Are Budget Deficits Excessive in the UK?

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Abstract

In this paper we apply a thorough cointegration analysis to annual time-series data for the U.K. in the period, 1948-1997, to examine whether government revenue and expenditure have been congruent. The data do provide considerable evidence of a cointegrated long-run relation between government revenue and expenditure in the U.K. during this period implying that the two have been congruent in the long-run. All the relevant series in our analysis are stationary in their first-difference and there are evidence of structural breaks in most of the series around 1973. The revenue and the expenditure series are cointegrated at the 5% level if allowance is made for changes in the slope and intercept in the revenue-expenditure relation after 1973 when tests for cointegration are made by the Engle and Granger and by Johansen’s multi-cointegration tests criteria.. Some of the results also imply that capital-flows are important in removing budget deficits in the short-run and a balance between revenue and expenditure can be achieved over a longer period.

Key Words: Unit Roots, Dickey - Fuller, Structural Breaks, Philip - Perron, Cointegration, Engle – Granger, Granger – Lee, Johansen, Capital - Flows

JEL Classification: H61 H62 C22 C52 O52
1. Introduction

A major issue that has been repeatedly debated in macroeconomics is the proper size of the government deficit (Barro, 1986). Are the budget deficits ‘excessive’? This issue has been generally examined within the framework of the intertemporal budget constraint (Hamilton and Flavin (HF, 1986). An interesting analysis of annual time-series data from 1962 to 1984 under such constraint concludes that the U.S. budget deficit has been a stationary stochastic process (HF, 1986). Other studies test for stochastic variation in real interest rate and instability in the estimates of parameters and point out that HF’s unit root (UR) tests suffer from misspecification (Wilcox, 1987). A correction for the bias due to specification errors enabled Kremers (Kremers, 1988, 1989) to alter the HF results and conclude that government sector deficit is incongruous with the intertemporal budget constraint.

More recently, some have concentrated on the use of cointegration (CI) tests to examine directly the relationship between government revenue and spending for different countries (Trehan and Walsh, 1988, 1990; Smith and Zin, 1988). They all conclude that the behaviour of the government revenue and spending is not incongruous with the intertemporal budget constraint.

The main motivation for writing this paper is to examine directly the cointegration of government revenue and spending in Britain. More specifically, using the annual time-series data at 1985 prices for the U.K. economy during the period 1948 - 1997, we analyse whether government revenue and expenditure and the real interest rate follow random walks or are stationary and whether government revenue and spending are cointegrated in a meaningful way. The other important motivation for analysing such an issue is to investigate the reason for the current British Chancellor of the Exchequer, Gordon Brown's repeated emphasis to ‘balance the book’ by following a prudent fiscal policy on a longer term. The Chancellor's point, in our view, fits
nicely in the context of the analysis of budget deficit in the U.K. in an intertemporal context. We believe such an analysis has not been attempted in the framework of a thorough CI analysis before. The data used in our paper have been obtained from various issues of the International Financial Statistics and from the Economic Trends Annual Supplement 1998.

The rest of the paper is organised as follows: section 2 explains the theory in the intertemporal context; section 3 explains the econometric methodology that has been used in our analysis; section 4 interpretes the results obtained from econometric estimation on the basis of annual time-series data for the U.K. for the period 1948 – 1997 and section 5 draws conclusions from our study.

2. Theory

Many governments run huge budget deficits to attain a variety of targets including raising the growth rate. Such targets could be financed by capital inflows as domestic savings fail to match required investment to attain the planned growth rate. On the other hand, governments also face intertemporal budget constraints (BC) which can be written as:

\[ R_t + B_t = G_t + (1 + d_t)B_{t-1} \quad \ldots \quad (1) \]

where \( R_t \) stands for government revenue, \( B_t \) for bond sales, \( G_t \) for government expenditure excluding interest payments on debt and \( d_t \) stands for one-period interest rate.

This BC for period \( t \), therefore, can be rewritten as a first-order non-homogeneous difference equation in \( B_t \):

\[ B_t = (1+d_t)B_{t-1} + (G_t - R_t) \quad \ldots \quad (2) \]

and can be solved by iteration as follows:

\[ B_1 = (1+d_1)B_0 + (G_1 - R_1) \quad \Rightarrow \quad B_2 = (1+d_2)B_1 + (G_2 - R_2) \Rightarrow \]

\[ B_2 = (1+d_1)(1+d_2)B_0 + (1+d_2)(G_1 - R_1) + (G_2 - R_2) \quad \ldots \quad (3) \]
Following the iterative procedure in the manner indicated above, one can derive

\[ B_n = (1+d_0)(1+d_n) \cdots (1+d_1)B_0 + (1+d_0) (1+d_n) \cdots (1+d_2)(G_1 - R_1) \]
\[ + (1+d_0) \cdots (1+d_3)(G_2 - R_2) \cdots + (G_n - R_n) \cdots \] \text{(4)}

Equation (4) can be rewritten for the present stock of bonds, \( B_0 \):

\[ B_0 = \sum r_t(R_t - G_t) + \lim (r_tB_n) \cdots \] \text{(5)}

where \( r_t = \prod b_s \), the product running over all values of \( s \) from 1 to \( t \) and \( b_s = 1/(1+d_s) \) \ldots \text{(5a)}\) and \( \lim \) means the value in the limit as \( n \) tends to infinity. The equation (5) implies that when \( \lim(B_n) \) tends to zero, the present stock of bonds \( B_0 \), equals the present value (PV) of government budget surplus. The equation (5) for intertemporal BC shows that the textbook definition of budget deficit as \((G_t + d_B_{t-1} - T_t)\) where \( T \) is tax no longer relevant. The limiting value of \( r_nB_n = 0 \) eliminates the possibility of the government financing its deficit by issuing new debt. If this limiting value is not equal to zero, the government is 'bubble financing' its expenditure in which old debt is financed by new debt and the deficit is 'too large'(i.e. Ponzi game, Blanchard and Fischer, 1989). Therefore, available, relevant data must support the condition that \( \lim(r_nB_n) = 0 \). To check if they do, we need to check if the variables \( R \) and \( G \) follow random walks in which \( \lim(r_nB_n) = 0 \) i.e., if \( E[ \lim(r_nB_n |(R,G) ] \) follow random walks with no drift \ldots \text{(6)}\)

