

# **Decision-Making and the Role of Feedback in Complex Tasks**

**By**

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Many others have provided assistance along the way. Embarking on any study relating to judgment and decision-making should, at some stage, make one aware of their own processes and biases. Apparently simple questions, such as which areas of existing research should be highlighted and which inevitably excluded, while maintaining a narrative with sufficient integrity, take on particular significance. My thanks therefore go to everybody who



tempered any biases which I may have had, contributing, I hope, to some degree of rational coherence in the pages which follow.

Sheridan Taylor

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## **NOTE ON THE INCLUSION OF PUBLISHED WORK**

Chapter 8 describes an empirical study of behaviour and decision-making in financial markets. This study was published in the Journal of Behavioral Finance (Taylor & Taylor, 2016).

In order to achieve a high degree of authenticity, a financial simulation platform was used for the study enabling access to actual historical market and other data. That platform had been developed by, and is the property of, Alpha-Q Investment Management (AQIM), an investment management company engaged in the development and application of quantitative techniques for asset management. The platform is used by that company primarily to test market trading algorithms and for general simulation. Considerable quantities of historical data are accessible via the platform, including historical closing prices for a large number of securities and asset classes as well as extensive fundamental data relating to companies listed on the major exchanges. This data, together with all of the simulation software used for the experiment, is the property of AQIM.

The use of actual historical data, although anonymised in order to avoid easy identification, added richness to the study as the experimental environment reflected actual outcomes within markets. With the sophisticated software platform providing the mechanism through which the required data could be accessed and manipulated, the exact specification for the study, including its objectives and methods, were devised entirely by the author of this thesis. The same applies to analysis of experimental results data generated.

## GENERAL ABSTRACT

This thesis investigates the process of decision-making in relation to complex tasks and considers the important role which dynamic information and real-time feedback play in shaping response behaviour and adaptation within such environments. Through empirical studies, the thesis explores the extent to which decision-makers can be said to act rationally when challenged by complex decision-making environments. Evidence relating to demand for information and the impact of feedback on behaviour is provided with two studies: The first uses a simulated auction platform to examine behaviour within overlapping auctions of short duration with close-to-identical items and minimal participation costs. Mouse tracking is used to capture data on relevant interactions of participants with the simulated online platform, including switching behaviour independent of bidding. The resulting data suggests that participants did behave in a manner consistent with utility maximisation, seeking to acquire the item at the lowest possible price and showing no bias in terms of auction preference. The impact of fixed-price offers in the form of a “Buy it Now” option is also examined with some evidence that participants again seek, and respond to, current information when deciding on their bidding strategy.

The second study is a test of the impact of real-time feedback and demand for information within the context of financial markets. The study again uses a novel simulated environment which provides access to considerable amounts of relevant data which participants can choose to access. In addition, participants are exposed to regular feedback with regard to their own performance. Overall, demand for information is found to be dependent upon the type of feedback received and its context. Decision-makers then appear to behave objectively, apparently seeking the latest available information to support current

decisions, although investor style is found to be important in determining overall trading propensity.

The thesis starts by considering a number of the foundations and pathways which run through the judgment and decision-making literature. It is not a complete description, review or analysis of all of the prevailing lines of enquiry. Nevertheless, it seeks to achieve coherence in terms of bringing together some of the key themes dealing with risky choice under conditions of uncertainty and ambiguity. The field of judgment and decision-making is inevitably vast; its scope owing much to the fact that it transcends individual disciplines. The emergent behavioural sciences thus draw together important strands from various sources, notably Economics and Finance. In many areas, psychological traits can be applied to explain inconsistencies which are found in classical theory of rational behaviour. The recognition of behavioural traits has thus contributed greatly to the evolution of decision-making theories under conditions of uncertainty and ambiguity which are, in many cases, substantially more adaptable and robust than early normative theories of rational behaviour.

The classical approach to rational decision making within Economics, together with some theoretical and empirical challenges to it, are considered in Chapter 1. It is here that we are introduced to the Rational Man. Like the mythical creatures found in Classical Antiquity, the Rational Man does not actually exist in the real world; he is nevertheless central to the concept of utility maximising rational choice which provided much of the foundation of Economics. Developments of expected utility theory (EUT) are considered, including its replacement of expected value, and the formalisation of rational behaviour within the context of axioms. When those logical axioms apply, decision-makers can be said to behave *as if* they are utility maximisers. The chapter ends with some empirical evidence, showing the

types of approaches often used to explore rational decision-making. Some violations of EUT are explored, both in relation to notional gambles and consistency with regard to revealed preferences. Chapter 2 extends the narrative by considering rational decision-making in cases where there is no objective information about possible outcomes. Subjective utility theory (SEU) is then introduced, describing objective functions based upon preferences derived from combined utility and probability functions. The implications of the Allais' and Ellsberg paradoxes are discussed, along with some possible solutions. It is here that we explore the concepts of uncertainty and ambiguity in more details and consider some theoretical formulations for addressing them.

Chapter 3 covers the significant contribution to decision-making under conditions of uncertainty provided by Prospect Theory and, later, Cumulative Prospect Theory (CPT). Their evolution from the pioneering work of Markowitz is discussed within the context of reference points relative to which outcomes can be evaluated. The significance of stochastic dominance and rank dependence are explored. By this stage, we have examined numerous theories which have fundamentally transformed standard EUT into much more flexible and adaptable frameworks of rational choice. The core concepts of utility maximisation remain yet the initial, strictly concave utility function describing diminishing marginal utility is now substantially replaced by more complex weighted preference functions.

From this theoretical base, the process of choice reduction and the application of heuristics in decision-making are considered. We again describe axiomatic behaviour compatible with rational choice. Therefore, decision-makers faced with multiple choices about which there may be little or no objective information about likely outcomes can nevertheless develop rational beliefs and expectations which can then be applied to reduce

complex tasks to more manageable proportions. As well as considering these aspects from the point of view of actual choices, we also consider the processes by which decisions are taken. Thus, process tracing methods are introduced into the discussion. The chapter also explicitly considers the role of feedback in decision making. This includes a consideration of Bayesian inference as a process for updating probabilistic expectations subject to new information.

From considering theoretical formulations from which we can judge rational behaviour, Chapter 5 looks at evidence for sub-optimal decision-making and bias. Bias with regard to probability assessments are considered along with empirical evidence of bias in relation to intertemporal discounting. Sunk cost bias is also considered as a clear example of irrational behaviour, leading in to a specific discussion about a number of persistent behavioural biases identified within financial markets. As an introduction to later chapters, this also covers the basic theoretical principles of market efficiency and evidence that real markets fail to adhere to those principles in important ways.

Chapter 6 and 8 describe the empirical studies with Chapter 7 providing a more detailed introduction to the financial markets experiment, considering aspects of market efficiency, models of behaviour and other empirical evidence.

# CHAPTER 1

## THE CLASSICAL PARADIGM

### 1.1. Decision-Making Under Conditions of Uncertainty

Decision making is a cognitive process of varying complexity usually conditioned by input signals interpreted and processed in accordance with internal cognitive perceptions of external interdependencies from which the set of perceived choices is derived. Usually, choices are made within a context of risk and uncertainty. Certain decision tasks, such as choosing between a square of milk or plain chocolate, can be trivial, involving limited choice and offering very little variability in terms of perceived outcome. Such tasks may be considered by the decision-maker to be of marginal or transient consequence with little cognitive resource being applied actively to the choice selection. Many other decisions, while being commonplace or routine in terms of frequency, may carry far greater consequence and significance and offer many alternative choices which may, individually or collectively, involve much greater uncertainty with regard to outcome. Further, certain decisions may be unitary in terms of outcome and effect. For example, the decision to try a new brand of coffee is likely to have a limited, relatively transient and non-consequential future impact on the individual even if the brand, or even the product, is subsequently rejected. Neither the decision-maker's overall state of well-being nor his scope for future action is necessarily compromised beyond repair. Other decisions, such as the degree subject pursued at university, can be far more significant in terms of enabling or precluding future potential options, for example, in terms of possible careers. Such decisions therefore feed, unwittingly or consciously, into a broader matrix of sequential decisions which then become interlinked due to their inter-temporal framing of available choice sets. Consequently, more complex

decisions usually impose much greater cognitive load, potentially influencing broader perceptions, uncertainty and ambiguity (Kirschner, 2002; Paas, Renkl & Sweller, 2004; Polisson, Quah & Renou, 2017).

From these few examples alone, it may be inferred that certain decision-making tasks which many people face with some regularity can be perceived as complex and of significant consequence. Often, such tasks are associated with, and influenced by, various forms of feedback. Feedback, in this context, relates to all data, stimuli and cues which inform the cognitive process, thereby contributing to the development of a perception of the environmental framework within which such decisions are made (Serman, 1989; Wickens, 1987). Therefore, from a cognitive perspective, how feedback is presented, framed, perceived and processed can shape decision bias and choice outcome (Isen & Means, 1983; Kleinmuntz, 1993 ; Checkel, 2008). Consequently, it is apparent that assessing the degree and nature of uncertainty associated with higher level decision tasks might also be perceived as being a process of varying complexity. Even if risk and uncertainty are defined narrowly in terms of likely deviation of outcome from expectations, the range and scope of decision-making tasks obviously varies significantly and, in the complex form, stretches practical ability to make anything other than generic assessments of potential outcomes. Thus, while one-off decisions of transient importance may appear to be bounded in terms of the impact of potential ranges of outcomes, interrelated or interdependent choices are likely to be much less well defined, both in terms of scope for possible outcomes and the uncertain timing and variability of those outcomes (Chang & Kim, 2017).



## 1.2. Summary of Early Approaches to Decision-Making

As a consequence of its fundamental importance to most areas of human endeavour, many disciplines have considered the process of decision-making and its implications for explaining and predicting behaviour. Quite naturally, economists have long embraced the study of individual and collective decision-making, developing models and theories largely compatible with other tenets of the discipline. Psychologists have also pursued the topic, originally in the form of learning and behavioural expression and, more latterly, through the lens of cognition and neuroscience. Consequently, the processes through which decisions are made have enhanced the study of observable outputs, in the form of actual choices made, the two combining to make a significant contribution to current behavioural sciences.

Essentially from its inception as a formal discipline, the concept of rational behaviour lay at the heart of Economics. This advanced the proposition that various economic agents (consumers, businesses and governments) would pursue choices which maximised defined objectives. From this basis, theories were developed designed to explain and prescribe the way in which these different agents functioned and inter-depended (Barber, 2010; Hunt, 2002; Screpanti & Zamagni, 2005). Yet, while early economists debated what was and what should be (a normative approach), it was largely left to mathematicians such as Fermat, Pascal and Bernoulli to consider what could be, the latter, in particular, prompting changes in the way economists thought by exposing contradictions and paradoxes arising from prevailing consensus, and laying at least some of the foundations for what has now become a more robust concept of judgment and decision making under conditions of uncertainty (Daston, 1995). It is, therefore, from within Economic doctrine, albeit influenced by and, to some

extent, responsive to developments and inputs from outside the discipline, that we find the most highly developed early models of rational behaviour and choice.

In the earliest formulations, price was generally seen as a key determinant of rational economic behaviour (Friedman, 1953). Since income is the price of labour, and income provides the means to extend choice through selective consumption, monetary gain was initially proposed as the primary objective of a Rational Man, just as profit maximisation was seen as the only rational goal for private companies (Robbins, 1929). From that basis, it appeared logical that early theories of decision-making in the face of uncertainty should be expressed and framed in the form of potential monetary pay-offs from various gambles with uncertain outcomes. Thus, rational consumers would seek to allocate their available resources in an optimal manner, maximising the expected pay-off from the choices made. As a result, money would only be exchanged for goods if the perceived expected value derived from those goods was at least equal to the perceived value of the money expended on them (Marschak, 1938). This concept of expected value can easily be expressed mathematically in the following form;

$$EV = \sum_{i=1}^n p_i x_i, \quad \text{Equation 1.}$$

where  $p_i$  and  $x_i$  denote the probability and amount of money (value) associated with each available outcome  $i$ . Assuming an income constraint, the objective function is maximised by allocating available income in such a way that the aggregate independent payoffs ( $p_i x_i$ ) derived from the available range of options ( $i = 1 \dots \dots \dots, n$ ) is maximised.

This seemingly simple proposition implies some important tenets about rational behaviour: Essentially, the Rational Man is assumed to behave in a manner consistent with objectively logical decisions based upon current expectations; past decisions and outcomes should not affect these current decisions other than through the shaping of probabilistic assumptions. The rational decision-maker is essentially assumed to apply unbiased Bayesian (stochastic) principles in forecasting the outcomes of current choices. Therefore, the assumed probability of the likelihood of an event derives from some past knowledge of conditions which may influence that event. On this basis, decision-makers are expected to modify their expectations with regard to the likelihood of events as more evidence pertaining to those events becomes available (Anderson, 1998). While rational decision-makers are therefore free to differ in terms of their expectations, they are nevertheless assumed to be motivated by the prospect of greater monetary wealth as it is through this route that greater consumption becomes possible (more is therefore always better than less).

This apparently non-controversial conclusion nevertheless proved problematic when translated into the realm of games offering various uncertain or ambiguous potential outcomes. Even within very simply framed propositions, inconsistencies and paradoxes were found to exist. In essence, wealth expressed in monetary terms assumes a linear scale implying that equivalent differences between monetary amounts should be considered equally attractive by rational decision makers, regardless of their absolute magnitude and impact on total wealth. However, the proposition that an increase in total wealth from £50 to £100 is likely to result in the same sense of satisfaction as an increase from £1,000,000 to £1,000,050 is open to challenge as the latter level implies some degree of existing comfort while the former represents an essential improvement in general subsistence levels of income or wealth. The impact on the change in quality of life from the same absolute increase in wealth of £50 is

therefore unlikely to be viewed equally across the two states. Similarly, based upon a purely numeric scale, a doubling of income implies a doubling of the satisfaction derived from that income. Such a proposition again ignores the existence of satiation, which implies that the desire to consume more of any particular product is both limited and finite.

