_H) - h_H\right],$$
$$t_1^* = \frac{N^*}{2}\frac{t^* h_L^*}{\tau^* w}, \qquad t_3^* = \frac{N^* t^*}{2}\left[\frac{\Gamma_t^* \tau^*}{\Gamma_{TR}^*(\tau^* - 1)}(h_L^* + h_H^*) - h_L^*\right].$$

Taking the inverse yields

$$\begin{bmatrix} \widehat{w} \\ \widehat{t^*} \\ \widehat{t} \end{bmatrix} = \frac{1}{D_{BH}} \begin{bmatrix} t_2 t_3^* & -t_1^* t_2 & -t_1 t_3^* \\ w_3 t_2 & w_2 t_{1-} w_1 t_2 & -t_1 w_3 \\ w_2 t_3^* & -t_1^* w_2 & t_1^* w_3 - w_1 t_3^* \end{bmatrix} \begin{bmatrix} \tau_1 & \tau_1^* \\ 0 & \tau_3^* \\ \tau_2 & 0 \end{bmatrix} \begin{bmatrix} \widehat{\tau} \\ \widehat{\tau}^* \end{bmatrix},$$

and the following derivatives follow:

$$\frac{\widehat{w}}{\widehat{\tau}} = \frac{(t_2\tau_1 - t_1\tau_2)\, t_3^*}{D_{BH}} \qquad \frac{\widehat{w}}{\widehat{\tau}^*} = \frac{t_2\,(t_3^*\tau_1^* - t_1^*\tau_3^*)}{D_{BH}}$$
$$\frac{\widehat{t}}{\widehat{\tau}} = \frac{w_2 t_3^* \tau_1 + (t_1^* w_3 - w_1 t_3^*)\,\tau_2}{D_{BH}} \qquad \frac{\widehat{t}}{\widehat{\tau}^*} = \frac{w_2\,(t_3^*\tau_1^* - t_1^*\tau_3^*)}{D_{BH}} = \frac{w_2}{t_2}\frac{\widehat{w}}{\widehat{\tau}^*}$$
$$\frac{\widehat{t^*}}{\widehat{\tau}} = \frac{w_3(t_2\tau_1 - t_1\tau_2)}{D_{BH}} = \frac{w_3}{t_3^*}\frac{\widehat{w}}{\widehat{\tau}} \qquad \frac{\widehat{t^*}}{\widehat{\tau}^*} = \frac{w_3 t_2 \tau_1^* + (w_2 t_{1-} w_1 t_2)\tau_3^*}{D_{BH}}.$$

Here we derive a sufficient condition so that the sign of the determinant D_{BH} is negative.

$$\begin{aligned} D_{BH} &= [t_2 w_1 - w_2 t_1]\, t_3^* - t_1^* t_2 w_3 \\ &= -\frac{tt^* NN^*}{8(\tau-1)(\tau^*-1)\Gamma_{TR}^*\Gamma_{TR}} \left[\tau\Gamma_t(h_L + h_H) - (\tau-1)\Gamma_{TR} h_H\right] \\ &\quad \times \left\{ \begin{array}{c} \frac{\Gamma_t(h_L+h_H)Nwa(\widetilde{z})\widetilde{z}}{\zeta(\widetilde{z})} \frac{[\tau^*\Gamma_t^*(h_L^*+h_H^*) - (\tau^*-1)\Gamma_{TR}^* h_L^*]}{[\tau\Gamma_t(h_L+h_H) - (\tau-1)\Gamma_{TR}h_H]} \\ +\tau^*\Gamma_t^*(h_L^*+h_H^*)\left(N^*\int_0^{n_L^*} wa(s)ds + \frac{N^* wa(\widetilde{z}^*)\widetilde{z}^*}{\zeta(\widetilde{z}^*)}\right) \\ -(\tau^*-1)\Gamma_{TR}^* h_L^* N \int_{\widetilde{z}}^{n_H} a^*(s)ds \end{array} \right\}. \end{aligned}$$

Define $C \equiv \tau\Gamma_t(h_L + h_H) - (\tau - 1)\Gamma_{TR}h_H$, let $\Gamma^* = \Gamma^*_{TR} = \Gamma^*_t$, and use the trade balance condition to replace $N\int_{\widetilde{z}}^{n_H} a^*(s)ds$, D_{BH} reduces to:

$$\begin{aligned} D_{BH} &= -\frac{tt^*NN^*w}{8(\tau-1)(\tau^*-1)\Gamma_{TR}} \\ &\quad\times\left\{\begin{array}{c} \dfrac{\Gamma_t(h_L+h_H)Na(\widetilde{z})\widetilde{z}}{\zeta(\tilde{z})}\left[\tau^*(h_L^*+h_H^*)-(\tau^*-1)h_L^*\right] \\ +\tau^*(h_L^*+h_H^*)C\dfrac{N^*a(\widetilde{z}^*)\widetilde{z}^*}{\zeta(\tilde{z}^*)}+C\tau^*h_H^*N^*\int_0^{n_L^*}a(s)ds \\ CN^*h_L^*\left[\int_0^{n_L^*}a(s)ds-(\tau^*-1)\int_0^{\widetilde{z}^*}a(s)ds\right] \end{array}\right\}. \end{aligned}$$

All terms in the braced bracket are unambiguously positive except the squared bracketed term since for $h_L^* \leqslant h_H^*$, $\widetilde{z}^* \geqslant n_L^*$. As n_L^* depends on h_L^*, a sufficient condition for the determinant to be negative is that the skill gap in North is not too large and northern tariffs are not too high, rendering the last expression in squared brackets small so that $D_{BH} < 0$ follows.