To derive testable connotation, interest rate can be assumed stationary with unconditional mean = \( d \). Adding and subtracting \( dB_{t-1} \), equation (1) can yield

\[ R_t + B_t = G_t + (1+d)B_{t-1} +(d-t-d)B_{t-1} \implies E_t + (1+d)B_{t-1} = R_t + B_t \ldots \] \text{(7)}

where \( E_t = G_t + (d_t - d)B_{t-1} \ldots \text{(7a)}\)

The equation (7) is valid for each period. Solving equation (7) in the same way as before, one can obtain

\[ B_{t-1} = \sum b^{t-1}(R_{t-1} - E_{t-1}) + \lim(b^{t-1}B_{t-1}) \ldots \] \text{(8)}\) where the summation runs from zero to infinity; the limit is evaluated at \( j = \infty \) and \( b \) is obtained from (5a) for the mean value of the interest rate.
To exploit the random-walk properties, the equation (8) can be rewritten in terms of first-differences denoted by a D before each variable:

\[ G_t + i_{t+1} = R_t + \sum b^{j+1}(D^jR_{t+j} - DE_{t+j}) + \lim(b^{j+1} B_{t+j}) ......(9) \]

Let GV indicate the left hand side (LHS) of equation (9), which is total government expenditure including transfer payments and including interest payments on debt; and let \( R_t \) and \( G_t \) follow random walks with drifts \( \alpha_1 \) and \( \alpha_2 \) respectively:

\[ R_t = \alpha_1 + R_{t-1} + \varepsilon_{1t} ......(10a) \] and \[ E_t = \alpha_2 + E_{t-1} + \varepsilon_{2t} ......(10b) \]

Then GV follows from equation (9) as:

\[ GV_t = \alpha + R_t + \lim (b^{j+1} B_{t+j}) + \varepsilon_t ......(11) \]

where \( \alpha = \sum b^{j+1}(\alpha_1 - \alpha_2) - \alpha = (1+d) (\alpha_1 - \alpha_2) ......(11a) \)

and where \( \varepsilon_t = b^{j+1}(\varepsilon_{1t} - \varepsilon_{2t}) .....(11b) \)

If the limit in equation (11) is zero, then we have the following regression equation:

\[ R_t = a + bGV_t + \varepsilon_t ......(12) \]

The null hypothesis to be tested is \( H_0: b = 1 \) and that \( \varepsilon_t \) is stationary. The latter requires \( R \) and \( GV \) to be cointegrated. The economic implications of the acceptance of the null hypothesis will be that the government deficits are not excessive but that the expenditure and revenue are congruent with each other implying, in turn, prudent management of the fiscal machinery by the Chancellor of the exchequer. The acceptance of the null hypothesis of cointegration, in turn, would imply that this compatibility of expenditure and revenue holds in the long run and the relation between the two is not spurious or a fluke.

In what follows we estimate appropriate equations applying time-series methodology to annual data at 1985 prices for the U.K. economy in the period 1948 - 1997. The techniques of time-series modelling are vital for investigating stationarity properties of variables, for searching possible existence of structural breaks in the data, checking on cointegration of variables and for
consequent estimation of error correction models. Therefore, we adopt time-series methods to test all the variables in our study - R, GV, d, df ( = government budget deficit), B ( = real value of government bonds), RPB ( = real interest payments on government bonds)) and G – for stationarity with and without allowing for structural breaks; to test whether R and GV are cointegrated for a meaningful relation like (12) to exist; and to test whether the parameter b in equation (12) equals unity. The following section explains the methodology we adopt for econometric estimation of our theory as outlined above.

3. The Time-Series Methodology

Testing for Unit Roots (URs)

The first step in time-series modelling is to check each relevant series for stationarity. Tests for stationarity check whether one can reject the null hypothesis of URs. Whether a series, say, $X_t$, has unit roots in it or not can be tested by estimating the equation:

$$X_t = \alpha + bX_{t-1} + ct + u_t \ldots(13i),$$

where $t$ is the time trend variable which would take, in our sample, the value 1 for the year 1948, 2 for 1949 and so on. The null hypothesis of URs can then be tested by testing $H_0: b = 1$ against the critical values of the Dickey - Fuller (DF) $t$ - distribution. An improved version of the UR test assumes the error term, $u_t$, in (13i) as a moving average (MA) process and , accordingly, estimates the equation (14):

$$DX_t = \alpha + bX_{t-1} + c_1DX_{t-1} + c_2DX_{t-2} + \ldots c_kDX_{t-k} + \delta t + u_t \ldots(14),$$

where $k$ is the selected length of lag. The null hypothesis of URs is then stated as $H_0: b = 0$ and tested against the critical values of the augmented Dickey - Fuller (ADF) $t$ - distribution. Selection of lag length, $k$, can be made by more than one criterion. We have selected the Akaike Information criterion (AIC) (Maddala, 1992, p.502, 550) and the Ng and Perron criterion (Ng and
Perron, 1997, Perron, 1997). Both amount to using F-tests computed for comparing the residual sums of squares (RSSs) of the equations with and without the additional lags. On the AIC, the equation without, say, m additional lags is selected if the F-value computed as the ratio of the difference in RSS/m to RSS/(n-k) is less than the ratio: (n-k)/(n+k-m). Ng and Perron (1997) and Perron (1997), on the other hand, simply recommend a general to specific modelling approach. This approach involves looking at the significance of the t-statistic of the last lag added at each stage and at the significance of the F-statistic computed for comparing the equation including the additional lag with the equation excluding the additional lag.