### **1.2.1. The St Petersburg Paradox**

A practical challenge to the prevailing theory of monetary return was expressed mathematically in the form of the St Petersburg Paradox. The Paradox, originally presented by Bernoulli (1738), proposed a type of lottery, or game of chance, using a fair coin. The game proceeded on the basis that a coin is tossed until a head appears. If the head appears on the first toss, the lottery pays 1 monetary unit and the game ends. Should a head be returned on the second toss, the lottery pays 2 monetary units, 4 monetary units if it occurs on the third toss, with each subsequent toss of the coin leading to a doubling of the lottery pay-out until a head is tossed, at which point the game ends and the lottery pays out its cumulated value. The question posed by Bernoulli related to the price which any rational person would be willing to pay in order to enter such a lottery.

The Paradox derives from the fact that there is no obvious solution to the problem unless other assumptions are made. In theory, the player's expectations range from one monetary unit (a head is returned on the first toss, terminating the game) to infinity (no head is ever returned). In between those extremes, the player faces the prospect of various payoffs with associated, albeit rapidly diminishing, probabilities determined in accordance with binomial theory. The prospects of substantial payoffs thus become vanishingly small, but not zero.

The response to the Paradox, proposed initially by Bernoulli himself in answer to his own question, involved imposing an effective upper boundary on the possible returns. The apparent solution lay in the proposition that the broader value of money is derived not simply in accordance with its quantum but in proportion to the satisfaction or pleasure which stems from the uses to which it can be put (Gigerenzer & Selten, 2002). In effect, therefore, a psychological component to decision-making was being inferred. The contention was that, just as satisfaction from consumption is subject to diminishing returns, so too is the change in satisfaction derived from ever increasing amounts of money. Therefore, while increasing amounts of money afforded greater choice, both in terms of absolute levels and breadth of consumption, there could only be a finite level of overall consumption which any individual could undertake (Brito, 1975). Consequently, since the psychological value of money is now measured not solely by its quantum, but on the basis of the satisfaction which can be derived from its allocation to consumption, the perceived psychological value of money must itself ultimately be finite. This is nothing more than an extension of the economic “law” of diminishing marginal utility to money itself, an apparently reasonable transformation since the value of money depends directly upon its economic usage rather than its own existence as specie, especially in fiat form.

Bernoulli’s concept of utility not only broke the dependence upon expected monetary value as the basis for rational decision-making under conditions of uncertainty but also replaced the assumption of linearity of effect by describing a concave utility function. In mathematical terms, the Expected Value function described in Equation 1 was thus replaced by an Expected Utility function of the form;

$$EU = \sum_{i=1}^n p_i x_i^a, \quad \text{Equation 2.}$$

where the  $a$  term describes the marginal utility function of the decision-maker. Values for  $a$  less than 1 define a concave utility function of the general form shown in Figure 1. The objective measure of money, based upon its quantum, is now replaced by a subjective value of utility which changes according to levels of consumption. In addition, the rate of decline in marginal utility per unit consumed, defined by  $a$ , can vary from one decision-maker to the next (Allen, 1933).

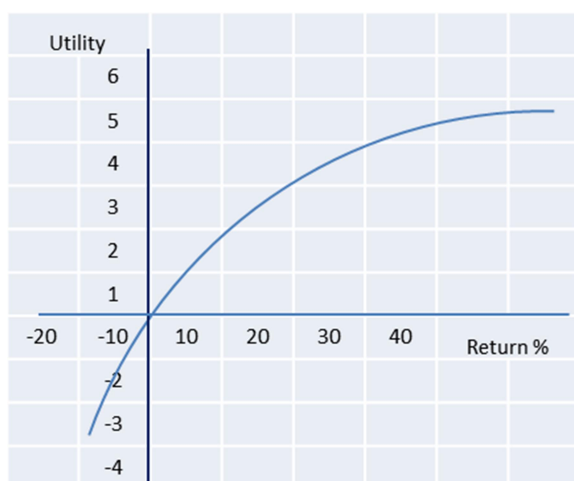


Figure 1. A representative utility function, monotonically increasing with diminishing marginal utility

Expected utility theory marked a significant advance in approaches to decision-making, not least in allowing both risk-seeking and risk-averse behaviour to be accommodated within the same framework. The essential concept of rational behaviour is maintained with decision-makers deemed to allocate their resources in such a way that expected utility is maximised; in general terms, more is still preferable to less. With regard to the St Petersburg Paradox, the theory also provided the basis for a solution as it enabled an upper bound to be applied to the problem. In essence, as marginal utility or the rate of increase in total utility, declines, a point is eventually reached at which marginal utility is close to zero. Based upon that assumption,

the upper bound offered by the Paradox is no longer infinite but becomes, to all intents and purposes, a finite amount determined by marginal utility. Within the parameters of the Paradox, this can then be translated in a straightforward manner to derive a maximum monetary amount a rational decision-maker would be prepared to pay to enter such a lottery. For example, if the threshold for maximum utility derived from any level of wealth is assumed to be £2<sup>24</sup> (£16.8m), with gains over and above this threshold deemed to be “meaningless” (close to zero in terms of marginal utility), then the solution to the Paradox becomes;

$$\sum_{n=1}^{24} 2^{n-1} \binom{1}{2} + \sum_{n=25}^{\infty} 2^{24} \binom{1}{2} = 13 \quad \text{Equation 3.}$$

In this case, the player, possessing the required appetite for risk, expects to receive £13 from this lottery, which then becomes the rational breakeven price for participation (Shafer, 1988).

### 1.3. Concave Utility Functions and Loss Aversion

A concave utility function has important properties in relation to risk preferences. Essentially, the curve suggests generalised risk aversion; as consumption increases, the utility-adjusted “value” of that consumption becomes increasingly discounted by diminishing marginal utility. Consequently, it is perfectly rational for risk preferences to change at different point along the utility function (Friedman & Savage, 1948; Pratt, 1964; Rabin, 2000).

A further important property of a concave utility function is that it implies loss aversion whereby rational decision-makers do not view gains and losses of equal magnitude as

offsetting propositions. This can be illustrated by considering a notional, fair gamble offering a 50% chance of a 10% gain or a 50% chance of a loss of 10%. The expected payoff from such a gamble is, of course, zero ( $0.5 \times £10 + 0.5 \times -£10 = 0$ ). The *ex post* utility of a zero return is also zero. However, from the slope of the curve representing the utility function shown in Figure 1, it can be seen that the prospective impact on utility of the two possible outcomes is not equal. From the chart, it is seen that a gain of 10% increases overall utility by 1 *utile* while a 10% loss reduces utility by 3 *utiles*. We are therefore faced with the following proposition based upon the respective probabilities of the outcomes and the change in utility implied by those outcomes;

$$U = (0.5 \times 1) + (0.5 \times -3) = -1 \quad \text{Equation 4.}$$

From Equation 4, we see that the combined effect of the two potential outcomes is a decline in overall utility meaning that a 50:50 gamble with equivalent payoffs is a less attractive proposition than the certainty of losing and gaining nothing (i.e. rejecting the gamble and maintaining the status quo). Therefore, while the expected monetary returns from the gamble and doing nothing are the same (zero), the combined expected utility is lower in the case of accepting the gamble.

Loss aversion, as a rational psychological trait, also finds support from basic mathematical principles if outcomes are viewed as rates of return. This stems from the fact that the compounding effect of a sequence of a gain followed by a loss of equal magnitude, with all gains and losses expressed in percentage terms, leads to degradation of the compounded sequence. For example, assuming base capital of £100, a repetitive sequence of a



10% gain followed by a 10% loss results in the following compounding sequence ( $t_0$  .....  
 $t_n$ );

£100.00<sub>t<sub>0</sub></sub>, £110.00<sub>t<sub>1</sub></sub>, £99.00<sub>t<sub>2</sub></sub>, £108.90<sub>t<sub>3</sub></sub>, £98.01<sub>t<sub>4</sub></sub>.....

This result can be generalised to state that, the rational expectation of the residual return of a compounded series generated from a large number of trials with individual, independent returns drawn randomly from a normal distribution with the characteristics of  $M = 0$ ,  $SD = x$ ;  $x > 0$ , is a value less than zero.

### **1.3.1. Utility, Revealed Preference & Axioms**

Rational behaviour based upon utility maximisation enhanced the basic economic theory of supply and demand as consumers were assumed to rebalance choices based upon changes in income and / or relative prices in accordance with their marginal utilities (Allen, 1934). Nevertheless, practical issues remained in terms of objectively measuring the strengths of preferences. Thus, while the rankings of rational decision-makers could be determined by observation, there was apparently no cardinal scale which could be imposed upon these ordinal rankings. As a result, it was not clear how comparisons could be made between consumers. Therefore, while it may be assumed that all consumers would pursue the same objective of maximising their individual utilities, the basic economic problem of satisfying infinite demand with finite resources in the most efficient manner remained unresolved. Essentially, given an existing allocation of resources in a resource-constrained world, aggregate utility is only maximised if there is no other allocation of those same resources

capable of increasing utility still further. Unless there is a mechanism through which such reallocations can be measured with some degree of objectivity, it becomes impossible to compare alternative allocations across the whole economy.

Samuelson (1938) attempted to deal with this problem through the concept of revealed preference and linking this with prevailing prices. The concept postulated that relative preferences are revealed by the actual choices of rational decision makers. Therefore, if sufficient combinations of relative prices for two goods are presented to the Rational Man, it should be possible to develop an extensive locus of points at which he is found to be indifferent between the two goods. Such locus points can then be extrapolated to create continuous indifference curves (assuming perfect divisibility of goods). At each point along such a curve, the rational consumer derives equal levels of utility and so should be indifferent between the various combinations offered to him in accordance with the combinations represented by a particular indifference curve.

In theory, consumers are assumed to face a range of indifference curves representing higher and higher levels of aggregate consumption as they extend outwards (more is again preferable to less). Therefore, curves representing higher levels of consumption also represent higher levels of utility. If an income constraint is then applied to the indifference curve space, a single point of tangency can be established representing a position of maximum utility from the consumption of the two goods in that combination (Figure 2.).



Figure 2. Indifference curves with a fixed income constraint

To illustrate; the indifference curves labelled *a* and *b* represent potential consumption combinations of goods *x* and *y* in relation to which the consumer is indifferent. In other words, a rational consumer would be equally happy to consume any combination of *x* and *y* shown on any indifference curve as each combination confers exactly the same level of marginal utility. When all marginal utilities are positive, higher levels of consumption result in higher levels of utility, shown by indifference curves expanding to the right. Therefore, indifference curve *b* is said to “dominate” indifference curve *a* as, at each point along the curve, it represents a higher level of absolute consumption, and hence utility, than any level or combination available from curve *a*. The straight line forming a point of tangency with indifference curve *b* represents the income constraint, the exact slope and position being determined by the relative prices of *x* and *y*. Therefore, given that level of income and prevailing relative prices, the consumer can consume either 9 units of *x* and no units of *y*, or 9 units of *y* and no units of *x*. Any point along the income line represents possible combinations, assuming that consumption of the goods is perfectly divisible (i.e. fractional consumption is possible). Since the rational goal is to maximise utility, the consumer chooses the maximum level of consumption possible in

accordance with the highest indifference curve for his level of income (curve  $b$ ). The point of tangency between the income line and the indifference curve  $b$  therefore represents the optimal level of consumption given that level of income. In this example, the consumer would choose to consume 4.5 units of  $x$  and 4.5 units of  $y$ . By extension, utility can also be related directly to relative prices thereby enabling ordinal ranking preferences to be translated into monetary-equivalent values, potentially providing a basis to compare utilities across consumers.

A further process whereby subjective utility could be transposed to objective values was explained in a seminal work by von Neumann and Morgenstern (1944) based upon axioms which must hold if behaviour is wholly rational and consistent with maximising expected utility. The result is that the strength of a rational decision-maker's preferences with regard to "sure options" could be measured. Von Neumann and Morgenstern thus defined the expected utility from one lottery  $L_i$  selected from a set of lotteries  $\mathbf{L}$  as follows;

$$EU(L_i) = \sum_k u(O_{ik}) \cdot p_{ik} \quad \text{Equation 5}$$

where  $O_{ik}$  is the outcome of lottery  $L_i$  which has the associated probability,  $p_{ik}$ . Based upon an assumption of consistency across preferences for all lotteries,  $L_i$  derived from the set  $\mathbf{L}$ , the full set of axioms then takes the following form:

- *Completeness*: rational decision makers can express a clear preference across all paired comparisons such that A is preferred to B ( $A > B$ ), A is inferior to B ( $A < B$ ), or there is indifference between A and B ( $A = B$ ).

- *Transitivity*: expressions of preference must be internally consistent. Therefore, if  $A \geq B$  and  $B \geq C$ , then  $A \geq C$  (if  $A = B$  and  $B = C$  then  $A = C$ ).
- *Continuity*: given  $A \geq B \geq C$ , then there must exist a combination of  $A$  and  $C$  such that  $B$  is considered to be at least as attractive as the combination of  $A$  and  $C$ . The continuity axiom implies that no potential outcome is so negative that a decision-maker would not be willing to assume a gamble where that outcome was possible if a more favourable outcome could also be achieved with a sufficient level of probability. Decision-makers are therefore assumed to be sensitive to the probability profiles of various lottery outcomes from which they then develop orders of preference capable of being represented by a continuous, cardinal preference function.
- *Independence*: given  $A \geq B$ , and assuming a probability  $p \in (0,1)$ , then  $pA + (1 - p) C \geq pB + (1 - p) C$ . Therefore, in the event that two lotteries are combined with a third, the expressed preference between those two lotteries, when presented independently, is maintained. Independence therefore implies that two lotteries with the same probability of an identical outcome will be evaluated independently of the decision-maker's opinion of that outcome; in other words, lotteries with some identical characteristics will be evaluated solely on the basis of the characteristics which differ.