Consequently,

$$\begin{aligned} \frac{\widehat{w}}{\widehat{\tau}} &= -\frac{Nt^*tNN^*}{8D_{BH}}\frac{\left[\tau^*\Gamma_t^*(h_L^*+h_H^*)-(\tau^*-1)\Gamma_{TR}^*h_L^*\right]}{(\tau^*-1)(\tau-1)\zeta(\tilde{z})\Gamma_{TR}^*\Gamma_{TR}} \\ &\quad\times\left\{\begin{array}{c} \left[\Gamma_t(h_L+h_H)-h_H\Gamma_{TR}\right]\tau\zeta(\tilde{z})\int_{\widetilde{z}}^{n_H}a^*(s)ds \\ +\Gamma_t(h_L+h_H)wa(\widetilde{z})\widetilde{z} \end{array}\right\} \end{aligned}$$

$$\frac{\widehat{t}}{\widehat{\tau}} = \frac{1}{D_{BH}}\frac{NN^*t^*}{8\zeta(\tilde{z}^*)\zeta(\tilde{z})}\left\{\begin{array}{c} -\frac{h_L^*}{\tau-1}\zeta(\tilde{z}^*)N\int_{\widetilde{z}}^{n_H}a^*(s)ds\left[\zeta(\tilde{z})\frac{\tau}{w}\int_{\widetilde{z}}^{n_H}a^*(s)ds\right] \\ -\frac{1}{\tau-1}\frac{1}{\Gamma_{TR}^*}\zeta(\tilde{z}^*)\left[a(\widetilde{z})\widetilde{z}\right]\left[N\int_{\widetilde{z}}^{n_H}a^*(s)ds\right]\times \\ \left[\Gamma_{TR}^*h_L^*-\tau\dfrac{\tau^*}{(\tau^*-1)}\Gamma_t^*(h_L^*+h_H^*)\right] \\ +\left[\dfrac{\tau^*}{(\tau^*-1)}\dfrac{\Gamma_t^*}{\Gamma_{TR}^*}(h_L^*+h_H^*)\right]\times \\ \left\{\begin{array}{c} \left[\zeta(\tilde{z}^*)N^*\int_0^{n_L^*}a(s)ds+N^*a(\widetilde{z}^*)\widetilde{z}^*\right]\times \\ \left[\dfrac{\tau}{\tau-1}\zeta(\tilde{z})\int_{\widetilde{z}}^{n_H}a^*(s)ds-wa(\widetilde{z})\widetilde{z}\right] \end{array}\right\} \end{array}\right\}$$

$$\begin{aligned} \frac{\widehat{t^*}}{\widehat{\tau}} &= \frac{w_3}{t_3^*}\frac{\widehat{w}}{\widehat{\tau}} \\ &= -\frac{\tau^*\Gamma_{TR}^*(\tau^*-1)w}{\zeta(\tilde{z}^*)t^*\left[\Gamma_t^*\tau^*(h_L^*+h_H^*)-\Gamma_{TR}^*(\tau^*-1)h_L^*\right]}\left[\zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*}a(s)ds-a(\widetilde{z}^*)\widetilde{z}^*\right]\frac{\hat{w}}{\hat{\tau}} \end{aligned}$$

$$\begin{aligned} \frac{\widehat{t^*}}{\widehat{\tau}} &= -\frac{\left[\zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*}a(s)ds-a(\widetilde{z}^*)\widetilde{z}^*\right]\tau^*N^*N^*Ntw}{8\zeta(\tilde{z}^*)\zeta(\tilde{z})(\tau-1)\Gamma_{TR}D_{BH}}\times \\ &\quad\left\{\zeta(\tilde{z})[\Gamma_t(h_L+h_H)-\Gamma_{TR}h_H]\tau\int_{\widetilde{z}}^{n_H}a^*(s)ds+\Gamma_t(h_L+h_H)wa(\widetilde{z})\widetilde{z}\right\} \end{aligned}$$

$$\frac{\widehat{w}}{\widehat{\tau^*}} = \frac{\tau^* N N^* N^* t^* t w C}{8(\tau-1)(\tau^*-1)\zeta(\tilde{z}^*)\Gamma_{TR}\Gamma^*_{TR}D_{BH}} \times$$
$$\left\{ \begin{array}{c} \Gamma^*_t(h^*_L+h^*_H)\left[\zeta(\tilde{z}^*)\int_0^{n^*_L} a(s)ds + a(\widetilde{z}^*)\widetilde{z}^*\right] \\ -h^*_L\Gamma^*_{TR}\zeta(\tilde{z}^*)\left[\int_0^{n^*_L} a(s)ds + \int_0^{\widetilde{z}^*} a(s)ds\right] \end{array} \right\}$$

Let $\Gamma^* = \Gamma^*_{TR} = \Gamma^*_t$ we obtain

$$\frac{\widehat{w}}{\widehat{\tau^*}} = \frac{\tau^* N N^* N^* t^* t w C}{8(\tau-1)(\tau^*-1)\zeta(\tilde{z}^*)\Gamma_{TR}\Gamma^*_{TR}D_{BH}} \times$$
$$\left\{\zeta(\widetilde{z}^*)\left[h^*_H\int_0^{n^*_L} a(s)ds - h^*_L\int_0^{\widetilde{z}^*} a(s)ds\right] + (h^*_L+h^*_H)a(\widetilde{z}^*)\widetilde{z}^*\right\}$$

$$\frac{\widehat{t}}{\widehat{\tau^*}} = -\frac{N N^* N^* t^* w \tau^* a(\widetilde{z})\widetilde{z}}{8D_{bnh}\Gamma^*_{TR}(\tau^*-1)\zeta(\tilde{z})\zeta(\tilde{z}^*)} \times$$
$$\left\{ \begin{array}{c} \Gamma^*_t(h^*_L+h^*_H)\left[\zeta(\tilde{z}^*)\int_0^{n^*_L} a(s)ds + a(\widetilde{z}^*)\widetilde{z}^*\right] \\ -h^*_L\Gamma^*_{TR}\zeta(\tilde{z}^*)\left[\int_0^{n^*_L} a(s)ds + \int_0^{\widetilde{z}^*} a(s)ds\right] \end{array} \right\}$$