The tests for unit roots can be biased if any structural breaks in the series are ignored (Phillip and Perron, 1988, Perron, 1989, Zivot and Andrews, 1992, Andrews, 1993). Structural breaks in series are expected to occur due to changes taking place in the economy resulting from internal or domestic events as well as due to changes forced on the economy by external events or "shocks". Change of political party in power, a bad harvest, etc., can be named as common examples of domestic shocks and the oil price rise of the early seventies, the three-day working week introduced in response to the former can be named as examples of external shocks. The unit root tests to take structural breaks at known and unknown time points into account can be done by estimating the following types of equations (Ghatak, 1996, 1997). For breaks assumed at known and estimated times, the following dummy variables can be used:

\[ D_1 = 1 \text{ for } t > T_B, \ 0 \text{ otherwise} ; \ D_2 = 1 \text{ for } t = T_B + 1, \ 0 \text{ otherwise}; \]
\[ D_3 = t-T_B \text{ for } t > T_B, \ 0 \text{ otherwise}; \ D_4 = t \text{ for } t > T_B, \ 0 \text{ otherwise} \]

where \( T_B \) is the time of break. The equation to be estimated to test for unit roots in the series, \( X_t \), under possible breaks at a known point, \( T_B \) is then:

\[ X_t = a + bX_{t-1} + ct + c_1DX_{t-1} + ... + c_kDX_{t-k} + d_1D_1 + d_2D_2 + d_3D_4 + u_t \ldots (13ii) \]

The null hypothesis, \( H_0: b = 1 \) can then be tested by using the ADF-t statistics and by the Phillip-
Perron (PP) statistics (Phillip and Perron, 1988). We have used the critical values of these statistics provided in Perron (1989) for different values of the location of the break points, $d = t/T_B$. PP values are estimated as $T(b-1)$ where $T$ is total number of years included in the series. Perron's studies have been modified to have time of break, determined by the data themselves and not by assumption (Andrews, 1993, Zivot and Andrews, 1992, Ng and Perron, 1997, Perron, 1997). The equation to be estimated for this purpose is:

$$X_t = a + bX_{t-1} + ct + c_1DX_{t-1} + ... + c_kDX_{t-k} + d_1D_1 + d_3D_3 + u_t \ldots \ldots \ (13iii)$$

The idea behind equation (13iii) is to search for the year of break which maximises the ADF - $t$ - value in absolute terms. The results of unit root tests with and without structural breaks are given in Table 1 and they will be interpreted in Section 4.

**Testing for Cointegration (CI)**

Non-stationary variables included in a regression can not be assumed to be meaningfully related in the long run unless they are cointegrated. Existence of CI among non-stationary but integrated variables means that a linear combination of these non-stationary variables is stationary. The existence of a long run equilibrium relation in the context of CI means that specific pairs of non-stationary but integrated series can move together without drifting too far apart, although individually they may wander off extensively due to non-stationarity. Existence of CI can be checked in a number of ways - all calculable from least squares regression. We will discuss, in what follows, the DF - $t$ values for Engle and Granger cointegration test (EGC), (Engle and Granger, 1987), the extended CI tests of Granger and Lee (1991), a simple test using the Durbin Watson statistic of the cointegration regression (CRDW) (Engle and Granger, 1987) and Johansen's test for multicointegration (Johansen, 1988).

The DF test for CI (abbreviated as EGC in our text), recommended by Engle and Granger(1987) runs in two stages. In stage one, the CI regression is estimated by adding a linear trend to it, which
in our illustration will be:

\[ R_t = a + bGV_t + ct + \varepsilon_t \quad \ldots \quad (12a) \]

and in stage two, testing the residuals from the least squares regression (12a) for stationarity i.e., by applying DF tests to

\[ D\varepsilon_t^* = \beta \varepsilon^*_{t-1} + \Gamma_1 D\varepsilon^*_{t-1} + \Gamma_2 D\varepsilon^*_{t-2} + \ldots + \Gamma_k D\varepsilon^*_{t-k} + v_t \quad \ldots \quad (12b) \]

where \( \varepsilon^* \) are the residuals from regression (12a). The null hypothesis of CI between \( R \) and \( GV \) can then be stated as \( H_0 : \beta = 0 \) which can be tested against the critical values of the DF \( t \) distribution.

As the tests for CI are, in essence, tests of stationarity in the residuals of the CI regression, one can also check for possible structural breaks in the CI regression at known and estimated time points (Ghatak, 1998). There will be a strong case for testing for CI under structural changes if some or all series involved in the CI regression exhibit breaks in the trend and/or in the rate of growth in the first stage of testing for unit roots as indicated in equations (13) above. For example, the CI regression (12a) can exhibit changes in the intercept and / or in the slope, which can be accommodated as:

\[ R_t = a + (a^* - a)D_t + bGV_t + (b^* - b)D_5 + \varepsilon_t \quad \ldots \quad (12c) \]

where \( D_5 = \) values of \( GV \) for \( t > T_B \), 0 otherwise. Again \( T_B \) can be either assumed exogenous or it can be estimated on the basis of data. The results of CI tests with and without structural changes are given in Table 2 and they will be interpreted in Section 4.