It is worth noting that some elements of this derivation of axiomatic behaviour were not entirely new. *Completeness* and *transitivity* had been referenced by Samuelson (1938), for example, who also proposed *Non-Satiation*, such that more consumption is always preferred to less and *Convexity*, a further mathematical expression of diminishing marginal utility. According to this axiom, combinations of any two goods would be preferred to the consumption of a single good. All such combinations, in fact, would be considered superior

since substitution of one good held in abundance would result in less of a reduction in marginal utility than the marginal gain in utility resulting from allocation to the new good.

The axioms provide formal expressions for the conditions implied by utility maximisation as an objective. It can be seen that certain axioms are logically conditional upon others. For example, without *completeness*, *transitivity* cannot be assumed. Similarly, *continuity* depends upon the validity and existence of the preceding axioms. The important insight provided by von Neumann and Morgenstern lies in the recognition that Rational Man could order the probability combinations of all available states, making it possible to express utility in monetary form. For example, if a rational agent is found to be indifferent between a 50:50 chance of winning £10.00 or nothing versus a guaranteed sum of £7.00, the following can then be derived:

$$U(\text{£}7.00) = .5U(\text{£}10.00) + .5U(\text{£}0.00) = .5(10) + .5(0) = 5 \quad \text{Equation 6.}$$

The utility of £7.00 is therefore found to be the equivalent of 5 *utils*. Consequently, it is now theoretically possible to transform certain preferences and equivalences into cardinal form based upon the perceived value of money.

Certain preferences may not, of course, be transformed directly with such ease in the absence of some objective equivalence in the choice set. For example, a preference for tooth ache over back strain does not readily lend itself to such a transformation. Determining a monetary amount which provides compensation for either affliction, however, could solve such a problem. It is thus the reference back to the common scale which monetary equivalence provides which forms the basis for calibrating the utility scale, thereby transforming utility

itself into a cardinal measure. The result of von Neumann and Morgenstern's formulation of cardinal utility meant that it was now possible to order risky preferences as easily as riskless prospects, at least in principle. Expected utility theory thus became more complete and, equally crucially, empirically testable within a coherent framework, an essential consideration from a behavioural perspective (Edwards, 1954).

#### **1.4. Empirical Evidence**

While theoretical discussions relating to the possibility of measuring marginal utility and divining utility functions remained extensive and robust (Friedman & Savage, 1948, *op cit*; Vickrey, 1945), various efforts were also made to investigate behaviour empirically. Such efforts naturally tested evidence for classical rational behaviour, revealed preference and the existence of behavioural consistency based upon the axioms of expected utility. Testing rational behaviour in simple choice tasks often employed series of gambles offering straightforward pay-offs defined by assumed probability distributions. Examples of such explorations were undertaken by Preston and Baratta (1948) and Mosteller and Noguee (1951), both of which are examined below. Choice consistency explored through revealed preference, a direct test of Samuelson's (1938) concept for assessing rationality from observed behaviour, was in turn examined in an important paper by Sippel (1997), also considered below.

While Preston and Baratta represents a seemingly straightforward experiment, its specification and interpretation of probabilistic choice is flawed. The data is therefore presented and interpreted in accordance with the original findings of the authors and then reinterpreted on the basis of the correct specification of the choice outcomes. The work is included here in order to illustrate how apparently simple propositions can be subject to

misinterpretation in terms of expected utility and expected payoff formulations if not correctly framed. Mosteller and Nogee pursued the same general theme but with a more robust specification.

#### **1.4.1. Testing Mathematical Probability and Monetary Value: Preston & Baratta**

In an attempt to investigate consistency between the psychological interpretation of mathematical probability and monetary value, Preston and Baratta (1948) conducted an experiment using a purpose-built game of chance. The game was played by a total of 50 participants, allocated to 20 groups, either in pairs or in groups of 4. Each group was presented with a pack of 42 cards with each card representing a unique opportunity to win a number of points (the prize) at pre-determined odds (declared probabilities). Six levels of prizes were offered (5, 50, 100, 250, 500 and 1000). These were combined with 7 unique probabilities of success (0.01, 0.05, 0.25, 0.50, 0.75, 0.95 and 0.99). Each individual prize was combined with each probability in the notional gamble. The 6 prizes times the 7 probabilities therefore made up the total of 42 cards presented to the players, each possible combination of prize and probability appearing exactly once in each pack of cards.

Each player was awarded an ‘endowment’ of 4000 points at the start of the game. The packs of cards were sorted randomly using Tippett’s random sampling number tables<sup>1</sup>, with each pack used in the game sorted in an identical manner in order to ensure the same sequence was followed by each group. Players faced each other around a table and the experimenter turned over the first card from the pack, reading the contents of the card (the prize and its associated odds) aloud. The opportunity offered by the card was then auctioned to the highest

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<sup>1</sup> For a description of Tippett’s random sampling numbers, see Gage (1943)



bidder from amongst the participants at that table. There could only be one winner at a particular table in each round, therefore<sup>2</sup>. The successful bidder then rolled a dice in an effort to win a monetary prize equivalent to the value contained on the card. The dice score needed to win the prize reflected the probability associated with the gamble (again, the details of this were not disclosed). If the roll of the dice were successful, the player won the value of the prize less the level of his bid. If it were unsuccessful, the player paid for his bid from his pot of accumulated 'funds' comprising the initial endowment plus any cumulative gain, if any, from previous rounds. This procedure was repeated 42 times with every card in the pack utilised. The player with the highest cumulative gain at the end was declared the winner, receiving an actual prize of either candy, cigarettes or cigars.

No formal statistical analysis was applied to the data with the authors relying primarily upon inspection. Unfortunately, the analysis conducted by Preston and Baratta was based upon an erroneous interpretation of the propositions contained in the game. Consequently, a number of the conclusions appear to be flawed. A reinterpretation of the main results, discussed below, does, however, shed some light upon issues relating to psychological interpretations of declared probabilities associated with risky gambles. Nevertheless, certain caveats remain due to the specification of the experiment itself.

Preston and Baratta presented their findings by comparing mathematically based expected values for each of the 42 prospects along with mean winning bids from the 20 separate games. On that basis, average winning bids were found to exceed mathematically-based expected payoffs in every case where probabilities were low ( $< .25$ ), regardless of the size of the

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<sup>2</sup> No details are given on the exact bidding process used, for example, open outcry versus single highest bid. Such specifications may have implications for sequential learning for the purposes of bidding strategies as well as gaming effects. Since this procedural information is not disclosed, no reliable conclusions with regard to possible behavioural impact or biases can be drawn.

potential prize. This was reversed for probabilities of .25 and over, where average winning bids were all below the applicable mathematical expectation. The size of the potential prize did not appear to affect ratios of mean winning bids to expected values, with the authors concluding that all prizes with small probabilities of success were psychologically overvalued while those with higher probabilities were systematically undervalued. They further concluded, based upon extrapolation of the data, that an “indifference point” exists at which the mean of successful bids would equate with mathematical expectations, which they inferred existed at probability levels at some level below .25. No detail is given on the distribution of winning bids as only mean data is used. Similarly, the distribution of non-winning bids and evidence for any competitive auction effects were not considered at all. In addition, since no statistical analysis was applied to the data, no significant conclusions could be drawn concerning any possible intra-group differences. The primary data on which the analysis was based is replicated in Table 1.

As can be seen from the Table, for probabilities below .25, the ratios of mean winning bids to mathematical expected value all exceed unity. This is reversed for probabilities of .25 and above, with all ratios less than one. There are no obvious, consistent differences in the data based upon level of the prize within each probability level.

Table 1. Replication of table of results reported in Preston and Baratta (1948)

Mathematical Expectations ( $E$ ), Mean Successful Bids ( $V$ ), and Ratio of Bid to Expected Payoff ( $R$ ), For Each Play

Probability	Metric	Prize					
		5	50	100	250	500	1000
0.01	E	0.05	0.50	1.00	2.50	5.00	10.00
	V	0.51	4.44	4.86	12.75	19.59	59.96
	R	10.20	8.88	4.86	5.10	3.92	6.00
0.05	E	0.25	2.50	5.00	12.50	25.00	50.00
	V	0.98	2.66	5.52	27.27	27.85	85.4
	R	3.92	1.06	1.10	2.18	1.11	1.71
0.25	E	1.25	12.50	25.00	62.50	125.00	250.00
	V	1.06	10.21	14.74	35.53	114.95	231.25
	R	0.85	0.82	0.59	0.57	0.92	0.93
0.50	E	2.50	25.00	50.00	125.00	250.00	500.00
	V	1.93	21.84	41.56	110.47	242.80	488.50
	R	0.77	0.87	0.83	0.88	0.97	0.98
0.75	E	3.75	37.50	75.00	187.50	375.00	750.00
	V	3.73	29.98	71.88	168.30	304.70	716.40
	R	0.99	0.80	0.96	0.90	0.81	0.96
0.95	E	4.75	47.50	95.00	237.50	475.00	950.00
	V	3.41	37.71	73.48	161.00	397.80	790.75
	R	0.72	0.79	0.77	0.68	0.84	0.83
0.99	E	4.95	49.50	99.00	247.50	495.00	990.00
	V	3.67	41.72	84.25	226.35	384.20	913.18
	R	0.74	0.84	0.85	0.91	0.78	0.92

*Note:* The data is presented in the same format as that used in the original article by Preston and Baratta. The mathematical expected payoff ( $E$ ) is shown for each combination of probability and prize along with the mean value of the successful bids ( $V$ ). The ratio of mean value of bid to mathematical expected payoff is also shown ( $R$ )

There is, however, a significant flaw in the presentation and interpretation of the results arising from an incorrect framing of the propositions on which mathematical expected values

were derived. Based upon the description of experimental procedures, each card used in the experiment presented a single risky gamble. For example, one such gamble would have been presented in the following equivalent form:

50% chance to win \$500

It is on the basis of that presentation that the mathematical expected value was then calculated as \$250 ( $.50 \times \$500$ ), as shown in the appropriate row and column of Table 1. However, the appropriate description of the probability distribution associated with that gamble, which follows the actual stated specification of the experiment, is as follows:

50% chance to win ( $\$500 - \$Bid$ )

50% chance of winning nothing; certain loss of  $\$Bid$

The proposition therefore entails two prospects which, as described earlier, are deemed to be evaluated separately by the rational decision-maker.

In the formulation presented by Preston and Baratta, the effects of bid costs are ignored, effectively assuming the equivalent of free plays with the game aspects and risks of competitive bidding removed. Allowing for the impact of bid costs, a bid made at the mathematical expected value would result in a combined expected outcome of zero for each prospect. Consider, for example, the prospect of a .75 chance of winning \$500. This results in the following proposition, assuming a maximum bid equal to the mathematical expectation for the cost-free proposition:

$$\text{Bid} = .75 \times \$500 = \$375$$

The full, combined proposition can then be expressed as:

$$(\$500 - \$375) \times .75 - (1 - .75) \times \$375$$

$$= \$125 \times .75 - \$375 \times .25$$

$$= \$93.75 - \$93.75 = \$0$$

The probabilistic expected net gain in monetary terms is therefore exactly equal to the probabilistic loss, making the overall mathematical expected return zero. It has been shown earlier that, given a concave utility function, the sum of the utilities associated with uncertain gains and losses of equal magnitude is less than zero. Therefore, the proposition that equilibrium bids should equate with mathematical expected values of a costless gamble would, based upon the parameters of this experiment, imply the assumption of linear, rather than concave, utility functions for the rational decision-maker over the specified range of values, making them effectively indifferent, or risk seeking, with regard to equivalent gains and losses.

A more appropriate interpretation of expected value would assume that the maximum willingness to pay, and hence the highest rational bid, is the level which equates the overall expected return from the joint prospects with the bid level. Such a proposition conforms with the concept of equity within games of chance. This balance is achieved, in all cases, when the

maximum bid is equal to one half of the cost-free mathematical expected value. Again taking the example of a .75 chance of winning \$500, the following example is derived:

$$\text{Bid} = \frac{1}{2} (\$500 \times .75) = \frac{1}{2} (\$375) = \$187.50$$

The full, combined proposition for expected value is then expressed as:

$$(\$500 - \$187.50) \times .75 - (1 - .75) \times \$187.50$$

$$= \$312.50 \times .75 - \$187.50 \times .25$$

$$= \$234.38 - \$46.88 = \$187.50$$

Therefore, at that level of bid, the combined expected return from the two prospects is exactly equal to the level of maximum rational bid. Maximum bids exceeding this level would result in an expected return lower than the level of bid submitted based upon the two separate prospects, consistent with risk-seeking behaviour, or some other bias. Maximum bids below this equilibrium level would provide a mathematical expected return greater than the bid level submitted, consistent with risk and loss aversion. The effect of reinterpreting each of the 42 propositions on the basis of the mathematical expected return from the evaluation of each prospect for each proposition is shown in Table 2.