$$\frac{\widehat{t^*}}{\widehat{\tau^*}} = \frac{t w w N^* N \tau^*}{8D_{bnh}\zeta(\tilde{z}^*)\zeta(\tilde{z})\Gamma_{TR}(\tau-1)(\tau^*-1)} \times$$
$$\left\{ \begin{array}{c} N^*\zeta(\tilde{z})\left[\tau\Gamma_t(h_L+h_H) - (\tau-1)\Gamma_{TR}h_H\right] \times \\ \left[\zeta(\tilde{z}^*)\int_0^{n^*_L} a(s)ds + a(\widetilde{z}^*)\widetilde{z}^*\right]\left[\int_0^{n^*_L} a(s)ds + \int_0^{\widetilde{z}^*} a(s)ds\right] \\ +\Gamma_t(h_L+h_H)Na(\widetilde{z})\widetilde{z}\left[\zeta(\tilde{z}^*)\int_0^{n^*_L} a(s)ds + \zeta(\tilde{z}^*)\tau^*\int_0^{\widetilde{z}^*} a(s)ds - (\tau^*-1)a(\widetilde{z}^*)\widetilde{z}^*\right] \end{array} \right\}$$

Here we derive the slopes of the curves in Figure 3 and show that the NBC curve is the steeper of the two negative curves in Figure 3b2.

Slope of TB-curve:

$$\left(\frac{\widehat{w}}{\widehat{t}}(t^*)\right)_{\text{TB}} = \frac{\zeta(\tilde{z}^*)\zeta(\tilde{z})Nth_H}{D_H} > 0$$
$$\left(\frac{\widehat{w}}{\widehat{t^*}}(t)\right)_{\text{TB}} = -\frac{\tau\zeta(\tilde{z}^*)\zeta(\tilde{z})N^*t^*h^*_L}{\tau^* w D_h} < 0$$

Slope of SBC-curve:

$$\left(\frac{\widehat{w}}{\widehat{t}}\right)_{\text{SBC}} = -\frac{\tau\zeta(\tilde{z}^*)\zeta(\tilde{z})Nwt}{(\tau-1)\Gamma_{TR}D_H}\left[\tau\Gamma_t(h_L+h_H) - (\tau-1)\Gamma_{TR}h_H\right]$$

Slope of NBC-curve:

$$\left(\frac{\widehat{w}}{\widehat{t^*}}\right)_{\text{NBC}} = \frac{\zeta(\tilde{z}^*)t^*\left[\Gamma^*_t\tau^*(h^*_L+h^*_H) - \Gamma^*_{TR}(\tau^*-1)h^*_L\right]}{\tau^*(\tau^*-1)\Gamma^*_{TR}\left[wa(\widetilde{z}^*)\widetilde{z}^* - \zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*} wa(s)ds\right]}$$

For $\zeta(\tilde{z}^*) > a(\widetilde{z}^*)\widetilde{z}^*/\int_0^{\widetilde{z}^*} a(s)ds$, both the NBC-curve and the TB-curve are negatively sloped with the NBC-curve the steeper of the two. This implies the following inequality

$$\begin{aligned} & -\frac{[\tau^*\Gamma_t^*(h_L^*+h_H^*)-\Gamma_{TR}^*(\tau^*-1)h_L^*]}{\Gamma_{TR}^*N^*(\tau^*-1)\left[\zeta(\widetilde{z}^*)\int_0^{\widetilde{z}^*}a(s)ds-a(\widetilde{z}^*)\widetilde{z}^*\right]} \\ < & -\frac{\zeta(\widetilde{z})\tau h_L^*}{[\zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^*+\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}+\zeta(\widetilde{z}^*)\zeta(\widetilde{z})N^*\tau\int_0^{n_L^*}a(s)ds]}, \end{aligned}$$

where the left hand of the inequality is the slope of the NBC-curve, while the right hand side reflects the slope of the TB-curve. After some manipulation we obtain

$$\begin{aligned} & [h_H^*\tau^*+h_L^*]\,\zeta(\widetilde{z}^*)Na(\widetilde{z})\widetilde{z}+\tau^*(h_L^*+h_H^*)\zeta(\widetilde{z})N^*\tau a(\widetilde{z}^*)\widetilde{z}^* \\ & +[h_H^*\tau^*+h_L^*]\,\zeta(\widetilde{z}^*)\zeta(\widetilde{z})N^*\tau\int_0^{n_L^*}a(s)ds-\zeta(\widetilde{z})\zeta(\widetilde{z}^*)\tau h_L^*N^*(\tau^*-1)\int_0^{\widetilde{z}^*}a(s)ds \\ > & \ 0, \end{aligned}$$

provided that the skill gap in North is not too marked.

The relative shift of both curves, holding t^* constant, is determined by:

$$\begin{aligned} & \frac{\tau_1^*w_3-\tau_3^*w_1}{w_1w_3}= \\ & -\frac{\left\{\begin{array}{c}\left[\zeta(\tilde{z}^*)\zeta(\tilde{z})\tau N^*\int_0^{n_L^*}a(s)ds+\zeta(\tilde{z}^*)Na(\widetilde{z})\widetilde{z}\right]\left[\zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*}a(s)ds+\zeta(\tilde{z}^*)\int_0^{n_L^*}a(s)ds\right] \\ +\left[\tau^*\zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*}a(s)ds+\zeta(\tilde{z}^*)\int_0^{n_L^*}a(s)ds-(\tau^*-1)\,a(\widetilde{z}^*)\widetilde{z}^*\right][\zeta(\tilde{z})\tau N^*a(\widetilde{z}^*)\widetilde{z}^*]\end{array}\right\}}{(\tau^*-1)D_H\left[a(\widetilde{z}^*)\widetilde{z}^*-\zeta(\tilde{z}^*)\int_0^{\widetilde{z}^*}a(s)ds\right]} \end{aligned}$$

which is positive (negative) if $\zeta(\tilde{z}^*)$ is sufficiently large (small).