Granger and Lee (1991) proposed a "deeper" test for multico integration which they applied to sales and production data of U.S. industries which are influenced by inventory considerations. The test for a deeper level of cointegration seems particularly appropriate for considerations of inventory but they recommend this extended method for application to other economic time-series as well. In this extended Granger - Lee method (GLC), there are four steps. In the first two, the residuals from the CI regression are tested for stationarity using the DF-t value; in the third step an
additional regression is estimated:

\[ R_t = \alpha + \beta_1\varepsilon_{t-1} + \beta_2\varepsilon_{t-2} + \beta_3\varepsilon_{t-3} + \ldots + \omega \ldots (14) \]

where \( \varepsilon_t \) are the residuals from CI regressions (12) without any structural changes or from equation (12c) with structural changes as the case may be; and the fourth step is to test the residuals from the regression of \( R_t \) on the lagged \( \varepsilon \)'s for stationarity. The length of lags to be taken into account can again be determined by the criterion of general to specific modelling. The rationale behind using the Granger and Lee method in our case study can be given as follows:

\[ R_t - GV_t = Ddf_t \]

and if government spending, \( GV \), and government deficit, \( df \) are both stationary in the first difference, then, government revenue, \( R_t \) and government spending, \( GV \), will be cointegrated. The results of the GLC test will be discussed in Section 4.

CRDW is a simple test by using the value of the DW statistic in the relevant CI regression and testing the null hypothesis, \( H_0: DW = 0 \) against the critical values given in Engle and Yoo(1991). Critical values of CRDW for different numbers of variables and sample sizes, 50, 100 and 200 are available in Engle and Yoo (1991). For sample sizes of 50, the critical values of CRDW at 1%, 5% and 10% levels of significance are 1.00, 0.78 and 0.69 respectively for two variables. If the estimated value of CRDW is greater than the critical value for the chosen level of significance, the hypothesis of CI between the variables in question can not be rejected. The results of the CRDW test are given in Table 2a.

Johansen's method of testing for multico integration is based on a different econometric methodology from that of Engle and Granger (1987). Johansen (1988) derives maximum likelihood estimators of the CI vectors for an autoregressive process with independent Gaussian errors and then derives a likelihood ratio test for the null hypothesis that there is a given number of CI vectors, \( r \). This method involves estimation of vector autoregression models (VARs) in the relevant variable, for example, our \( R_t, GV_t \) and their first differences, \( DR_t \) and \( DGV_t \).
likelihood ratio test statistic is then derived from selected eigen values of the product moment matrix. Using a single period lag, for the sake of convenience, for the VAR model, the following illustrations can be given in the vector - matrix notation:

$$DX_t = \sum \delta DX_{t-1} + AX_{t-1} + e_t ....(15)$$

In our illustration, X_t is a column vector of order 2, (R_t, GV_t)'. The coefficient matrix A is of the order 4x4 composed of coefficients of R_{t-1} and GV_{t-1} in the two equations of the VAR model. If the rank of the A matrix is 4, then the vector of variables, R_t, G_t are stationary; if the rank of A is zero, the level terms have no effect and a model in first differences is more appropriate and if the rank of A is 0 < r ≤ 4, then there are 4xr matrices, γ and α such that A = γα'. This implies, in turn, that there are r cointegrating vectors e_t = α'X_t.

The results of Johansen's test will be given in Table 2b and will be discussed in section 4.

**Error Correction Models (ECMs)**

The existence of CI between two non-stationary but integrated variables implies that the trends of these variables are linked and, therefore, that the dynamic paths of these variables must be related in some way to the current deviations from the equilibrium relationship between them (Enders, 1995, p.355). In the ECMs (Engle and Granger, 1987), lagged residuals from the long-run CI regression serve as a measure of short-run dynamics. According to the Granger Representation Theorem, cointegrated variables which are integrated of order one (I(1)) can always be represented in an error correction model (ECM). An ECM in the simplest form for the variables under our study, R and GV can be written as

$$DR_t = \rho e_{t-1} + bDGV_t + u_{1t} .......(15i),$$

where e are the residuals from the CI regression (12). These lagged residuals stand for the error correction (EC) term as they measure the extent of drift from the structural relation which will be corrected in the long-run. For the ECM to be a meaningful representation, the coefficient of the
EC term should turn out to be negative and statistically significant. The negative sign of $\rho$ ensures that the short-run deviations from the long-run cointegrated relationship are temporary. The parameters estimated from an ECM equation such as (15i) are the short-run parameters. In the ECM formulation of the regression with the dummy variables, the intercept dummy, $D_1$ does not exist as it disappears on taking first difference. So only the dummy variable for slope changes appears. The relevant ECM is, therefore,

$$DR_t = \rho \varepsilon_{t-1} + bDGV_t + (b^* - b)DD_5 + u_{2t} \ldots (15\text{ii})$$

where $\varepsilon_{t-1}$ are the lagged residuals from the regression (12c). The estimation of ECMs gives the researcher an opportunity to compare the long-run and the short-run parameters. In the present study, the ECM formulation will show the short-run relation between government revenue and government expenditure and whether the government budget is balanced in the short-run. Testing the null hypothesis $H_0$: $b = 1$, from the ECM estimation, we can check whether revenue and expenditure are in line with each other.