Table 2. Reinterpretation of the tabular results reported in Preston and Baratta (1948)

Mathematical Expectations Assuming Zero Bid Cost ( $E$ ), Maximum Willingness to Pay Based Upon Mathematical Expectation of Net Successful Bid Costs ( $Bmax$ ), Mean Successful Bids ( $V$ ) and the Ratio of Maximum Expected Payoff to Bid ( $R$ ) For Each Play

Probability	Metric	Prize					
		5	50	100	250	500	1000
0.01	E	0.05	0.50	1.00	2.50	5.00	10.00
	Bmax	0.03	0.25	0.50	1.25	2.50	5.00
	V	0.51	4.44	4.86	12.75	19.59	59.96
	R	20.40	17.76	9.72	10.20	7.84	11.99
0.05	E	0.25	2.50	5.00	12.50	25.00	50.00
	Bmax	0.13	1.25	2.50	6.25	12.50	25.00
	V	0.98	2.66	5.52	27.27	27.85	85.4
	R	7.84	2.13	2.21	4.36	2.23	3.42
0.25	E	1.25	12.50	25.00	62.50	125.00	250.00
	Bmax	0.63	6.25	12.50	31.25	62.50	125.00
	V	1.06	10.21	14.74	35.53	114.95	231.25
	R	1.70	1.63	1.18	1.14	1.84	1.85
0.50	E	2.50	25.00	50.00	125.00	250.00	500.00
	Bmax	1.25	12.50	25.00	62.50	125.00	250.00
	V	1.93	21.84	41.56	110.47	242.80	488.50
	R	1.54	1.75	1.66	1.77	1.94	1.95
0.75	E	3.75	37.50	75.00	187.50	375.00	750.00
	Bmax	1.88	18.75	37.50	93.75	187.50	375.00
	V	3.73	29.98	71.88	168.30	304.70	716.40
	R	1.99	1.60	1.92	1.80	1.63	1.91
0.95	E	4.75	47.50	95.00	237.50	475.00	950.00
	Bmax	2.38	23.75	47.50	118.75	237.50	475.00
	V	3.41	37.71	73.48	161.00	397.80	790.75
	R	1.44	1.59	1.55	1.36	1.67	1.66
0.99	E	4.95	49.50	99.00	247.50	495.00	990.00
	Bmax	2.48	24.75	49.50	123.75	247.50	495.00
	V	3.67	41.72	84.25	226.35	384.20	913.18
	R	1.48	1.69	1.70	1.83	1.55	1.84

*Note:* The data adjusts the original data shown in Table 1 to take account of the full probabilistic outcomes of the notional gambles, including bid costs. This creates a new mathematical net expected payoff ( $Bmax$ ) which is then used to derive the ratio of mean bid to expected payoff ( $R$ ).

The Table is presented in the same generic form as that replicated from Preston and Baratta (1948) employing a similar layout and method. The original expression of mathematical expected value ( $E$ ) is displayed along with the adjusted measure taking account of the joint probabilistic outcomes of the propositions ( $Bmax$ ). The average of the winning bids is replicated for each prospect ( $V$ ). In this case, however, that winning average bid is divided by the new measure,  $Bmax$ , in order to derive the ratio ( $R$ ).

Examining the data in Table 2 leads to some conclusions which differ from those implied by the original interpretation of the data. In each case, the highest winning average bids exceed mathematical expected values. While this effect is again more exaggerated for low probabilities ( $p < .25$ ), the propensity to overbid is not reversed at higher probabilities but is present in every prospect. The findings are potentially consistent with the proposition that low probabilities are over-weighted systematically by a significant margin. This cannot, however, be generalised across all probabilities due to the likely presence of other behavioural effects and biases introduced into the experiment, some of which are discussed below. The tendency to overbid occurs regardless of the scale of the potential prize, as found originally. Overall, such behaviour is consistent with risk seeking rather than risk aversion, implying a convex utility function versus the concave function assumed by classical expected utility theory. The findings do not support the proposition that prospects were assessed separately with the possibility that the framing of the experimental presentation contributed to a general willingness to overpay. This would again conflict with the description of Rational Man in a close-to-perfect world, able to compute accurately all prospects presented to him. In fact, one possibility for the results is that the participants were actually relatively poor in terms of computational skills, tending to simplify the prospects to such an extent that they more closely



approximated the mathematical basis assumed in the original experiment, adjusted for some random bias.

As mentioned before, further detailed analysis of the results is not possible as all of the data was not presented. In particular, it could have been informative to examine the distribution of all of the bid data in order to ascertain whether the tendency to overbid was a general trait attributable to the majority of participants. The more difficult proposition is that of disentangling apparent willingness to pay from effects and behavioural biases which may have been introduced as a result of the auction and gaming elements present in the experiment (Thaler & Johnson, 1990). For example, each player in each of the 20 groups would have been aware of the localised performance of their immediate competitors. Therefore, as the game progressed, there could well have been a tendency to bid more aggressively simply due to the competitive need to secure the chance of winning a prize, almost on any terms, while, at the same time, denying that same opportunity to the other immediate players. This effect could have been pronounced since there was a single ultimate prize with no penalty for losing the game by a narrow or wide margin. While the data do not suggest that competitive pressures varied by level of prize, it is not clear whether individual bidders who had failed to secure the opportunity of a prize in earlier rounds of the game became more aggressive in later rounds.

While reservations exist in relation to the construction of the experiment and the interpretation of the data, some tentative conclusions can nevertheless be drawn. Regardless of causation of bias, the overall behaviour displayed by the participants cannot be considered objectively rational on the basis of classical theory which deems any and all biases to be irrational. Further, it does appear likely that psychological probability differs from its

objective mathematical equivalent, although this behavioural bias is far from proven due to the ambiguity associated with the meaning of the results.

#### **1.4.2. Testing Utility-Based Consistency Across Risky Choices: Mosteller & Noguee**

A further experiment designed to measure utility, with some conceptual similarities to that of Preston and Baratta was undertaken by Mosteller and Noguee (1951). A small sample of 15 participants, divided into 3 groups, took part in the experiment, which was conducted over a period of 4 months, comprising several sessions. Two-thirds of participants were students enrolled at Harvard University while the remaining 5 were recruited from the local National Guard. As might be expected, these two groups revealed markedly different characteristics in terms of income expectation, employment prospects, educational achievement and family net wealth. The game for the experiment was based upon the card game of poker but used dice instead of playing cards. Participants could generate their own poker hand by rolling 5 dice, the outcomes of which were translated into a type of poker hand.

The game was played in multiple rounds. At the start of each round, participants were shown pre-prepared stimulus cards representing a hand of poker. In addition, a potential prize was displayed along with an associated cost for gambling against the declared hand. The lowest cost for playing the game was set at 5c. Mosteller and Noguee provided an example of such a stimulus card as follows:

44441

\$10.00 : : 5c

The top line represented a hand at poker, expressed in numeric form. This was the target hand to beat for those deciding to accept the gamble. The second line indicated the prize on offer to successful participants along with the fixed cost of playing the game. All of the hands represented by the stimulus cards and the corresponding fair odds were selected by the experimenters from the full set of possible hands. Fair odds were computed for every hand in the set. Various offers were used in relation to each hand, ranging from attractive to very unattractive propositions based upon fair bets. The arbitrary groups of different hands and various different offers were combined to create the full series of games. The order of games within a series was randomised before the cards were presented to the participants. Play was then continuous with no time lags or intervals introduced between series.

Once a card was presented, participants were asked if they wished to play against that hand on the basis of the terms shown; they could either accept or decline. All decisions were recorded. Unlike the Preston and Baratta experiment, there was no auction involved in order to determine the single player to pursue the gamble and hence no behavioural biases related to competitive auctions were introduced. All participants were free to decide to play or pass based upon the conditions presented.

In the event that participants elected to play against the prepared hand, they would roll the 5 dice. If the roll of the dice resulted in a poker hand superior to that of the stimulus card, they won the game, receiving the prize indicated. If the result were equal or inferior to the stimulus card hand, they lost, incurring a penalty of 5c. Participants accepting the gamble could roll the dice in any order. Once the first subject had rolled the dice, none of the decisions to play or pass could be revoked. At the start of the game, each participant was given a \$1.00 “endowment” as seed capital. This endowment was in the form of coloured poker chips, the

currency used for the game. Additional chips were passed to winning players, according to the prize won, with chips removed in the case of losses. In any single game, it was possible for any number of the participating players to win, or none at all, depending on the outcome of their rolls of the dice. At the end of the game, all of the chips held by players were cashed in for actual money.

The experimenters structured the game in three parts. The initial 3 sessions were used to teach the participants about the game, with each group meeting after the third session in order to learn how to compute probabilities with regard to any hands. In addition, all participants received handouts containing all of the true odds which they were required to keep for all subsequent sessions. In addition, the participants were given important feedback on the collective outcomes from the previous sessions. They therefore saw how often particular hands had been played, the results from subsequent rolls of the dice and information on the expected number and percentage of wins. Participants then discussed these results with the experimenter and could also ask questions about the probability calculations and their meaning. This initial phase of the experiment, which Mosteller and Noguee labelled “*Uncertainty*” therefore performed an important educative role while also providing feedback on outcomes.

The second part of the experiment proceeded with the participants now in full possession of information relating to probabilities for every hand they might play. The experimenters referred to this phase of the experiment as “*Known Risk*”. In each case, the series of hands used in the *Uncertainty* condition were repeated in the *Known Risk* domain. The experimental purpose was now to derive data from which utility curves could be estimated. This required an extensive array of propositions to be presented to participants, ranging from some so

unattractive that most players refuse to play most of the time to others in which most of the players are prepared to play. Since the propensity to play or pass is assumed to be individual-specific, actual results from earlier sessions were used to tailor series offerings to each group to ensure that the maximum amount of information could be derived for each individual within the group. The data derived from the later series in the *Known Risk* phase then became the basis for deriving individual utility functions.

The final phase of the experiment involved out-of- data tests of the derived utility curves in order to determine how well behaviour could be predicted. The *Known Risk* condition was therefore modified with the introduction of more complex propositions, in what the experiments labelled the “*Doublet*” situation. This entailed offering participants a choice of hands, for example:

22263 : : 20c

66431 : : 3c

This proposition offered participants the chance to bet 5c to play. If the playing participants beat the first hand (22263), they received 20 cents; if they failed to beat this hand but outperformed the second (66431), they would receive 3 cents. In the event of a failure to beat either hand, they lost the stake of 5 cents. Despite the double gamble, a single roll of the dice was still employed. Again, in each case, the participants had sheets containing all of the fair odds information for each proposition.

Mosteller and Noguee attempted to create utility curves for each subject by extrapolating points of indifference with regard to identical hands with varying risky propositions. It might

normally be expected that most participants would decline to play very unattractive offers but would play attractive ones. By presenting a sufficient array of risky prospects, it should then be possible to identify a general area, in terms of risky proposition, in which the decision to play or decline is likely to be close to 50:50. Based upon the individual data points for each subject, this indifference point was extrapolated.

Significant behavioural bias was found in the data. One subject exhibited such erratic behaviour that it proved impossible for any utility curve to be derived while a function covering only a limited span of values was possible for two students. Interestingly, all participants participated in the gambles, essentially rejecting the certain alternative of \$1 which would have been received simply by doing nothing (other than attending the experiment). Differences were observed between the student and National Guard groups, with the former showing general risk aversion, consistent with diminishing marginal utility, while the latter were strongly risk seeking, consistent with increasing utility from monetary return. Performance in the *Doublet* sessions indicated a high level of confusion resulting in inconsistent behaviour versus that predicted from utility curves. Although not part of the main experiment, participants were also presented with paired choices, comprising two gambles, from which they had to indicate which gamble they would prefer, thus providing a test of utility versus monetary value. In this regard, estimated utility curves proved more reliable predictors of choice for risk averse participants (the student groupings) than the more risk seeking participants.

Overall, Mosteller and Noguee concluded that the derivation of utility curves from simple experimental gambles was possible and that these curves could be translated into cardinal functions based upon reference points. However, considerable individual differences were

found with divergent behaviour between students and National Guards. Students, exhibiting general risk aversion, behaved more in accordance with classical expected utility theory, while the National Guard group clearly did not. Such findings therefore provided some support for the proposition of Friedman and Savage (1948, op cit) that utility curves could exist with inflection points, although no such individual examples were observed from the participants studied in this experiment. In aggregate, the derived utility curves proved not to exhibit the accuracy of prediction which the experimenters had hoped for, although general behavioural biases were observed. However, no evidence for the *Completeness* axiom was found with participants making contradictory preference choices, particularly with regard to more complex choices.

#### **1.4.3 Testing Decision-Making Consistency Across Available Choice Sets: Sippel**

A direct examination of consumer behaviour and revealed preference was undertaken in two controlled experiments by Sippel (1997). These experiments, involving a total of 42 university students, improved upon prior time-series studies which relied upon surveys of household weekly expenditure on consumption goods (Koo, 1963; Mossin, 1972). Such studies found inconsistent behaviour in terms of consumption patterns, although the significance of these findings is open to question as such outcomes could simply be the result of changing tastes over time. In contrast, Sippel's experiments tested choice behaviour at a single point in time, thereby eliminating the problem of potentially changing tastes. In addition, participants were tested in terms of their individual preferences, thus eliminating any group effects, with decisions observed and recorded, hence avoiding potential data bias as a result of inaccurate reporting.

Participants were presented with a list of 8 consumer goods, comprising various food and drink items as well as other goods of a recreational nature (magazines, video clips and a computer game). The goods were selected so as to appeal to a wide range of tastes but also to enable as fine a division of available notional income as possible, thereby enabling clear and precise preferences to be expressed in terms of quantities chosen. The non-food and drink items had time-related levels of consumption associated with them. For example, the video clips and magazines had implied consumption times of between 30-60 minutes, while the computer game had a running time of between 27.5 and 60 minutes. Participants were able to divide their income freely meaning that they could purchase a fraction of a video clip, if they so desired.

In order to derive meaningful data on preferences and to analyse the internal consistency of choices made, 10 different schedules were presented to the participants. The same 8 items were used in each schedule but the prices of the items and applicable income constraints were varied substantially. All prices and incomes were expressed in terms of nominal monetary units, with prices unrelated to those found in actual retail environments. Participants were required to select their 10 preferred baskets conditional upon the relative prices presented and within their income constraints.

The first experiment, comprising 12 participants, was structured to test homogeneity of preference. The approach adopted was to make 2 of the 10 choice sets identical in terms of relative price structure across all items. The two sets differed, however, to the extent that all of the prices in the second set, as well as the income constraint, were 15% higher than in the first set. This replication was made less obvious by varying budget constraints across all 10 series and separating the similar sets in the presentation sequence. The expectation, based upon



rational behaviour, was that participants would be consistent in their choices across the two similar sets by declaring the same preferences in each, both in terms of items included and their relative weightings within their respective consumption baskets.