Figure 1a: Trade policy equilibrium and public good provision in South – homogeneous populations

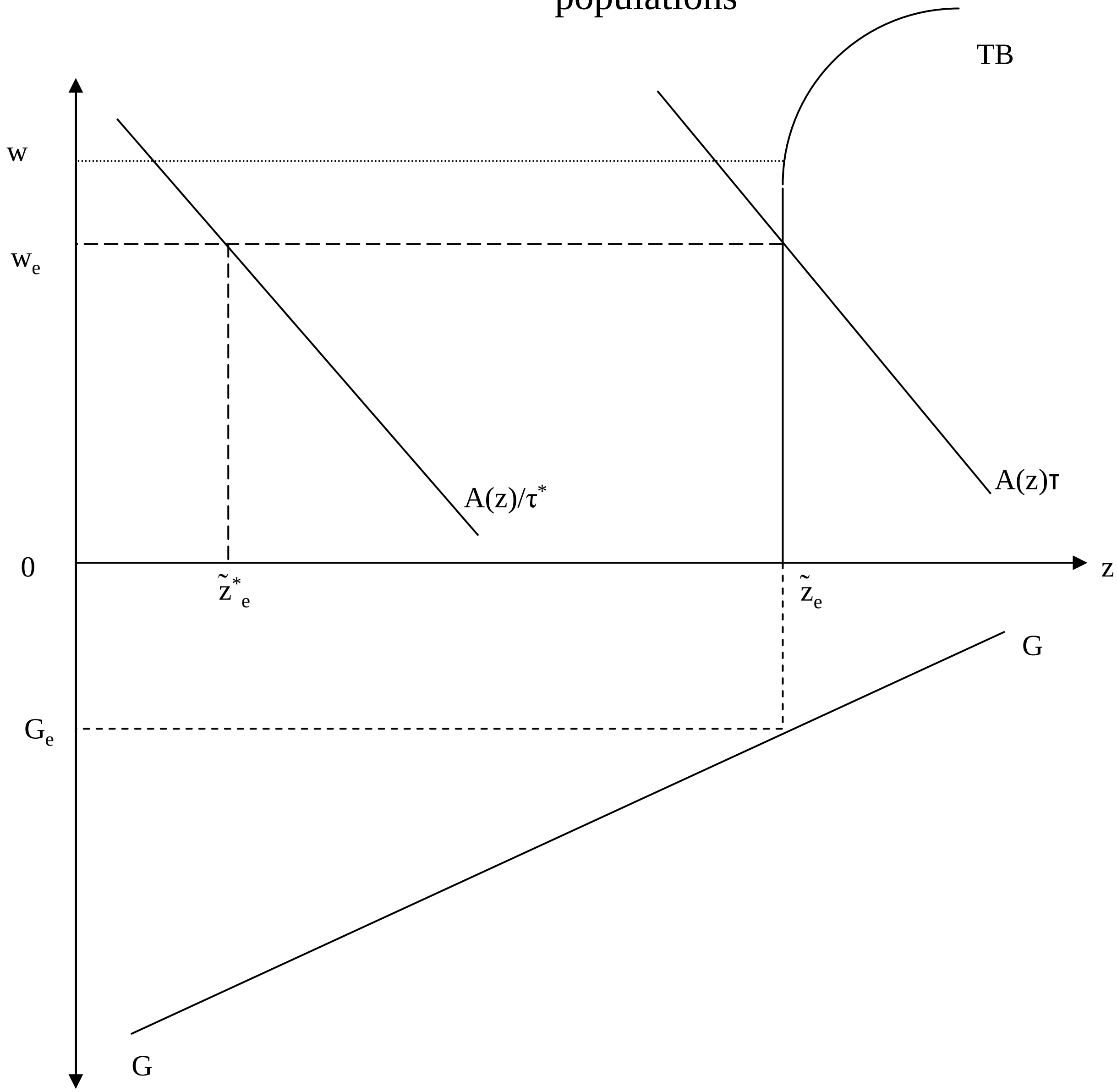

Figure 1b: Trade policy equilibrium and public good provision in North – homogeneous populations