The ECMs emphasise the use of the "equilibrium error" (as measured by the lagged residuals) which arises from the concept of cointegration meaningfully related to the long-run and yet not distract from the short-run dynamics. More complicated versions of the ECMs include lagged values of first differences of the variables on both sides of the equation and the validity of these lags can be checked in the usual way by looking at the significance of the ordinary t-value of the additional lag at each stage and at the significance of the F-value for joint significance of all the lags. The modified ECM to include a lag in the variables under our study can be written as:

$$DR = \rho \varepsilon_{t-1} + bDGV + b_1DGV(-1) + c_1DR(-1) + u_{3t} \ldots (15\text{iii})$$

where (-1) means a lag of one period in the past. In the same way, one can also add the lagged first differences of variables to get the extended ECM including the dummy variables:

$$DR = \rho \varepsilon_{t-1} + bDGV + (b^* - b) DD_5 + b_1DGV(-1) + b_2DD_5(-1) + c_1DR(-1) + u_{4t} \ldots (15\text{iv})$$
The ECM forms suitable for Granger and Lee's extended approach to CI have two EC terms as they use the two residuals in their test for CI. The simple form, the form with additional lags and the ECM form with the dummy variables are respectively given by:

\[ DR = \rho_{11}\varepsilon_{1t-1} + \rho_{21}\omega_{1t-1} + b_{1}DGV + u_{1t} \quad ...(16i) \]

\[ DR = \rho_{12}\varepsilon_{2t-1} + \rho_{22}\omega_{2t-1} + b_{1}DGV(-1) + c_{1}DR(-1) + u_{2t} \quad ...(16ii) \]

and

\[ DR = \rho_{13}\varepsilon_{3t-1} + \rho_{23}\omega_{3t-1} + b_{DGV} + (b_{*} - b) DD_{5} + u_{3t} \quad ....(16iii) \]

\[ DR = \rho_{14}\varepsilon_{4t-1} + \rho_{24}\omega_{4t-1} + b_{1}DGV + (b_{*} - b)DD_{5}(-1) + b_{2}DD_{5}(-1) + c_{1}DR(-1) + u_{4t} \quad ...(16iv) \]

where \( \varepsilon_{it} \) are the residuals from the respective CI regressions, (12), (12), (12c) and (12c) \( i = 1,2,3,4 \) respectively and \( \omega_{it} \) are the residuals from the third stage regression (14), \( i = 1,2,3,4 \).

In addition to yielding a comparison between long-run and short-run parameters, the ECM forms can also be used for Causality tests. It involves estimation of complementary pairs of ECM equations and testing the significance of all error correction terms jointly. Tests of Granger Causality, however only strictly applies to bivariate causality. So, we write the complementary forms for only (15i) for EGC and (16i) for GLC. The complementary form interchanges the variables R and GV. Therefore, the complementary form for (15i) is:

\[ DGV = \rho^{i}\varepsilon_{r-1*} + b^{*} DR + u_{*1t} \quad ....(17i) \]

The complementary form for the GL approach can similarly be written by interchanging R and GV in the equation (16i):

\[ DGV = \rho_{11}\varepsilon_{1h-1} + \rho_{12}\omega_{1h-1} + b' DR + u_{1t} \quad ....(17ii) \]

where \( \varepsilon_{r*} \) are the residuals from the following CI regression:

\[ GV = a^{*} + b^{*} R + d^{*}t + u_{t} \quad ....(18i) \]

and \( \varepsilon_{1/t} \) and \( \omega_{h/t} \) are the residuals from (18i) and from
$D\varepsilon = \beta /\varepsilon_{t-1} + \chi D\varepsilon_{t-1} + \delta D\varepsilon_{t-2} + \ldots (18\text{ii})$ respectively.

The tests for causality can then be made by forming the null hypothesis: $|\rho|$

$+ |\rho_1| = 0$ in the EGC equations (15i) and (17ii) and by testing the null hypothesis: $|\rho_{11}| + |\rho_{21}|$

$+ |\rho_{11/}| + |\rho_{21/}| = 0$ for the GLC equations (16i) and (17i).

Evidence of government spending Granger - causing government revenue, will understandably be checked by the joint significance of $b$ and the significance of the error correction term(s) in equations (15i) and (16i) with DR as the explained variable (Granger, 1991). It should be mentioned in this context, however, that typical empirical examples of error correcting behaviour are formulated as the response of one variable, the dependent variable, to shocks of another, the independent variable. So the results of the stationarity tests indicating the order of integration, results of CI tests and the consequent estimation of the EC coefficient in the ECM of the dependent variable are often taken to be sufficient evidence for unidirectional causality running from the independent variable. The main results of estimation of various ECMs and the consequent causality tests are respectively given in Tables 4 and 5 and interpreted in section 4.

4. Interpretation of Results

Stationarity tests with and without structural breaks

The results of stationarity tests with and without structural breaks are given in Table - 1. In the stationarity tests without assuming structural breaks, values of the augmented Dickey - Fuller (ADF) - $t$ imply that government revenue, government expenditure, expenditure excluding interest payments on government bonds, government budget deficit, real value of bonds, real interest payments on bonds and discount rate are all stationary in their first difference. The search for breaks in the level and in the slope of the trend is made in the context of equation (13iii) and it reveals the highest DF - $t$- values in absolute terms for all series but two at 1973. The government expenditure
series shows evidence of breaks in the level and the slope of the trend at 1972 and in the series excluding interest payments on bonds there is no significant evidence of breaks. It is important to note that the discount rate series is stationary when allowance is made for breaks as our theory outlined above requires d to be stationary. The same is the case with the real bond series which is also stationary when allowance is made for structural changes. The PP statistics are also statistically significant when the DF–t values are numerically the highest and significant for the relevant years for each series. Structural breaks can be explained sensibly by the oil-price shocks, introduction of three-day working week in majority of industries and various other responses to the OPEC’s major oil-price rise in the early seventies.