The specification of the first experiment had two income effects. The first resulted naturally from varying income constraints across the 10 series. The second, more subtle, income effect resulted from the changes in relative prices. Such an effect arises if, for example, for any given fixed level of income, the price of one item falls while the prices of all other items are held constant. Since the consumer now requires less income to purchase the same quantity of items as before, the effect is equivalent to that of an actual increase in income with no prices changing and is therefore termed an income effect. In order to eliminate this income effect, the second experiment, comprising 30 participants, was designed so that any changes in relative prices were completely offset by changes in the budget constraint. In order to then test homogeneity of preferences, the various series were presented to the participants in such combinations that the basket selected in series one would be affordable in later series and so should again be selected if the decision making process is rational. As with the first experiment, relative prices were varied across choice sets ensuring that participants were unlikely to spot any obvious patterns in the data, which may then have resulted in strategic choices in later sets based upon prior expressions of preference.

Participants had unlimited time in the first part of the experiments during which they selected their preferred baskets. They could review and change selections as often as they wished, including returning to and modifying earlier baskets if required, the only exception being the first chosen basket in the second experiment which remained fixed as the reference point. In order to help the participants in their selection tasks, they were provided with

personal computers and software designed to aid their allocation process. Therefore, participants could select the required quantities of any item from the screen and their basket would update automatically showing income allocated, based upon prices and quantities, and residual income to be allocated. Should budget constraints be violated, a warning message would appear. As a further aid, the programme also informed the subject how remaining income could be allocated to various items in order to use up all available funds. The computational aspects of the experiment were therefore made much easier and less time consuming for the participants and also eliminated the possibility of arithmetical error. Changes to baskets could be made with ease, ensuring that participants were able to allocate most of their time to deciding their preferences rather than being side-tracked in terms of performing calculations. Once each of the 10 selections had been finalised, the experimenter selected the actual basket to be presented to the subject at random. The subject was then taken to the laboratory and invited to consume his goods.

Sippel's experiments provided a direct test of the practical feasibility of the axioms of expected utility as expressed through Samuelson's concept of revealed preference. The tasks presented to the participants in the two experiments represented a typical decision-making scenario envisaged by expected utility theory. Obvious biases found in other experiments were largely eliminated, particularly in relation to time series impacts on taste shifts. In relation to the first experiment, 11 out of the 12 participants violated the axioms of rational choice selection, implying that just one subject could be considered a utility maximiser. Each of the other participants showed inconsistency with regard to their choices of bundles in identical situations. The only circumstance in which such an outcome might be consistent with classical rational behaviour would be in the case of indifference between the chosen bundles.

In the second experiment, 22 of the 30 participants were also found to violate the strong axioms of revealed preference.

The findings of the experiment directly contradict the assumptions of classical demand theory. Examination of the data found that participants were far from random in expressing their preferences, spending a considerable time engaging in the selection process. Each of the participants revealed strong preferences for certain goods over others, representing clear expressions of taste, such that some goods were not chosen at all even at low prices. In other cases, participants quite willingly substituted one good for another based upon changes in relative prices. On that basis, it is reasonable to assume that participants all selected the bundles they genuinely believed they preferred at that point in time. A number of participants made repeated changes to their chosen baskets before settling upon their final choices. Even when these prior choices were considered, before amendments were made, no improvements in overall consistency were found. Sippel thus concluded that, while all of the participants were clearly motivated by the experiment, the axioms of revealed preference were nevertheless violated by the overwhelming majority.

## **1.5. Discussion**

Collectively, the concept of classical rationality built upon early models of expected utility theory faces some challenges based upon the studies referenced. Indeed, a number of the apparent biases identified from the studies are consistent with evidence from other empirical investigations of behavioural biases (Hershey, Kunreuther & Schoemaker, 1982; Knetsch, 1989; Kühberger, 1998; Kühberger, Schulte-Mecklenbeck & Perner, 1999; Levin, Schneider & Gaeth, 1998; Payne, 2005; Zeelenberg, 1999) and psychological perception

(Hertwig, Barron, Weber & Erev, 2004; Lant & Montgomery, 1987; Lattimore, Baker & Witte, 1992; March, 1988; Payne, Laughhunn & Crum, 1980; Sitkin & Weingart, 1995). In practice, decision-makers appear not to rank preferences consistently nor are they found to maintain internal consistency with regard to those rankings in accordance with expectations based upon the axioms of rational behaviour. Indeed, in both the cases of Preston and Baratta, and Mosteller and Noguee, participants appeared to have great difficulty in determining the true nature of risky outcomes when presented with more complex tasks. In addition, even when provided with objective odds, as in the Mosteller and Noguee experiment, decision makers still exhibited bias. Therefore, while general traits such as risk aversion or risk seeking could be inferred in some cases, these findings are so non-specific in terms of ability to forecast that it is unclear how much value they actually represent.

There is support for the proposition that the psychological interpretation of probabilistic uncertainty differs from that of the mathematical measure. However, the empirical evidence presented by Preston and Baratta in that regard was somewhat ambiguous as it would appear that any such effect was indistinguishable from those of other apparent behavioural biases. While it appears possible to derive *ex post* utility functions based upon specifically generated data points derived from observed behaviour, the evidence suggests that subsequent behaviour is still likely to diverge from that implied by those functions in terms of future expressions of preference. Even allowing for changes in tastes over time, such variability within a relatively short testing span makes that presumption less viable. Simply adjusting for the impact of psychological interpretations of risk may not be sufficient, therefore, to explain observed, apparently inconsistent behaviour. A similar challenge to the computational efficiency implied by basic utility theory is provided by the experiments conducted by Sippel. Collectively, therefore, the experiments suggest that decision-makers faced with uncertain

choices may be prone to error, bias and inconsistency. Notwithstanding, the next chapter considers some major advances in decision-theory including approaches to modelling uncertainty and ambiguity.

## CHAPTER 2

### EXTENDING RISK & UNCERTAINTY

#### 2.1 Subjective Expected Utility

The axiomatic approach to evaluating preferences across risky options considered in the previous chapter represented an important development in the proposition that rational decision-makers behave in a manner consistent with utility maximisation. Essentially, when these axioms remain intact (are not violated), the decision-maker can be said to act *as if* they are applying expected utility functions as described by classical rational behaviour (there is, however, no assumption that they are actually doing so). Nevertheless, while the theorems of von Neumann and Morgenstern (vNM) dealt with “sure options”, or “choice under risk” (lotteries over which objective probabilistic outcomes are assumed to be known), many uncertain choices inevitably require the decision-maker to make assumptions about probable outcomes. Therefore, when there is no objective information about the likelihood of particular outcomes, preferences are assumed to be determined based upon paired utility and belief-based probability functions. This proposition forms the basis of subjective utility theory (*SEU*), describing an environment in which choice under uncertainty is resolved by the rational decision maker on the basis of their own desires (as defined by their own utility function) and beliefs about the likelihood of particular desires being realised (their self-determined probability function). On this basis, rational behaviour stems, in part, from the consistent application of a decision-makers probabilistic beliefs.

While the proposition of subjective expected utility had been advanced elsewhere (Ramsey, 1931, de Finetti, 1937), Savage (1954) provided the most robust and formal

manifestation of the theory, again developing axioms of behaviour which, if complied with, would be consistent with rational utility maximising objectives. In accordance with this theory, decision-makers face options, similar to prospects or lotteries as described by vNM. However, these possible options are not assumed to have assigned objective probabilities but are framed on the basis that the likelihood of a particular outcome, or *consequence*, is conditional upon a particular *state* of the world. Consequences, can therefore be positive or negative from the perspective of the decision-maker while the *state* of the world which brings those *consequences* about is uncertain and beyond the control of the decision-maker (the decision-maker's choice should not, therefore, have any causal impact upon *state*). Savage described the various sets of *states* as *events*. On the basis of paired utility and probability functions, a decision-maker is then faced with a number of options in relation to which preferences are to be distilled. This is achieved by considering the full set of *acts* which describe all possible *consequences* as determined by all possible *states* of the world. From this it may be deduced that *acts* are functions which map the set of *states* to the set of *consequences* determined by those *states*. The rational decision-maker is assumed to resolve all dimensions of the problem, including evaluating all possible combinations of *states* and their associated *consequences*, resulting in final preferences, based upon desires (utilities) and beliefs (self-defined probabilities), which maximise utility.

The process can be summarised as follows: If we define  $\mathcal{S}$  as the set of possible *states* of the world and  $\mathcal{C}$  as the set of possible *consequences*, we can define  $f$  as a function mapping elements of  $\mathcal{S}$  to elements of  $\mathcal{C}$ . Therefore,  $f(s_i)$  represents the *consequence* of  $f$  given *state*  $s_i$  where  $s_i \in \mathcal{S}$  and  $s_i$  actually occurs. The subjective expected utility of  $f$  is then given by;

$$SEU(f) = \sum_i u(f(s_i)).P(s_i) \qquad \text{Equation 7}$$

The decision-maker's confidence in *states* contained in  $\mathcal{S}$  is represented by a probability function  $P$  while the strength of desire for each *consequence* in  $\mathcal{C}$  is determined by the utility function,  $u$ . The probability and utility function pair  $(P,u)$  create the subjective expected utility function  $SEU$  which defines preferences for all possibilities in  $\mathcal{F}$ .

Given acts  $f$  and  $g$  together with an event,  $E$ , we can define the *act* as comprising the following payoffs;

$$fEg(s) = \begin{cases} f(s) & \text{if } s \in E, \\ g(s) & \text{otherwise} \end{cases}$$

Therefore, the *act* pays off in accordance with the event  $f$ ,  $s \in E$ , else in accordance with  $g$ .

A strict preference order  $\geq$  is identical to a vNM order if *acts* are ranked on the basis of expected utility. This is the case if there exists a general payoff function across all *acts* such that;

$$f \geq g \text{ iff } \sum_s u(f(s)) \cdot p(s) \geq \sum_s u(g(s)) \cdot p(s)$$

Similar to the axiomatic approaches of vNM, Savage derived a number of axioms, including the equivalent of *completeness* (the ability to rank preferences across all choices, as in equation 9 above), *transitivity* and *continuity*, defined in the previous chapter. The equivalent of the *independence* axiom was captured by a “Sure-Thing Principal”, stating that if a decision-maker would make the same choice regardless of *states*, then the *state* becomes irrelevant and dominance applies. Therefore, even if the decision-maker has full knowledge of prevailing *states*, his “Sure-Thing” preferences will proceed in the same manner as if he has no information, or expectations, about the *states* at all. Just as with the *independence* axiom assumed by vNM, preferences between choices are thus assumed not to be affected by an irrelevant third option.

From the overall formulation, Savage contended that determining the preferences of decision-makers enabled their beliefs to then also be derived. Therefore, if preferences



become known in relation to a sufficiently rich set of choices, actual bets on events indicate which event the decision-maker considers to be more likely. Therefore, any ordering in which a preference  $X > Y$  is revealed implies that the decision-maker believes that  $X$  is more likely to pertain than  $Y$ .

A simpler method of deriving subjective probabilities was proposed by Anscombe and Aumann (1963) using a lottery with an objective probability distribution together with a subjective, uncertain lottery. The former could be any unbiased mechanical device which would return independent, random results within a finite domain; they suggested a roulette wheel. The second lottery was framed as a horse race with no *a priori* information about likely outcomes. This is therefore a compound lottery in which an outcome of one results in entry into the other which determines the ultimate prize. Anscombe and Aumann showed that the subjective probabilities adopted in relation to the horse lottery could be inferred from the preferences derived from the dual lotteries. As a result, the subjective expected utility function reflects combined expectations with regard to the objective probabilities of the roulette lottery, with different consequence functions, and the subjective probabilities associated with different states (the outcomes of the horse lottery).

The formulations of subjective expected utility provided an essential contribution to the understanding of decision-making under uncertainty as they addressed the wide range of practical choice problems about which there is no objective probabilistic information. The significant contribution of Savage lay in the extensive development of specific axioms, similar to those developed by vNM, enabling metrics for rational behaviour to be established. As we have seen, *SEU* maximisation requires that decision-makers derive preference orderings over all *consequences* of *acts* in all cases where *acts* are not equivalent (“non-null”). In addition, preference orderings must satisfy Savage’s Sure-Thing Principle, the equivalent of the *independence* axiom within vNM expected utility theory. Only when the full axiomatic conditions hold, the final necessary condition for *SEU* maximisation applies whereby preferences over contingent consequence functions induce an ordering of events based upon their perceived likelihood.

The existence of axioms, of course, allows normative assumptions to be tested empirically thereby addressing the issue of whether decision-makers really do behave *as if* they adhere to utility-based rationality. Perhaps inevitably, the axiomatic approaches to

expected utility and subjective expected utility have been subject to challenge based upon both empirical observation and paradoxes. Two well-known paradoxes are discussed briefly below. The first is the Allais' Paradox, implying violations of the core *independence* axiom, while the second Ellsberg Paradox presents an apparent violation of the axioms of *SEU* as decision-makers appear to prefer risk in situations where probabilities are objectively knowable over situations in which they are wholly ambiguous. The Sure-Thing Principle, in which equivalent components of choice options are disregarded, therefore proves somewhat problematic.

### 2.1.1 Challenges to Axioms (Allais' Paradox)

A challenge to the validity of the *independence* axiom stems from the Allais' Paradox which presents combinations of notional risky gambles in the following form;

A: receive £1,000,000 with certainty.

B: receive £5,000,000 with probability .1,  
receive £1,000,000 with probability .89,  
receive nothing with probability .01.

A further proposition is then presented:

C: receive £1,000,000 with probability .11,  
receive nothing with probability .89.

D: receive £5,000,000 with probability .10,  
receive nothing with probability .90.

Allais conjectured that the majority of decision-makers preferring option A to B were also likely to prefer option D to C.