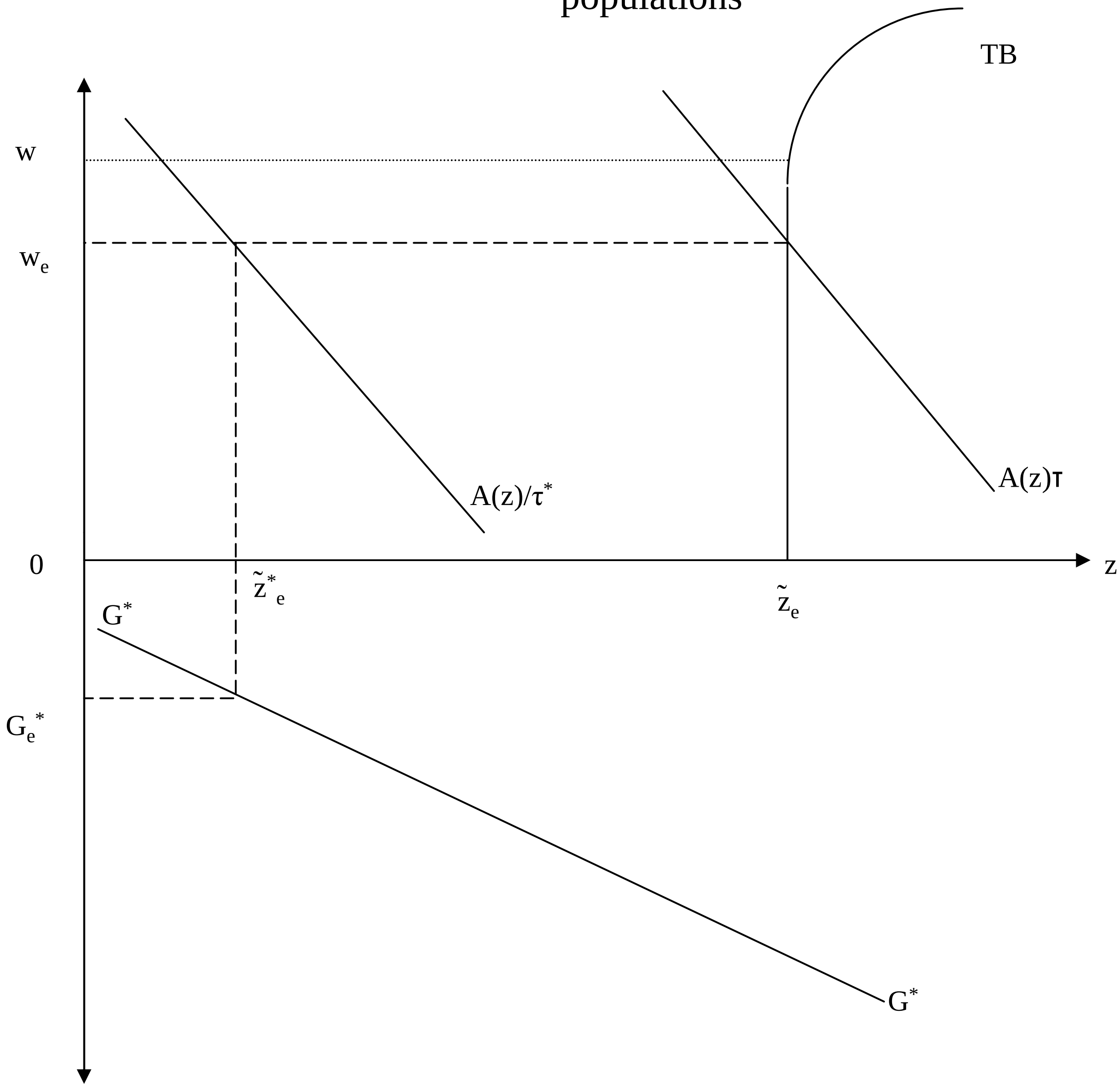

Figure 2. Budget-neutral liberalization and unilateral reduction in southern tariff, dτ<0.