**Cointegration regression**

The results of CI tests are given in Table 2. There are two sets of the DF-t values given for each regression, one for the Engle–Granger two-stage test (EGC) and the other for the Granger and Lee’s extended test (GLC). The DF-t value for EGC is insignificant at the 5% level but it is significant at the 10% level using the EY critical value, -2.90, calculated for a sample size of 50 and for two variables (Engle and Yoo, 1991, p. 127). The DF-t value for the deeper test, GLC, is significant at the 5% level, the critical value for 50 observations and two variables being -3.29. So the hypothesis of no cointegration between government revenue and government spending can be rejected by these tests. The value of the CRDW for CI regression of government revenue on government expenditure is insignificant. However, this is not surprising as the CRDW statistic is not asymptotically similar to the EGC or the GLC test statistics. According to Engle and Yoo (EY) (1991, p.128), the Durbin Watson statistic is not a very good test for CI because the discrepancy between the critical values for different systems remain significant even in samples of size 200. As already pointed out in our methodology section, consistency of approach requires that we must take possible breaks into account. As revealed by our stationarity tests there are breaks in the GV
series in the year 1973 and the revenue series has the highest numerical ADF-t-value for 1973. So, we modify our CI regression to include dummy variables for changes in the intercept and in the slope after the year 1973. These results are also given in the same table. In the extended CI regression with the dummy variables, the EGC DF-t value is significant at the 5% level; the CRDW value improves and both the GLC and the CRDW are close to the respective critical values at the 10% level. The significance of the DF-t values (and of CRDW) for CI is more difficult to achieve when one has a larger number of explanatory variables and a smaller sample (a smaller sample). The critical values given in Engle and Granger (1991, p.103) are simulated for 100 observations and, so, they are smaller. The respective critical values prepared by Engle and Yoo (1991, pp.126-128) are larger as they are prepared for various sample sizes starting from 50 and the critical values increase for a larger number of variables but decrease for a larger sample. As we find that in our sample size of 50 the DF-t and the CRDW values improve substantially when we increase the number of variables (adding the dummy variables, that is), we can not reject the possibility that cointegration of the revenue and expenditure can be achieved in a longer run, that is, in a larger sample. This seems to be a sensible conclusion to us in the light of the following admissions by the authors of the Representation Theorem: That the CI "analysis leaves many questions unanswered...There is still no optimality theory for such tests and alternative approaches may prove superior" (Engle and Granger, 1991, p.102).

Estimates of Johansen’s CI test–statistics for equations (12) and (12c) are given in Table 2b. The estimates being obtained from micro TSP version 4.3, we report the eigen - values, the trace test statistic, the p-value and the optimum lag. The p-value of a test of course indicates the probability value or the smallest significance level at which the relevant null hypothesis can be rejected and the optimum length of lag (opt.lag) has been selected by the AIC. For the revenue – expenditure relation ignoring structural changes, the trace test statistic, 17.23, is insignificant at the
5% level but significant at the 10% level, the respective critical values being 17.95 and 15.66 (Osterwald–Lenum, 1992, p.472). So the hypothesis of no cointegration vector can be rejected only at the 10% level in favour of the alternative hypothesis of one cointegrating vector. The results of Johansen’s CI tests for equation (12c) are promising and the null hypothesis of no cointegration vector can be rejected at the 5% level of significance in favour of the alternative hypothesis of one CI vector. The estimated value, 61.27 being greater than the relevant critical value, 48.28 (Osterwald–Lenum, 1992, p.472). So we take the revenue expenditure relation with dummy variables for changes in the slope and the intercept after 1973 as the long–run cointegrated relation. As already revealed, this relation is cointegrated at the 5% level also by the EGC test.

The main results of estimation of revenue - expenditure relations as given in equations (12) and (12c) are given in Table 3. The estimates of the relevant ordinary t-values given in Table 3 indicate a statistically significant drop in the autonomous component of revenue independent of government expenditure and indicate that there is a strong and statistically significant influence of government expenditure on government revenue supporting our theory outlined in section 2. The influence of government spending also significantly went up after 1973, as can be expected in response to the shocks and the total influence of government spending is complete, as the coefficient becomes 0.7793+0.2869 = 1.0662 and the null hypothesis of b=1 cannot be rejected (the t-value being only 0.3889). This, therefore, supports the contention that government revenue and expenditure are congruent. This, in turn, implies that budget deficits are not excessive and that the budget is balanced. The F-value for testing the joint significance of the two dummy variables for structural changes after 1973 is also significant at the 5% level. As already pointed out, revenue and expenditure are cointegrated at the 5% level when the dummy variables are added and at the 10% level when dummy variables are excluded. So, we move on to estimate the error correction forms of the equations (12) and (12c). The ECM of equation (12) will be required for making
causality tests.

Estimates of the ECMs

We have estimated the simple and the extended ECM equations of all cointegrated (or nearly cointegrated) regressions. So, the two ECMs for the original regressions without allowance for structural changes after 1973, equations (15i) and (15iii) and the two ECMs for the regression adding dummy variables, that is equations (15ii) and (15iv). The estimates of the parameters indicate that all the EC terms have the correct negative sign and all but two of them are significant at the 5% level; the EC coefficients in (15i) and in (15ii) are clearly significant at the 10% level and they are very close to the 5% critical value of the ordinary t, which lies in between -2.021 for 40 and -2.00 for 60 degrees of freedom respectively. In the extended ECMs with additional lags, the significance of the EC term(s) improves substantially and the significance of the dummy variable for slope change is retained. Of course, the dummy variable for changes in the intercept gets eliminated from all first difference forms. The F-values for testing the joint significance of the additional variables and/or the additional lags are all significant at either the 5% or at the 1% level.

We can now make a useful comparison of the estimates of the short-run with the respective long-run parameters of our revenue - expenditure relations.