If it is assumed that decisions relating to risky gambles are made on the basis of probability weighted final outcomes, rather than the manner in which those outcomes are generated, then these respective choices are in conflict and are inconsistent with expected utility theory. This can be shown as follows:

The preference for option A over B implies:

$$U(\pounds 1\text{m}) > .1U(\pounds 5\text{m}) + .89U(\pounds 1\text{m}) + .01U(\pounds 0)$$

adding  $[.89U(\pounds 0) - .89U(\pounds 1\text{m})]$  to both sides results in:

$$U(\pounds 1\text{m}) + .89U(\pounds 0) - .89U(\pounds 1\text{m}) > .1U(\pounds 5\text{m}) + .89U(\pounds 1\text{m}) + .01U(\pounds 0) + .89U(\pounds 0) - .89U(\pounds 1\text{m})$$

reducing to:

$$.11U(\pounds 1\text{m}) + .89U(\pounds 0) > .1U(\pounds 5\text{m}) + .9U(\pounds 0\text{m})$$

The expression immediately above is, of course, the equivalent representation of prospects C and D. Therefore, anybody preferring option A to option B should also prefer option C to option D if they are acting in accordance with expected utility theory. Empirical tests of the

Allais paradox were found to support the proposition of preference reversals, although the strength of the effect was found to be influenced by context and framing (Burke, Carter, Gominiak & Ohl 1996; Camerer, 1989; Conlisk, 1989; Huck & Müller, 2012; Kahneman & Tversky, 1979; MacCrimmon & Larsson, 1979; Machina, 1987; Morrison, 1967; Moskowitz, 1974; Oliver, 2003; Slovic & Tversky, 1974).

The Paradox violates the *independence* axiom of expected utility due to the assumptions regarding *common consequence*. According to this construct, it is assumed that equal outcomes added to two initial prospects should have no effect on original preferences as the equal outcomes must also be equivalents in terms of utility and therefore offset one another. *Independence* therefore implies that differentiating components of prospects should be evaluated independently of the other common prospects available. This proposition can perhaps be explained more clearly by decomposing and reformulating the original propositions as follows:

A: receive £1,000,000 with probability .89,  
receive £1,000,000 with probability .11.

B: receive £1,000,000 with probability .89,  
receive nothing with probability .01,  
receive £5,000,000 with probability .10.

Expressing the second propositions in a similar way, we derive:

C: receive nothing with probability .89,

receive £1,000,000 with probability .11.

D: receive nothing with probability .89,  
receive nothing with probability .01,  
receive £5,000,000 with probability .10.

If the common consequences (outcomes) are removed, as assumed by the *independence* axiom, option A reduces to a prospect of winning £1,000,000 with a probability of .11 (£1m, .11), which is identical to proposition C. Similarly, both B and D reduce to (£5m, .10). Both combinations can therefore be seen to offer the same choices. On that basis, the only rational combinations of preferences across both propositions are A:C or B:D, assuming any preference at all can be expressed in relation to options A and B .

A similar analysis to that described above was performed by Birnbaum (1999) showing that constant consequence paradoxes could also be decomposed on the basis of *transitivity*, *coalescing* and *restricted branch independence*. If all three of these properties remained inviolate, there would be no inconsistencies with regard to outcomes predicated upon expected utility theory.

*Coalescing* assumes that, for any gamble with two or more prospects (probability outcomes or branches) yielding identical outcomes, those branches can be combined by simply adding the associated probabilities. Therefore, the following option:

A: receive £100 with probability .20,  
receive £100 with probability .20,

receive £0 with probability .60.

coalesces to:

$A^l$ : receive £100 with probability .40,  
receive £0 with probability .60.

$A^l$  can therefore be termed the *coalesced* equivalent of A, hence the decision maker should be indifferent between A and  $A^l$ . Assuming that *transitivity* applies, a circumstance in which  $A^l > A$  is termed event-splitting (or branch-splitting), and is a violation of *coalescing*.

*Restricted branch independence* assumes that, in the case of two gambles with one identical probability outcome within one branch of each of the two gambles, those common outcomes can be changed in each gamble in an identical manner, thereby keeping the original preference unchanged. Therefore, given:

A: receive £100 with probability .10,      >      B: receive £200 with probability .10,  
receive £500 with probability .60,                      receive £500 with probability .60,  
receive £250 with probability .30.                      receive £0 with probability .30.

Applying the assumptions of *restricted branch independence* to eliminate the second branch common prospect then implies:

$A^l$ : receive £100 with probability .10,      >       $B^l$ : receive £200 with probability .10,  
receive £0 with probability .60,                      receive £0 with probability .60,

receive £250 with probability .30.

receive £0 with probability .30.

Applying these concepts to the original Allais paradox, and assuming no violations of any of the components, results in:

A: £1,000,000 with certainty. > B: .10 to win £5,000,000,  
.89 to win £1,000,000,  
.01 to win £0.

Applying *coalescing* with *transitivity* gives:

A<sup>I</sup>: .10 to win £1,000,000, > B: .10 to win £5,000,000,  
.89 to win £1,000,000, .89 to win £1,000,000,  
.01 to win £1,000,000, .01 to win £0.

From *restricted branch independence*, the following can then be derived:

A<sup>II</sup>: .10 to win £1,000,000, > B<sup>I</sup>: .10 to win £5,000,000,  
.89 to win £0, .89 to win £0,  
.01 to win £1,000,000, .01 to win £0.

Again applying *coalescing* with *transitivity* implies:

C: .11 to win £1,000,000, > D: .10 to win £5,000,000,  
.89 to win £0, .90 to win £0,

Therefore, since  $A$  and  $A'$  are equivalent, a rational individual is assumed to be indifferent between the two. Therefore, given  $A > B$ , then  $A' > B$  is also implied. The transition from  $A'$  to  $A''$  together with the transition from  $B$  to  $B'$  merely applies an equivalent transformation to each of the second branches. Therefore, due to *restricted branch independence*, it is expected that  $A'' > B'$ . Finally, *coalescing* branches with equivalent prospects on both sides results in the expectation  $C > D$ . Clearly, since the latter was found to be violated by the majority of participants, the implication is that one of the three assumptions regarding *transitivity*, *coalescing* or *restricted branch independence* must be incorrect. Such decomposition of risky propositions into axiomatic states serves the useful role of enabling specific tests to be applied under experimental conditions.

### **2.1.2 Ambiguity and the Ellsberg Paradox**

The Ellsberg Paradox challenges the assumptions of axiomatic behaviour contained within subjective utility theory. According to the latter, decision-making under conditions of uncertainty assumes that relative preferences between lotteries are uniquely determined by an individual's utility function combined with beliefs in relation to events (represented by a subjective probability function). The Ellsberg Paradox instead suggests that most decision-makers prefer gambles where objective probabilities are known to alternatives where there is genuine ambiguity.

Two problems are presented to illustrate this: In the first problem, two urns contain a known number of balls. In urn I, we are told that there are exactly 100 balls which are either red (R) or black (B), the actual mix is unknown. In urn II, we are told that there are exactly 50 red and 50 black balls. For a particular lottery,  $C_i$ , a ball is drawn at random from urn  $i$ ,  $i = I$  or  $II$ . The player receives \$100 if the ball selected at random is one of the two possible



colours. Ellsberg predicted that, since nothing is known about the number of red and black balls in urn I, most decision-makers would be indifferent with regard to their choice of colour. Since the number of red and black balls in urn II is the same, decision-makers should again be indifferent as to their choice of colour since there is an equal probability of returning either a red or a black ball. In other words;

$$R_I \sim B_I \text{ and } R_{II} \sim B_{II}.$$

However, while Ellsberg expected most decision-makers to be indifferent as to colour choice, he did not expect them to be indifferent regarding the choice of urns, instead preferring  $R_{II}$  to  $R_I$  and  $B_{II}$  to  $B_I$ . This is clearly contradictory since  $R_I \sim B_I$  implies equal subjective probabilities of 0.5, while the suggested preferences with regard to urns would imply a probability of less than 0.5 in relation to one colour.

In the second problem, there is a single urn containing 90 balls. Exactly 30 balls are red (R) while the remaining 60 balls are either black (B) or yellow (Y), the precise number of each being unknown. One ball is to be drawn at random from the urn. There are four lotteries offered;

$A_1$ : Receive \$100 if R, otherwise \$0

$A_2$ : Receive \$100 if B, otherwise \$0

$B_1$ : Receive \$100 if R or Y, otherwise \$0

$B_2$ : Receive \$100 if B or Y, otherwise \$0

Ellsberg predicted that most decision-makers would express a preference for  $A_1$  over  $A_2$  but would prefer  $B_2$  over  $B_1$ . Such behaviour contradicts Savage's Sure-Thing Principle, shown as follows;

Assume:  $f, f^{\sim}, g, g^{\sim}$  are lotteries and  $S$  is an event.

Then if, given  $S, f = g$  and  $f^{\sim} = g^{\sim}$  and, in the case  $\sim S$  (not  $S$ ),  $f = g$  and  $g = g^{\sim}$ , then  $f$  is preferred to  $f^{\sim}$  if and only if  $g$  is preferred to  $g^{\sim}$ .

The contradiction perhaps becomes most easily apparent if we represent the lotteries in tabular form;

Table 3. Representation of the Ellsberg Single Urn Lotteries

	30	60	
	R	B	Y
A1	100	0	0
A2	0	100	0
B1	100	0	100
B2	0	100	100

The probability associated with  $R$  is of course known ( $1/3$ ). The probability of any positive payoff associated with either  $B$  or  $Y$  is ambiguous and could range between  $2/3$  to zero for either (there are either 60 black balls and no yellows or vice versa). A rational decision-maker will therefore only prefer  $A_2$  to  $A_1$  on the basis that his subjective probability for  $B$  is higher than the known probability of  $R$ , meaning that he considers the likely number of black balls to be higher than the known number of red balls. If that is the case, he should then also prefer  $B_2$  to  $B_1$  since the aggregate of black and yellow balls must be greater than the sum of red and yellows. By the same logic, a preference:  $A_1 > A_2$  also implies  $B_1 > B_2$ . The same

conclusion is reached if we consider the problem assuming  $S = R \cup B$ . We then see that the outcomes for  $Y$  in the two paired lotteries represent *common consequences*. On that basis, the preference:  $A_1 > A_2$  must again imply  $B_1 > B_2$ .

The Ellsberg Paradox is not only inconsistent with the Sure-Thing Principle, but also violates First Order Stochastic Dominance. This axiom states that a random variable  $A$  is statewise dominant over another random variable  $B$  if, for any consequence or outcome  $x$ ,  $A$  gives at least the same likelihood of achieving  $x$  as  $B$ , and a higher probability of achieving  $x$  in at least one state. We may express this as:

$$P[A \geq x] \geq P[B \geq x] \text{ for all } x \text{ and } P[A \geq x] > P[B \geq x] \text{ for at least one } x.$$

Applying this to the lotteries, we know for certain that  $P(R) = 1/3$  and  $P(B \cup Y) = 2/3$ . Choosing  $A_1$  over  $A_2$  implies  $P(R) > P(B)$ . Therefore,  $P(R \cup Y) = 1 - P(B) > 2/3$ . A preference for  $B_2$  over  $B_1$  therefore violates the First Order Stochastic Dominance axiom given  $A_1 > A_2$ . The violations of axioms in accordance with the Ellsberg Paradox imply that decision-makers display aversion to uncertainty and ambiguity; when faced with a choice between gambles with objective probabilities and those with ambiguity, they prefer to bet on the former.

The Allais and Ellsberg Paradoxes challenge various axioms of normative theories of rational behaviour. While both call into question the validity of the core *independence* axiom, the latter has important implications with regard to the presence of ambiguity in decision choices. Both imply biases in the decision-making process such that, under certain conditions, individuals may not apparently behave in accordance with utility maximising objectives. Empirical evidence supporting the paradoxical outcomes inevitably led to

extensive reconsideration of the how decision-makers evaluated risk, ambiguity and pay-offs. In some cases, greater consideration was given to the process by which decision-makers arrived at choices. These efforts ultimately lead to even more robust theories of decision-making under conditions of uncertainty and ambiguity.

### **2.1.3. Possible Explanations for Axiomatic Violations**

Violations of normative theories of utility maximising behaviour appear to suggest that behavioural factors influence attitudes to uncertain choices. In effect, decision-makers appear to draw categorical distinctions between payoffs which are certain, possible, or highly improbable. The concept of a continuous utility function able to differentiate objectively between payoffs across all levels of risk is therefore open to question. The most obvious inconsistencies appear to lie at the extremes; either modelling potentially high, risky payoffs against certain alternatives or when considering potential outcomes with vanishingly small likelihoods.

In practice, decision-makers appear not to evaluate risky choices in isolation but apply some context with regard to consequences. The most obvious component of the latter appears to be related to changes in wealth states. Therefore, many individuals are prepared to play objectively unfair gambles in the form of lotteries offering extremely low probabilities of a high return as that high return is likely to be transformative in terms of wealth state while refusing to accept even small risks with relatively insignificant payoffs (Baillon & Bleichrodt, 2012). Conversely, already wealthy individuals may place greater importance upon preserving that wealth state, leading to a subjective over-discounting of risk. In such circumstance, risk preferences can be explained by the application of subjective functions

$w(p)$  which weight the probability  $p$  of risky events. The willingness to bet on a particular event  $E$  can then be defined by;  $w(p(E))$ , which may differ from  $p(E)$ .

Elements of this become clear if we re-examine the basic proposition advanced by Allais in which decision-makers are presented with the following paired choices;

A: receive £1,000,000 with certainty.

B: receive £5,000,000 with probability .1,  
receive £1,000,000 with probability .89,  
receive nothing with probability .01.

and

C: receive £1,000,000 with probability .11,  
receive nothing with probability .89.

D: receive £5,000,000 with probability .10,  
receive nothing with probability .90.

A preference for option D over option C is, of course, consistent with probability weighted expected payoffs assumed by expected utility theory; the expected payoff associated with option C being £110,000 while that of D is £500,000. This is, however, reversed in the case of preferring A to B; the expected monetary payoff from B is £1.39m (£500,000 + £890,000), while A guarantees a risk free return of £1m.