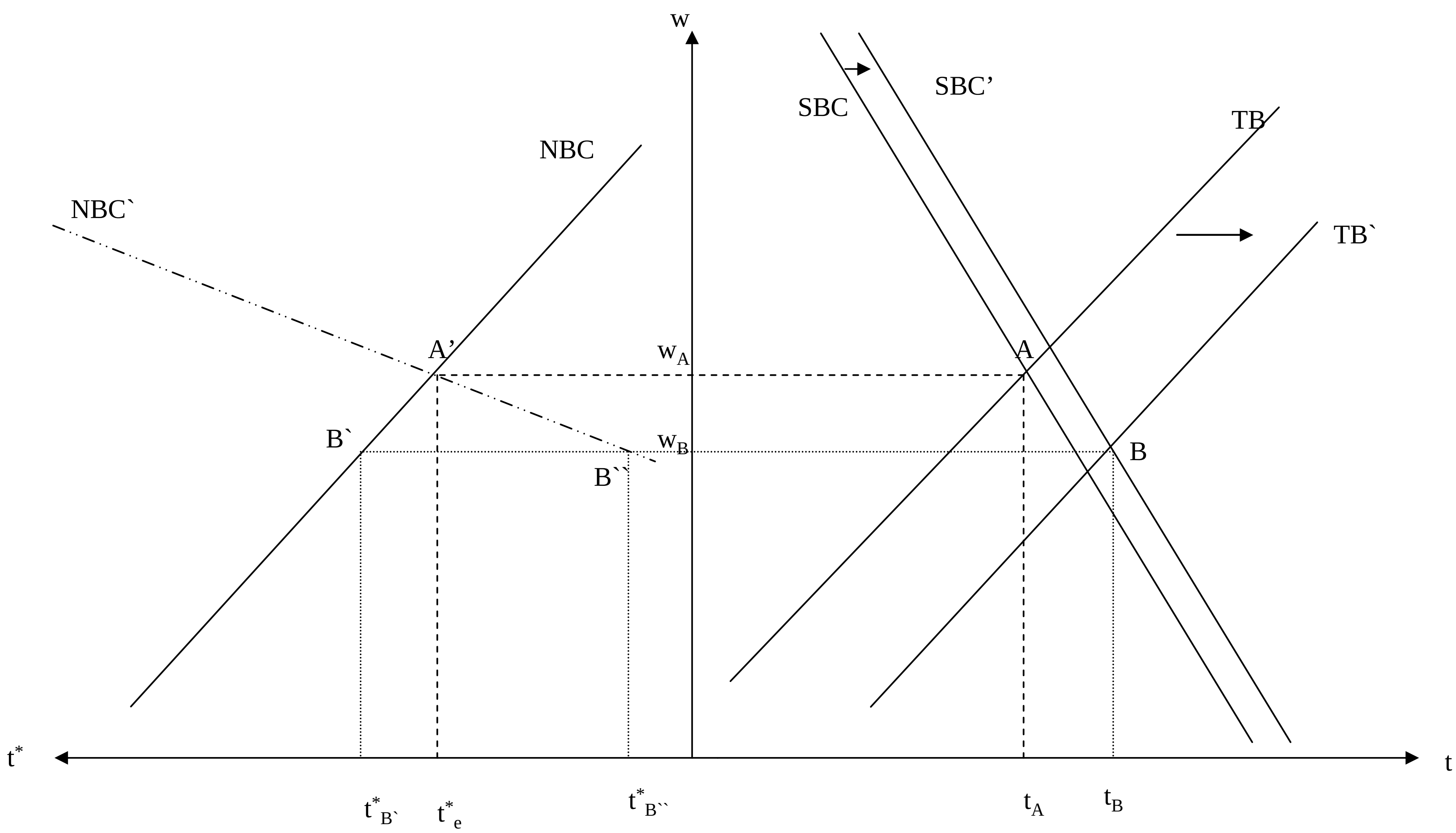

Figure 2. Budget-neutral liberalization, unilateral reduction in southern tariff, dτ<0.

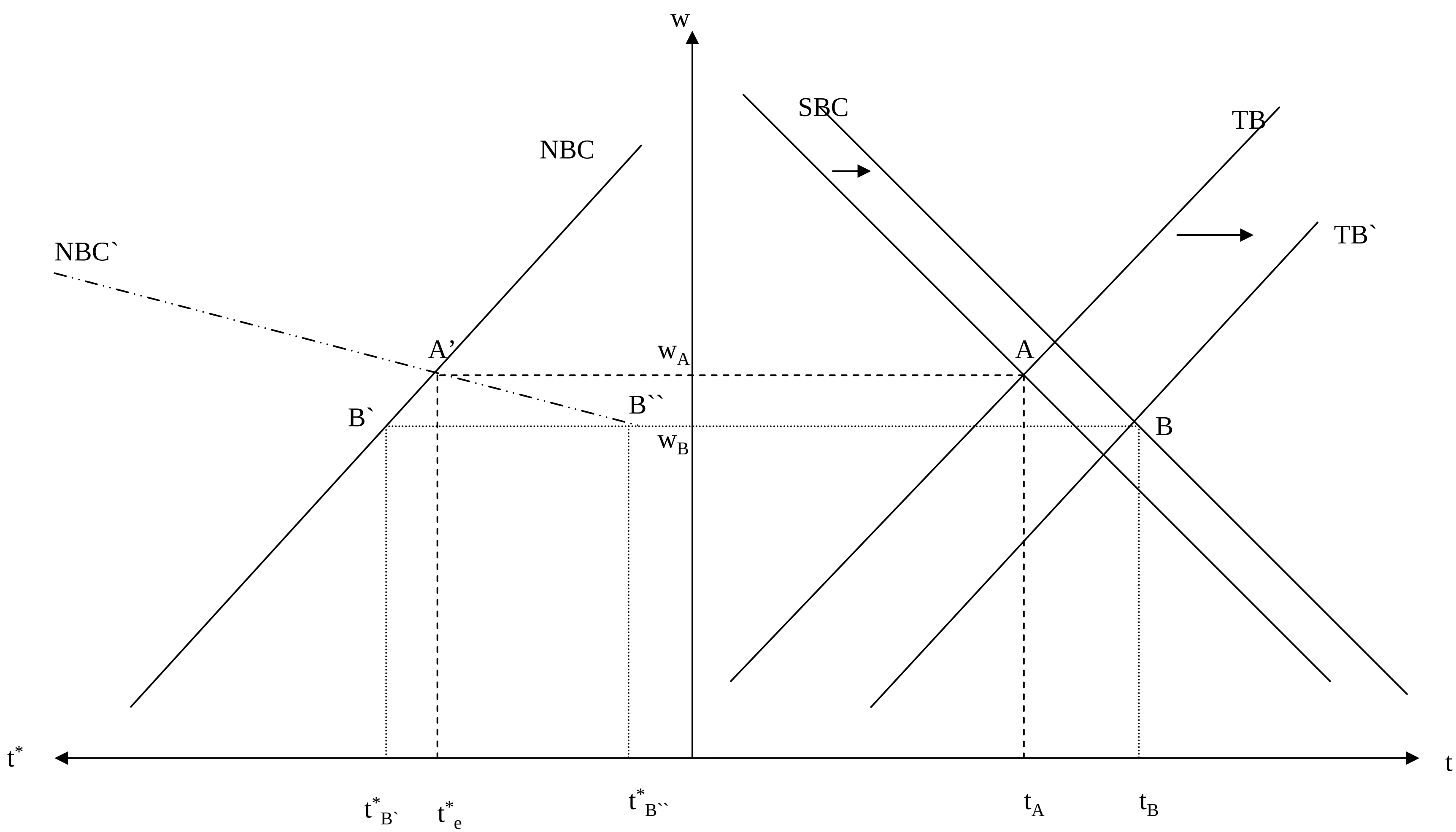

Figure 2. Budget-neutral liberalization and multilateral reduction in tariffs, $d\tau_M<0$.