The short-run values of parameters, as already pointed out, are turned out by the ECM forms and the long-run values by the corresponding CI regressions. Comparing the long-run estimates from (12) with the short-run estimates from (15i), we find that the short-run coefficient of government spending is 0.43 and it is significantly less than the long-run coefficient, 0.86, the estimated t-value for testing the null hypothesis of equality of the two coefficients is 6.4179. Comparing the estimates of (12c) with those of (15ii), we find that the short-run coefficient of government spending is 0.43 which only marginally increases to 0.432 after 1973 and the comparable long-run coefficient is 0.78 increasing to 1.0662 after 1973 and the difference between
the short and the long-run coefficients is significant at 5% level, the computed t-value being 2.07. Comparing the estimate of the long-run coefficient derived from (12) with that from extended ECM form (15iii), we confirm that the short-run estimate of the coefficient of government spending is significantly less than the long-run coefficient. The ordinary t-statistic of the additional lagged variable for revenue is significant and so is the F-statistic for the inclusion of lagged first differences (F=5.50 which is significant at the 1% level ) implying that the ECM form(15iv) including dummy variable is superior to the simple ECM (15ii) following the logic of the general to specific modelling criterion. The value and the t-statistic of the error correction coefficient both improve and the error correction coefficient is now clearly significant. The estimated short - run coefficient of government spending now is 0.46 and there is no significant change after 1973. The latter is not a surprising aspect of the results as we are concerned with the short-run here. The short-run coefficient is significantly lower than the long - run coefficient , the t-value for testing the relevant null being 7.3647. This discrepancy between the short-run and the long-run values of b is in accord with our theory outlined above. In the short-run, budget deficits do result ,the estimated value of the coefficient of expenditure being significantly less than 1; but in the long run, these deficits are not sustainable and the budget has to balance and this is demonstrated by a unitary coefficient of government spending in the main regression (12c). These results, in turn, therefore, point to the importance of capital flows to balance the budget in the long run. The adjusted $R^2$ in the ECM equation (15iv) is very low but the Durbin Watson statistic improves significantly (that is to 2.19) finally removing any signs of "spurious " regression. Other indications of the data supporting our theory are summarised here briefly. The adjusted $R^2$ is 0.9689 in the CI regression with the dummy variables and it is 0.9653 in the CI regression without the dummy variables. Both dummy variables are significant as judged by the respective t-values and by the F-value. The latter is computed as 5.0844 and the critical value of F for 2,46 degrees of freedom lies between 3.23 and
3.15 for 5% significance level. The significance of the dummy variables implies that the parameters changed due to the oil price shock in the early seventies which affected both government revenue and expenditure.

**Causality Tests**

Finally, the results of the bivariate causality tests in terms of the ECMs, (16i) and (17ii), that is by the GLC tests, provide evidence on Granger - causality running from government expenditure to government revenue but there is no evidence on reverse causality running from revenue to expenditure. However, as already noted in the t-value of the EC term in the EGC form (15i), the evidence for causality from government expenditure to revenue is insignificant at the 5% level but there is clearly no evidence on reverse causality in the ECM (17i).

**5. Conclusions**

Our study reflects the results of a thorough cointegration analysis. Our main findings are that government revenue and government spending are congruent in the long run. Budget deficits do appear in the short run but they are brought in line with government revenue in the long run. So, the annual time-series data for the U.K. for 1948-1997 donot provide evidence that budget deficits have been excessive in the U.K. economy. There have been changes in the level and slope of the trend in the revenue and the expenditure series around 1973. Accordingly, there have been some changes in the short-run and the long-run parameters around that date but the results still support the hypothesis of a balanced budget in the long-run. Therefore, the fiscal policies followed by the Chancellors have been prudent.
<table>
<thead>
<tr>
<th>Variables</th>
<th>ADF-t</th>
<th>Maximum ADF-t</th>
<th>PP Statistic</th>
<th>TB</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>-3.2164</td>
<td>-2.8571</td>
<td>-12.878</td>
<td>1973</td>
</tr>
<tr>
<td>DR</td>
<td>-3.7506*</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>DGV</td>
<td>-4.3015**</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>-1.0708</td>
<td>-6.8054**</td>
<td>-35.1959</td>
<td>1973</td>
</tr>
<tr>
<td>Dd</td>
<td>-5.2861**</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

* implies significance at 5% level and ** implies significance at 1% level

Table 2a. The Results of Cointegration Tests by the Engle and Granger, Granger and Lee and by the CRDW Criteria

<table>
<thead>
<tr>
<th>Variables</th>
<th>DF-t</th>
<th>k</th>
<th>CRDW</th>
</tr>
</thead>
<tbody>
<tr>
<td>R, GV</td>
<td>EGC</td>
<td>-2.6690</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>GLC</td>
<td>-3.5668*</td>
<td>2</td>
</tr>
<tr>
<td>R,GV,D1,D5</td>
<td>EGC</td>
<td>-3.8589*</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>GLC</td>
<td>-3.2290</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 2b. The Results of Multi-Cointegration Tests by Johansen’s Method