Loss aversion and the certainty effect, overweighting certain outcomes relative to those which are considered only to be objectively probable (Kahneman & Tversky, 1979), have been suggested as explanations for axiom violations. Bell (1982) has extended this to include

regret. Thus, when comparing risky options with those with certain outcomes, decision-makers consider the disappointment they might feel should the actual outcome from the risky choice fall short of probabilistic expectations. Both loss aversion and anticipated regret as a consequence of actual outcomes are thus assumed to apply a higher discount rate to risky prospects notwithstanding knowledge of objective probabilities. Loss aversion can also appear to align with regret in the form of opportunity cost. Thus, given the propositions presented by Allais, while there is no possibility of an actual loss being incurred, there is a clear potential opportunity cost in relation to the alternative foregone. Consequently, accepting the gamble represented by prospect B involves the opportunity cost associated with rejecting the certain receipt of £1m represented by prospect A. If the certain return of £1m is deemed by the decision-maker to be sufficiently transformative in terms of wealth states, the basis exists for the subjective preference for A over B notwithstanding the mathematically higher expected payoff associated with the latter. An issue remains in relation to how such behaviour can be generalised in model form.

One generalised framework for overcoming the restrictions of standard additive approaches has been proposed using a Choquet Integral to accommodate multi-criteria decision-making (Chateauneuf, Eichberger and Grant 2002). The model involves applying a non-extreme-outcome (neo-additive) function to model optimistic and pessimistic attitudes towards uncertainty, overweighting best and worst outcomes, while also applying weights to interactions between decision-criteria. The aggregate function (Choquet Integral) is defined with respect to a set function acting on all possible combinations of a set of criteria. As indicated, weights (defined as *fuzzy measures*) are applied across explicit decision criteria and also interactions between combinations of decision criteria. The latter provides significant flexibility when distinguishing between preferences.

A practical example of the process is provided by Li, Law, Vu, Huy quan and Rong, (2013), explained briefly as follows: a traveller selecting a hotel has identifies a number of criteria which will be used to arrive at a preference ordering between the various options. Assume that three criteria are deemed the most important: price; cleanliness and service. On that basis, four possible hotels are identified and the traveller has assigned utility values against each of the three criteria for each hotel. This results in the following;

Table 4. Hotel Choices With Utility Scores Assigned to Decision-Criteria

	Price	Cleanliness	Service
Hotel A	0.7	0.6	0.9
Hotel B	0.6	0.7	0.9
Hotel C	0.6	0.7	0.4
Hotel D	0.7	0.6	0.4

It might be concluded straight away that;  $A > D$  and  $B > C$ . In order to make a final decision, the pairs  $A \vee B$  and  $C \vee D$  must still be evaluated. In order to achieve full rankings, preferences for combined criteria might be considered. For example, assuming that service levels are high (A and B above), price may then be considered the most important secondary criterion. On that basis, Hotel A would be ranked higher than Hotel B. However, in the case where service is relatively poor (C and D), cleanliness may be considered more important than price. Applying these filters, we would then find the following preferences;  $A > B > C > D$ . Standard additive weighting models would not explain this outcome. For example, if  $w_p, w_c$  and  $w_s$  are taken to represent unequal weights applied to each of the three criteria, the preference for  $A > B$  implies  $w_p > w_c$ . However, the order  $C > D$  holds only when  $w_p < w_c$ . This therefore fails to explain the observed preference of the decision-maker.

The failure of the simple additive model stems from the fact that it assumes that the criteria are mutually independent. If, however, we allow utility-deriving interactions between

the criteria, a solution can be found. Given utilities associated with a set of  $N$  criteria,  $N = \{x_1, x_2, x_3\}$ , a *fuzzy measure*,  $\nu$ , represents weightings across all criteria;  $\nu(\{1\})$ ,  $\nu(\{2\})$  and  $\nu(\{3\})$  together with all interactions between the criteria;  $\nu(\{1, 2\})$ ,  $\nu(\{1, 3\})$ ,  $\nu(\{2, 3\})$  and  $\nu(\{1, 2, 3\})$ . The discrete Choquet Integral (CI) for each fuzzy measure,  $\nu$ , is then given by;

$$C_\nu(\mathbf{X}) = \sum_{i=1}^n x(i) [\nu(\{j \mid x_j \geq x_i\}) - \nu(\{j \mid x_j \geq x_{i+1}\})] \quad \text{Equation 8}$$

where  $x_{(1)}, x_{(2)}, \dots, x_{(n)}$  is an ordered (non-decreasing) permutation of the input  $\mathbf{x}$ .

Assuming the following with regard to fuzzy measures;

Table 5. Fuzzy measures across all choice criteria permutations

Criteria Groups	Weight
$\nu(\emptyset)$	0
$\nu(\{\text{price}\})$	0.4
$\nu(\{\text{cleanliness}\})$	0.3
$\nu(\{\text{service}\})$	0.75
$\nu(\{\text{price, cleanliness}\})$	0.2
$\nu(\{\text{price, service}\})$	0.9
$\nu(\{\text{cleanliness, service}\})$	0.6
$\nu(\{\text{price, cleanliness, service}\})$	1

Applying the weights above to the formula in Equation 8 gives;

Table 6. CI-derived combined weight utilities for each option

	Hotel A	Hotel B	Hotel C	Hotel D
$C(\cdot)$	0.73	0.7	0.59	0.58

On this basis, the preference ranks:  $A > B > C > D$  is derived in accordance with observed preferences.

Multi-criteria-decision models of the type described above are potentially promising approaches when clear decision criteria can be derived or inferred. In particular, they offer



solutions to more basic additive models which can fail to explain preferences across choices with multiple utility-inducing attributes. By allowing for the expression of general preferences within the context of ambiguity, the models relax Savage's Sure-Thing Principle. By allowing greater probability weights to be applied to the best and least favourable options, CI-derived models enable potentially problematic behaviours at the extremes of normal utility functions to be more easily accommodated. Similarly, a preference for certainty with minimum payoff over higher-potential payoff risky options can also be accommodated. Application of the models empirically is, however, a potentially complex task as, in many cases, attributes and criteria can be extensive. Nevertheless, attempts have been made to apply the approach to practical decision tasks (Büyüközkan, Feyzioğlu, & Göçer, 2018).

## **2.2. Discussion**

The current chapter discussed significant advances to expected utility theory, describing some key approaches to modelling uncertainty and ambiguity. Challenges to axiomatic approaches to behaviour were considered along with further developments of more complex models with elements relating in part to process as well as outcome. In that regard, multi-criteria-decision models were found to offer a flexible framework from which inferences about preferences relating to complex structures of criteria and their interdependencies could be derived. Such models offer significant advantages over and above simple additive probabilistic models and provide a basis for resolving some of the more challenging paradoxes which had called into question key assumptions of axiomatic behaviour.

Psychological explanations for apparently irrational behaviour and possible sources of bias were also considered. Reference was made to the possible role which prospective

changes in wealth states might play when decision-makers evaluate uncertain outcomes. The concept of changes in wealth, as opposed to its quantum, is present in the early literature, although it remained relatively underdeveloped for some time in terms of a robust framework. The concept itself implies that decision-makers may attach importance to reference points, evaluating preferences in relation to those reference points and more general goals. This has significance in terms of how utility might be expressed. The next chapter takes up this theme, covering one of the most significant advances in modelling behaviour in the form of Prospect Theory.

## CHAPTER 3

### PROSPECTS, CUMULATIVE PROSPECTS & REFERENCE POINTS

#### 3.1. Markowitz Value Function

In his seminal work on portfolio optimisation, Markowitz (1952) advanced the proposition that risky preferences should be framed not on the basis of total ending wealth but in relation to prospective changes from reference points. If starting levels of wealth are used as reference points then judgments about risky prospects would be made on the basis of expected outcomes in comparison with those points<sup>3</sup>. In general terms, expected final wealth ( $F$ ) is then a function of the starting reference point ( $r$ ) plus a prospective change,  $\alpha$ ; relative to that reference point;  $F = r + \alpha$ . Expected utility theory would assume that rational decision-makers choose risky options which maximise the expected value payoff in  $\alpha$ , conditional upon the available choice set. Assuming that  $r$  is a certain, risk-free alternative to any risky gamble  $x_i$ ; we can define the rational objective as maximising;  $EV(r + x_i) > r$ . In the event that  $r$  is not risk-free, but is independent in outcome with regard to  $x_i$ , then a further term,  $\Delta r^{\sim}$ , can be added. Expected ending wealth is then a function of expected payoffs to all  $x_i$  plus the starting value and expected payoff in  $r$ .

Markowitz proposed that loss aversion, risk-seeking and risk-aversion could be modelled within an overall value function. This was achieved by replacing the standard utility function, predicated upon quantum states of cumulative wealth, with a loss-averse value function defined in terms of changes in wealth measured against a reference point. This concept was

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<sup>3</sup> It is often assumed that reference points represent status quo positions, implying that they are certain alternatives against which risky choices are evaluated. This is not a necessary assumption as other reference points can be assumed. Indeed, Markowitz highlighted the general lack of theory relating to the location of reference points. Furthermore, starting wealth levels may not be risk free to the extent that they may themselves be subject to various outcomes according to states.

illustrated using a four-segment value function defining changing risky preferences conditional upon the magnitude of risky prospects. Thus, when outcomes are large, it was hypothesised that individuals would be risk-averse with regard to gains but risk-seeking with regard to losses. These risk preferences were expected to reverse with small to moderate outcomes. The value function then increased monotonically with three inflection points, transitioning from concave and convex segments along the curve, reflecting shifts in risk preferences as shown in Figure 3.

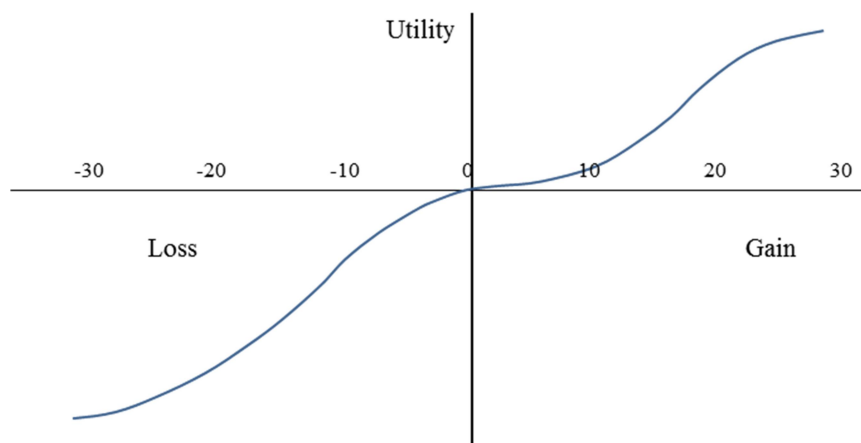


Figure 3. The Markowitz four-fold value function, depicting transitions in risky preferences.

Markowitz value functions are of course unique to individual decision-makers conditional upon their subjective probability beliefs which might render them either overly optimistic or pessimistic with regard to risky choices. However, despite laying these foundations, Markowitz undertook no formal empirical investigation of the four-fold value function and provided no derivation of possible decision-weights to explain over- and underweighting of risk based upon preferences. The role of subjective probability preferences was, however, discussed in general terms in a later work (Markowitz, 1968).

### **3.2. Prospect Theory & Rank-Dependent Utility**

The important foundations of Markowitz described above can be traced through to one of the most significant developments in approaches to behavioural decision-making in the form of Prospect Theory (Kahneman & Tversky, 1979; for a comprehensive critique and review, see: Wakker, 2010). Prospect Theory provides a systematic framework for describing decision-making under conditions of risk while simultaneously accommodating a number of observed behavioural biases with regard to subjective preferences. The theory is based upon two primary tenets; reference dependence and subjective weighting (“probability distortion”).

From a series of experiments, Kahneman and Tversky identified widespread violations of the axioms of expected utility theory, similar to those identified by Allais and others. In addition, they noted that preferences between prospects were influenced by changes in wealth rather than simply its quantum, consistent with the proposals of Markowitz. Similarly, individuals were found to underweight outcomes which were only probable when a certain alternative was presented (the “certainty effect”), implying categorical differentiation of payoffs based upon their perceived degree of likelihood. The same effect was found to trigger risk-aversion in relation to choices involving gains and risk-seeking behaviour in choices involving certain losses. Branch effects were also apparent as common components of prospects appeared to be ignored. Kahneman and Tversky termed this an “isolation effect”.

Kahneman and Tversky formulated a two-stage model in relation to simple prospects involving monetary outcomes. The first stage related to the mental editing (framing) of prospects, essentially reducing the components of various branch options contained within prospects into simplified forms. Such editing was assumed to include the identification of

reference points against which gains or losses could be assessed along with *coalescing*, whereby the decision maker aggregates individual probabilities associated with identical outcomes. It was further assumed that risky components of prospects would be separated from any risk-free components. Finally, individuals were assumed to disregard (“cancel”) components common across all prospects, thus reducing choice to the subset of differences between prospects. This editing stage was then assumed to lead to a second evaluation stage in which the filtered prospects are considered and ranked.

Similar to Markowitz, reference dependence is modelled formally by a monotonically increasing value function  $v : \mathbb{R} \rightarrow \mathbb{R}$  defined on the basis of changes in wealth states in which  $v(0) = 0$ . The function is illustrated in Figure 4.

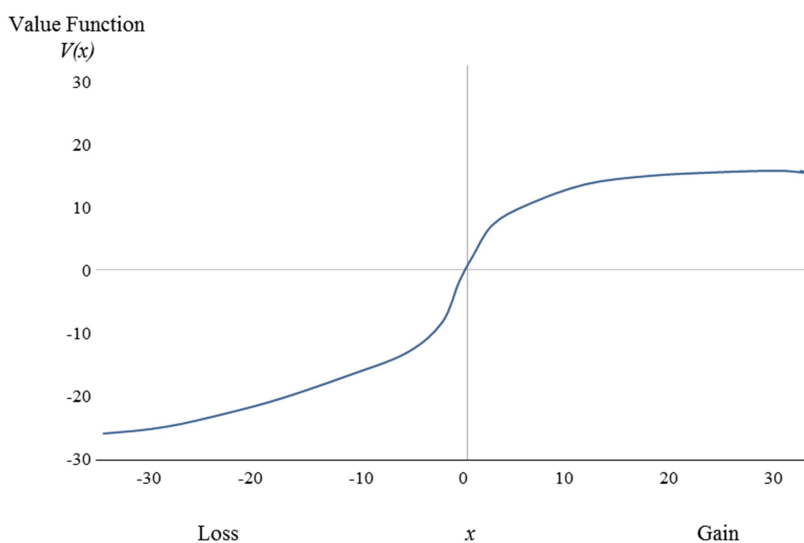


Figure 4. A representative prospect theory value function.