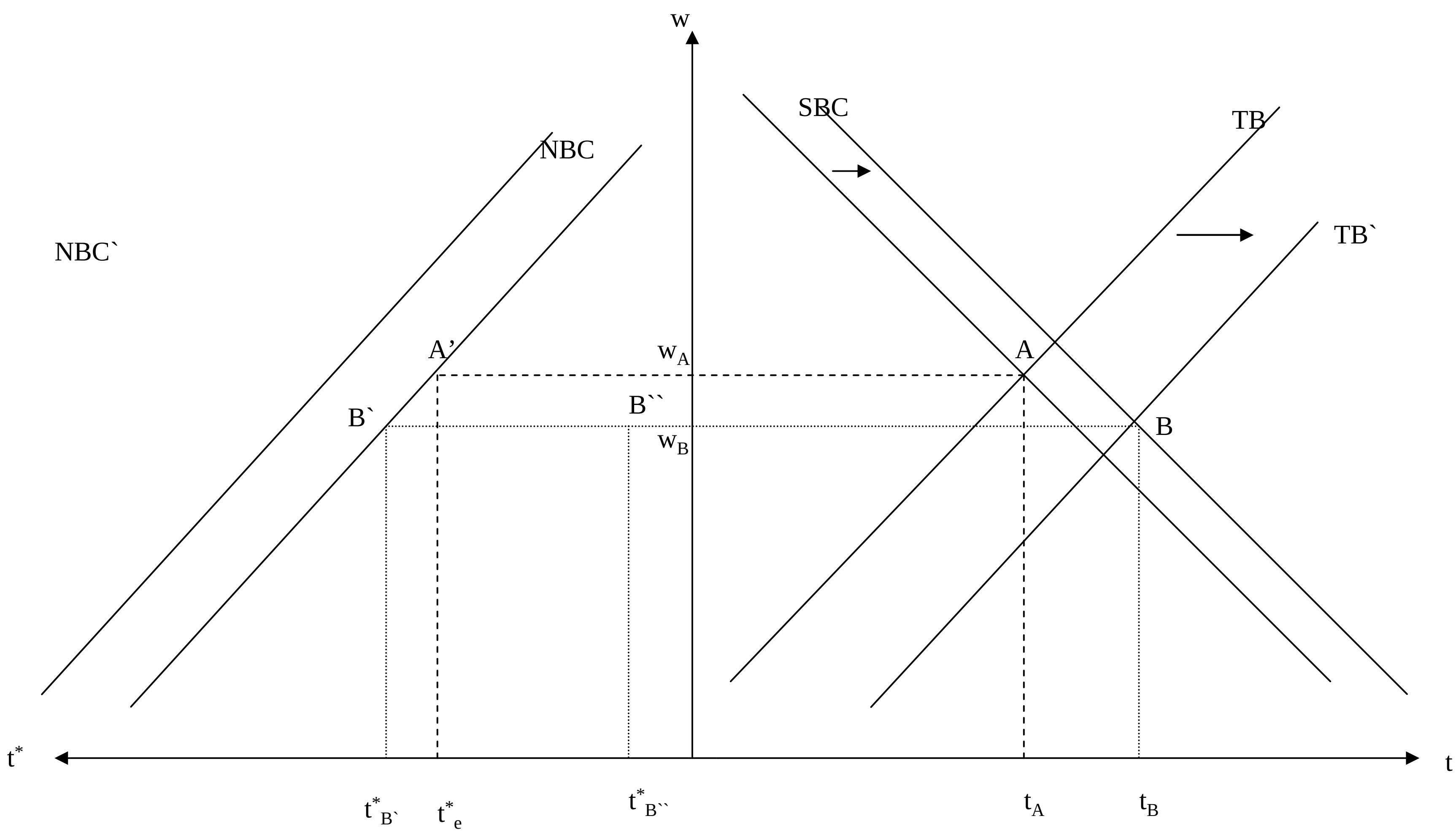

Figure 3a. Heterogeneous population: budget-neutral liberalization and unilateral reduction in southern tariff, dτ<0