Cointegration Regression (12) with Lag = 3

Length of Lag
<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</thead>
<tbody>
<tr>
<td>Eigen Value 1</td>
<td>0.22221</td>
<td>0.22434</td>
<td>0.22121</td>
<td>0.23046</td>
</tr>
<tr>
<td>Eigen Value 2</td>
<td>0.072902</td>
<td>0.14454</td>
<td>0.15605</td>
<td>0.13164</td>
</tr>
<tr>
<td>Null Hypothesis : $r=0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trace Test</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistic</td>
<td>14.38803</td>
<td>17.2269</td>
<td>16.78679</td>
<td>15.32679</td>
</tr>
<tr>
<td>$p$-Value</td>
<td>0.16045</td>
<td>0.0684</td>
<td>0.0791</td>
<td>0.12350</td>
</tr>
<tr>
<td>Null Hypothesis: $r \leq 1$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trace Test</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Statistic</td>
<td>3.33064</td>
<td>6.5571</td>
<td>6.7864</td>
<td>5.3636</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.0639</td>
<td>0.0091</td>
<td>0.0099</td>
<td>0.0184</td>
</tr>
<tr>
<td>Opt. Lag</td>
<td>1</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Cointegration Regression (12c)</td>
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<td></td>
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<tr>
<td>Length of Lags</td>
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</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Eigen Value 1</td>
<td>0.59103</td>
<td>0.42790</td>
<td>0.55235</td>
<td>0.62782</td>
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<td>Eigen Value 2</td>
<td>0.22503</td>
<td>0.31930</td>
<td>0.32347</td>
<td>0.45164</td>
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<td>Eigen Value 3</td>
<td>0.12017</td>
<td>0.15363</td>
<td>0.19484</td>
<td>0.28986</td>
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<td>Table 2b Continued</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lag</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Eigen Value 4</td>
<td>0.05874</td>
<td>0.04979</td>
<td>0.0669</td>
<td>0.10503</td>
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<tr>
<td>Trace Test</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Statistic</td>
<td>56.1797</td>
<td>44.1157</td>
<td>50.3389</td>
<td>61.2739 *</td>
</tr>
</tbody>
</table>
**Table 3** The Main Results of Estimates of Revenue – Expenditure Equations

<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimates of coefficients</th>
<th>DW</th>
<th>Adjusted R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>(12)</td>
<td>a    b    (a⁺-a)     (b⁺-b)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-18.83</td>
<td>0.87  -    -</td>
<td>0.34</td>
<td>0.9685</td>
</tr>
<tr>
<td>t-value</td>
<td>-0.68 38.81**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(12c)</td>
<td>52.63 0.78   -396.48   0.298</td>
<td>0.50</td>
<td>0.9731</td>
</tr>
</tbody>
</table>
t-value 1.12 12.58** -3.1565* 3.11*

F-value for comparing (12) with (12c) = 5.0844**

* and ** respectively imply significance at 5% and 1% significance levels

Table 4  The Main Results of Estimates of ECMs

<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimates of coefficients</th>
<th>DW</th>
<th>R²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EC term(s)</td>
<td>b</td>
<td>(b - b)</td>
</tr>
<tr>
<td>(15i) EG</td>
<td>-0.13</td>
<td>0.38</td>
<td>...</td>
</tr>
<tr>
<td>t-value</td>
<td>-1.96</td>
<td></td>
<td>4.026**</td>
</tr>
<tr>
<td>(15ii) EG</td>
<td>-0.15</td>
<td>0.40</td>
<td>0.003</td>
</tr>
<tr>
<td>t-value</td>
<td>-1.94</td>
<td>3.57</td>
<td>0.11</td>
</tr>
<tr>
<td>(15iii) EG</td>
<td>-0.15</td>
<td>0.37</td>
<td>-0.10</td>
</tr>
<tr>
<td>t-value</td>
<td>-2.30*</td>
<td>3.86**</td>
<td>-0.73</td>
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</table>

F-value for comparing (15i) with (15iii) is 11.9443**

(15iv) EG -0.211 0.43 -0.004 -0.12 -0.02 0.51 2.19 0.003

Table 4 Continued

| t-value | 3.84* -0.16 -0.90 -0.83 3.61 |
| F-value for comparing (15ii) with (15iv) is 5.0347* |

<p>| t-value |          |
| ρ₁₁     | ρ₁₂      |
| (16i) GL| -0.15 0.018 0.40 ... ... ... 1.15 -0.22 |</p>
<table>
<thead>
<tr>
<th>t-value</th>
<th>-2.12*</th>
<th>1.49</th>
<th>4.16**</th>
</tr>
</thead>
<tbody>
<tr>
<td>(16ii) GL</td>
<td>-0.16</td>
<td>0.011</td>
<td>0.38</td>
</tr>
<tr>
<td>t-value</td>
<td>-2.28*</td>
<td>0.96</td>
<td>3.89**</td>
</tr>
<tr>
<td>F-value for comparing (16i) with (16ii) is 3.39*</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(16iii) GL</td>
<td>-0.16</td>
<td>0.02</td>
<td>0.43</td>
</tr>
<tr>
<td>t-value</td>
<td>-2.13</td>
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<td>3.80</td>
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<tr>
<td>(16iv)</td>
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<td>0.01</td>
<td>0.45</td>
</tr>
<tr>
<td>t-value</td>
<td>-2.74*</td>
<td>1.17</td>
<td>3.93*</td>
</tr>
<tr>
<td>F-value for comparing (16iii) with (16iv) is 4.13*</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* and ** imply respectively 5% and 1% level of significance

Table 5  The Main Results of Granger - Causality Tests : From ECMs (15i) and (17i) and from (16i) and (17ii)

### Direction of Causality

<table>
<thead>
<tr>
<th>t-Values of EC Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>From GV to R: (15i)</td>
</tr>
<tr>
<td>Form R to GV (17i)</td>
</tr>
</tbody>
</table>

Table 5 Continued

<table>
<thead>
<tr>
<th>From GV to R (16i)</th>
<th>-2.12*</th>
<th>1.49</th>
</tr>
</thead>
<tbody>
<tr>
<td>From R to GV (17ii)</td>
<td>-1.44</td>
<td>-0.54</td>
</tr>
</tbody>
</table>

+ and ** respectively imply 5% and 1% significance levels
Bibliography


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