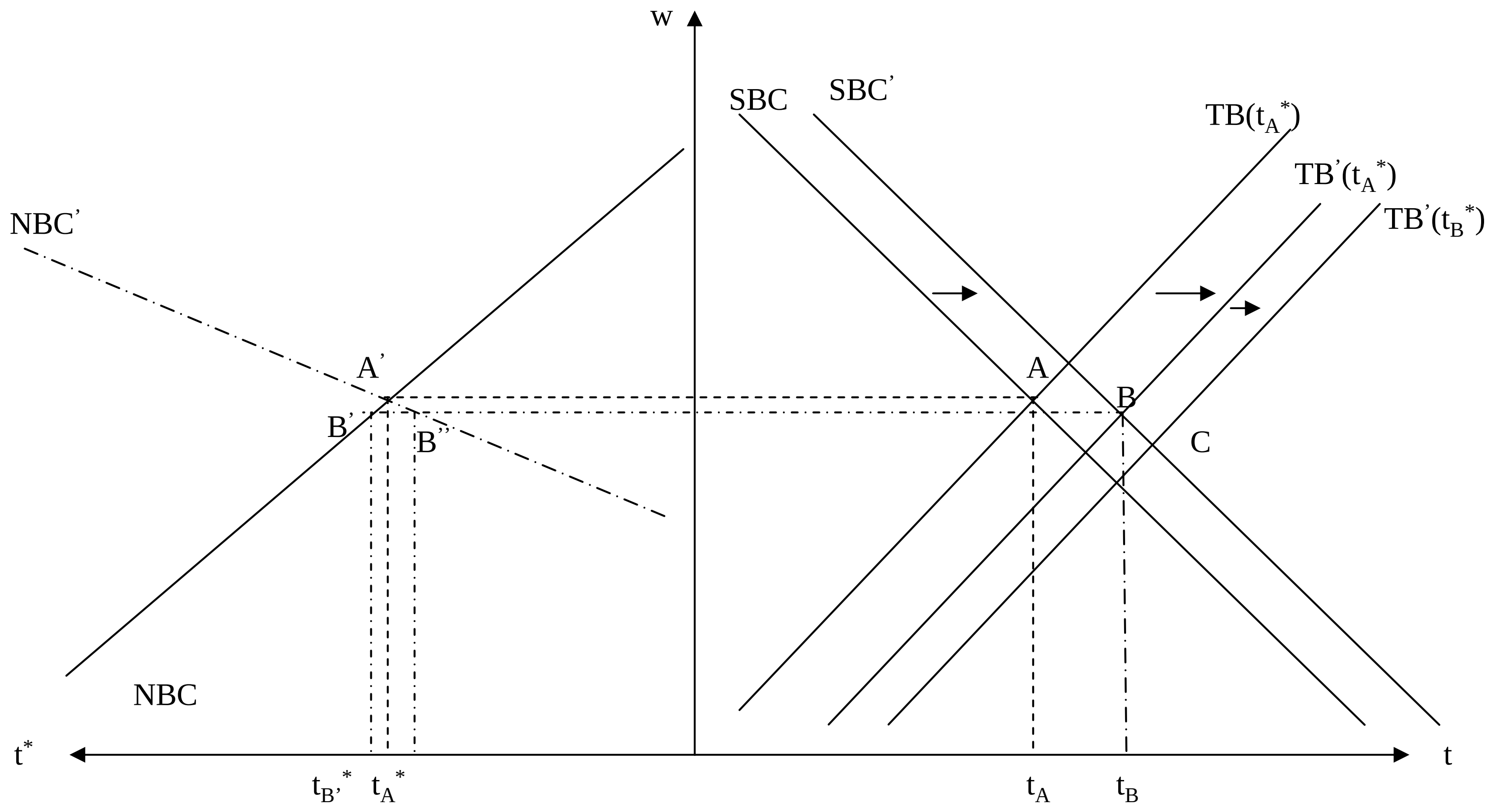

Figure 3b1. Heterogeneous population: budget-neutral liberalization and unilateral reduction in northern tariff, $d\tau^{*}<0$ for $\zeta(z^{*})$ sufficiently small

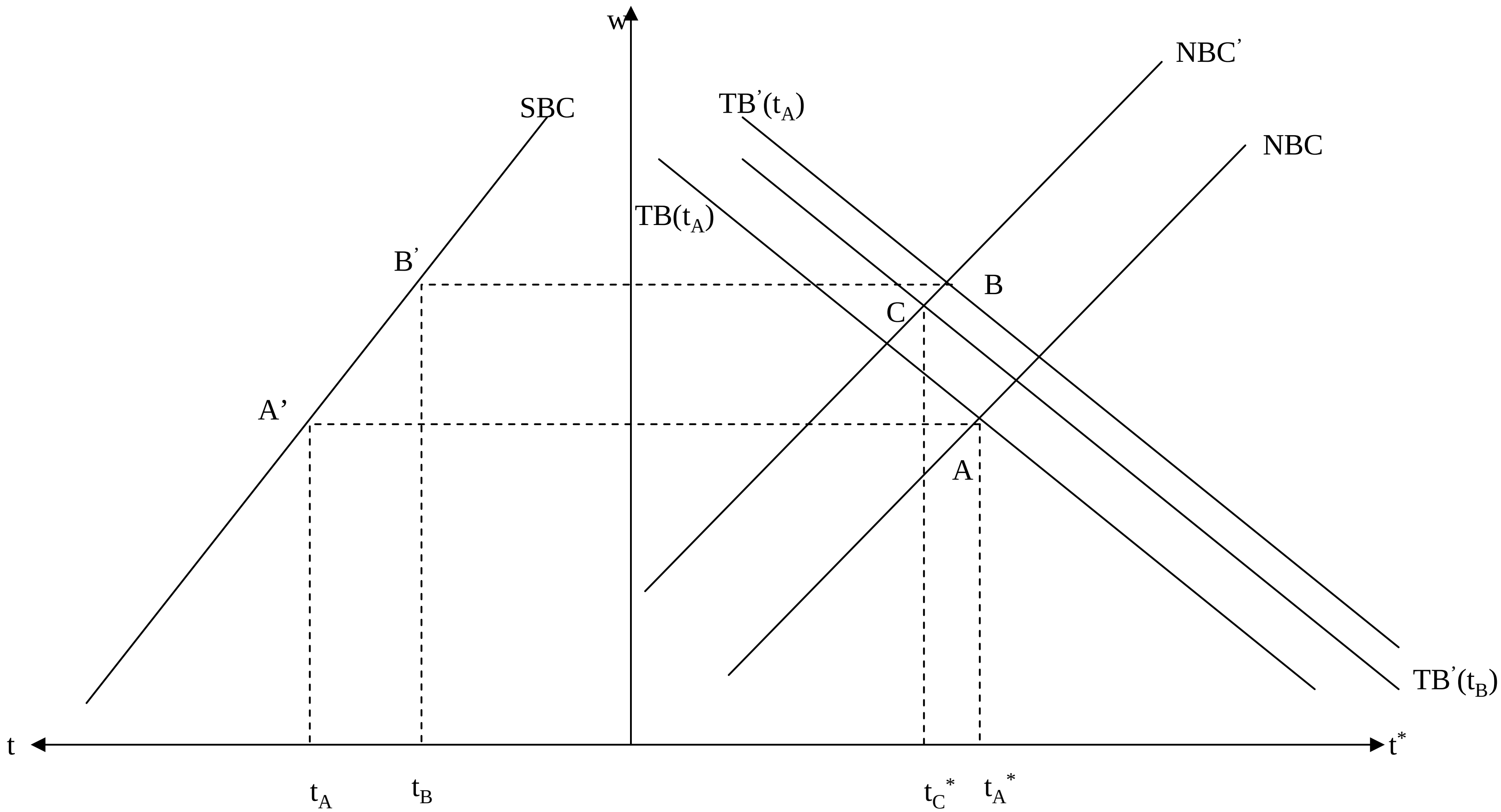

Figure 3b2. Heterogeneous population: budget-neutral liberalization and unilateral reduction in northern tariff, $d\tau^{*}<0$ for $\zeta(z^{*})$ sufficiently large