Diminishing marginal utility applies to gains while diminishing marginal disutility applies to losses. Loss aversion is defined for all  $x$  such that:  $v(x) < -v(-x)$ . The function differs in the domain of gains and losses so that  $v(x) = \tilde{v}(x)$  for all  $x \geq 0$  and  $v(x) = -\lambda\tilde{v}(-x)$  for all  $x < 0$ .

The degree of loss aversion is therefore captured by  $\lambda$ , and applies in all cases where  $\lambda > 1$ . Where general loss aversion exists, the slope of the value function is steeper in the domain of losses than for gains. Consequently, no rational decision-maker with loss aversion should accept a gamble offering  $(x, p = 0.5; -x, p = 0.5)$  over  $(x, p = 1)$  when  $x \neq 0$ . Similarly, if  $0 < |x| < |y|$ , then decision-makers should typically express the following preference:  $(x, 0.5; -x, 0.5) > (y, 0.5; -y, 0.5)$ , implying that overall aversion to risk is greater than that implied by the concave utility function alone.

In addition to a value function, Prospect Theory incorporates a weighting function which is assumed not only to reflect the perceived likelihood of an outcome but also the perceived impact an outcome might have on the desirability of prospects. Framed in this manner, weights are not necessarily assumed to obey probability axioms nor are they simply an expression of belief as to the likelihood of outcomes. The weighting function serves, however, to distort any underlying probability function so that;  $w(p(X)) \neq p(X)$ . The shape of the value function shown in Figure 4 above is then consistent with the proposition that low probabilities are overweighted ( $w(p) > p$ , for low  $p$ ), while moderate to high probabilities are underweighted ( $w(p) < p$ ). The overall weighting function is assumed to be an increasing function of  $p$  with;  $w(0) = 0$  and  $w(1) = 1$ . Assuming regular prospects, neither strictly positive nor strictly negative in outcome, the overall value of a prospect can then be shown as;

$$V(x_1, p_1, \dots, x_n, p_n) = \sum_{i=1}^n w(p_i) v(x_i) \quad \text{Equation 9}$$

The original formulation of Prospect Theory found considerable empirical support (Arkes & Blumer, 1985; Budescu & Weiss, 1987; Chang, Nichols & Schultz, 1987; Elliott & Archibald, 1989; Fiegenbaum & Thomas, 1988; Gregory, 1986; Meyer & Assuncao, 1990;

Payne, Laughhunn & Crum, 1984; Loewenstein, 1988). Nevertheless, under certain circumstances, preferences may violate the First Order Stochastic Dominance (FOSD) axiom. For example, if we are presented with two prospects:  $(10 + x, 0.5; 10, 0.5)$  and  $(10, 1)$ , where  $x > 0$ , the former must dominate the latter assuming FOSD. The rational decision-maker should therefore always express a preference for the risky gamble over the certain alternative since the risky gamble is guaranteed to return at least the same amount as the certain alternative while offering the potential of a higher return in at least one state. However, in accordance with Prospect Theory, the prospects are evaluated as follows:  $w(0.5)v(10 + x) + w(0.5)v(10)$  and  $v(10)$ . If the weighting function is such that  $w$  underweights objective probability ( $w(0.5) < 0.5$ ), then it is possible to find a value for  $x$  small enough that:  $w(0.5)v(10 + x) + w(0.5)v(10) < v(10)$ , thereby violating FOSD.

Birnbaum & Navarrete (1998) confirmed the violation with an experiment offering the following prospects:

$$A := (96, .85; 90, .05; 12, .10)$$

$$B := (96, .90; 14, .05; 12, .05)$$

Rational decision-makers would be expected to prefer  $B$ , ( $EV(B) = 87.7$ ) over  $A$ , ( $EV(A) = 87.3$ ). In the event, it was found that 70% of participants preferred  $A$  to  $B$ . However, it appears that the apparent “decision-errors” stemmed primarily from the framing of the prospects as the percentage of choice violations dropped to almost zero when the same prospects were subsequently presented in the following form, making comparisons easier;



$$A := (96, .85; 90, .05; 12, .05; 12, .05)$$

$$B := (96, .85; 96, .05; 14, .05; 12, .05)$$

Notwithstanding, potential violations of axiomatic behaviour caused by the weighting function remain possible. In fact, the proposition holds whether  $w(0.5) < 0.5$  or  $w(0.5) > 0.5$ . Therefore, monotonicity can only be guaranteed when  $w(0.5) = 0.5$  or, more generally,  $w(p) = p$  for all  $p \in [0, 1]$ . To that extent, the formulation of Prospect Theory allows for the possibility that FOSD will be violated in certain circumstances.

A general solution to empirical axiomatic violations of Prospect Theory (and also those of EU and SEU theories) is provided by models of rank-dependent utility (RDU). RDU assumes that the decision-weights assigned to prospects are a function of their ranking within the distribution of all feasible outcomes. Quiggin (1982) applied such a model to show that FOSD was eliminated by transforming cumulative probabilities into a new rank-dependent cumulative distribution assuming that the sum of the decision-weights equalled 1. One of the major distinctions between Prospect Theory and RDU, therefore, is that the former implies the transformation of each probability into a decision-weight, whereas the latter transforms the complete cumulative distribution. The process by which probability distortion occurs is therefore fundamentally different.

This can be explained by considering a lottery,  $L$ , paying prospects  $x_i$  with probability  $p_i$ . We assume a strict order such that  $x_1 < x_2 < x_3 < \dots$ , and all  $p_i \geq 0$ , subject to  $\sum_{i=1}^{\infty} p_i = 1$ . We then define a strictly increasing utility function as:  $u : \mathbb{R} \rightarrow \mathbb{R}$  along with a weighting function  $w$ , leading to the following general expression for decision weights;

$$\pi_i = w(\sum_{j=i}^{\infty} p_j) - w(\sum_{j=i+1}^{\infty} p_j) , i = 1, 2, 3, \dots \quad \text{Equation 10}$$

The rank-dependent utility of the lottery  $L$  unique to the decision-maker can then be expressed as;

$$RDU(L) = \sum_{i=1}^{\infty} \pi_i u(x_i) \quad \text{Equation 11}$$

Given that  $w : [0, 1] \rightarrow [0, 1]$  is strictly increasing, a rank-dependent cumulative distribution then exists which allows decision-weights,  $\pi_i$ , to be treated in the same way as probabilities. The result is a value function which reflects preferences across all prospects according to the weights assigned on the basis of the salient features of those prospects. Therefore, disproportionate weightings on the lowest and highest outcomes can be accommodated along with general preferences relating to risk aversion or risk seeking across the value function. RDU is therefore capable of accommodating behavioural biases without violating the axiom of FOSD.

### 3.3. Cumulative Prospect Theory

Evidence of potential axiomatic violations of Prospect Theory (PT), along with the practical solution proposed by rank-dependence models, led Kahneman and Tversky (1992) to develop a more refined model of PT based upon the two elements. Therefore, Cumulative Prospect Theory (CPT) emerged as a more robust framework for analysing decisions under risk and uncertainty. As with the original formulation, prospects are assumed to be evaluated relative to a reference point. Consequently, the normalisation of a reference point to zero enables losses to be defined as any negative outcomes relative to the reference point and gains

as positive outcomes against the same benchmark. Prospects are rank-ordered across a value function such that;

$$f := (x_1, P_1; \dots x_k, P_k; x_{k+1}, P_{k+1}; \dots; x_n, P_n) \quad \text{Equation 12}$$

where;  $x_1 < \dots < x_{k+1} < \dots < x_n$

As with PT, individuals are assumed to hold different preferences with regard to gains and losses. Negative and positive domains of  $f$  can therefore be represented as:

$$f^+ := (0, P_1 \cup \dots \cup P_k; x_{k+1}, P_{k+1}; \dots; x_n, P_n)$$

$$f^- := (x_1, P_1; \dots; x_k, P_k; 0; P_{k+1} \cup \dots \cup P_n)$$

where;  $P_1 \cup \dots \cup P_i$  defines a prospect with an outcome equal to or less than  $P_i$ .

The assumption of *sign dependence* means that *independence* can only be satisfied within each preference-ranked sign set. The preference weightings with regard to the positive and negative components of a prospect can be represented as;

$$\pi_i^+ = W^+(P_i \cup \dots \cup P_n) - W^+(P_{i+1} \cup \dots \cup P_n), \text{ for } i = k+1, \dots, n, \text{ and;}$$

$$\pi_i^- = W(P_1 \cup \dots \cup P_i) - W(P_1 \cup \dots \cup P_{i-1}), \text{ for } i = 1, \dots, k$$

$W^+$  and  $W^-$  are the separate weighting functions applied to prospective outcomes in the domains of gains and losses relative to the reference point. *Sign dependence* then implies that decision weights relating to purely positive or negative prospects sum to one. Therefore;

$$\sum_{i=1}^n \pi_i = \sum_{i=1}^n [W(P_i \cup \dots \cup P_n) - W(P_{i+1} \cup \dots \cup P_n)] = \quad \text{Equation 13}$$

$$W(P_1 \cup \dots \cup P_n) = W(S) = 1$$

However, such a summation of decision weights across mixed prospects does not necessarily hold meaning that, in such cases, the sum of weights can be less than one (termed *subcertainty*). Nevertheless, monotonicity is preserved by the strict rank-dependent ordering applied to each domain. It is now possible to represent a prospect under CPT as;

$$V(f) = \sum_{i=1}^k \pi_i^- v(x_i) + \sum_{i=k+1}^n \pi_i^+ v(x_i) \quad \text{Equation 14}$$

where  $v$  is a strictly increasing continuous value function with  $v(0) = 0$ .

The subjective value of a prospect described by the weighted probability measure  $p$  can then be shown as;

$$V(p) := \int_{-\infty}^0 v(x) \frac{d}{dx} (v(F(x))) dx + \int_0^{+\infty} v(x) \frac{d}{dx} (-v(1 - F(x))) dx \quad \text{Equation 15}$$

where  $v$  is a value function,  $w$  is the weighting function and  $\int_{-\infty}^x dp$  is a continuous, increasing cumulative distribution function for all values to  $x$ .

### 3.3.1. Determining Reference Points

While risk under CPT is derived from a combination of three elements: the individual's basic utility function; unique decision-weights; and loss aversion, the weighting and

reference-dependent value functions are predicated upon the location and nature of the reference point. Determining the reference point is actually critical to the theory as it is that which, along with rank-ordering of prospects within the positive and negative domains, ensures that FOSD is maintained despite *subcertainty* (the sum of decision weights being less than one with respect to mixed prospects). Yet, CPT, like its predecessor, PT, is silent about how reference points are actually derived.

In his earlier work, referenced at the start of this chapter, Markowitz spoke of “customary wealth” as a potential reference point but had no method of deriving this in the case that it diverged from starting wealth. For their part, Tversky and Kahneman (1991) also considered that a reference point would typically correspond with a “current wealth position”, although suggested that this could be modified by expectations or aspirations. In general, therefore, reference points were largely assumed to default to a status quo position or were considered to be psychological in nature. This lack of clarity clearly hinders empirical testing of a reference-dependent theory such as CPT as it allows too much latitude in terms of specification; the theory might be adapted to fit most observed outcomes.

In practice, there is ample scope for decision-makers to set single or even multiple reference points and to shift or modify them over time. Similarly, while CPT appears to imply that a status quo reference point is a certain alternative to risky prospects, there is no reason to assume that they cannot be stochastic. Indeed, stochastic reference points are assumed when those reference points involve expectations (Koszegi & Rabin, 2006). Since expectations can be assumed to involve uncertainty, prospects can then be considered as elements within lotteries offering an array of potential payoffs. The comparison can then be made with an alternative lottery which provides the relative reference points. Therefore, if we define a

lottery,  $L$ , offering  $N$  possible outcomes with associated probabilities,  $p$ ;  $L = (x_1, p_1, \dots, x_n, p_n)$ , subject to  $\sum_{i=1}^n p_i = 1$ , there exists a reference lottery,  $R$ , with  $M$  potential outcomes;  $R = (r_1, q_1, \dots, R_m, q_m)$ , subject to  $\sum_{j=1}^m q_j = 1$ . Decision-makers then evaluate the prospects contained in  $L$  against those found in  $R$  in the following manner;

$$U(L | R) = \sum_{i=1}^n p_i [u(x_i) + v(x_i | R)] \quad \text{Equation 16}$$

where,  $u(x_i)$  is the expected utility from outcome  $x_i$  and  $v(x_i | R)$  measures the utility (subjective value) associated with the gain or loss from outcome  $x_i$  measured relative to the outcome distribution in  $R$ .

Loomes and Sugden (1986), modelling disappointment aversion, applied such a model whereby  $x_i$  was assumed to be compared with a summary statistic from  $R$  such that;

$$v(x_i | R) = \mu (u(x_i) - (\sum_{j=1}^m q_j u(x_j))) \quad \text{Equation 17}$$

Koszegi & Rabin (2006) proposed a more general model in which each  $x_i$  is compared with every prospect in  $R$ ;

$$v(x_i | R) = \sum_{j=1}^m q_j \mu(u(x_i) - u(x_j)) \quad \text{Equation 18}$$

Therefore,  $x_i > x_j$  represents a gain over all such elements in  $R$ , while;  $x_i < x_j$  represents a loss. The assumption is that lottery  $L$  is then viewed on the basis of an assessment of all gains and losses relative to the elements in  $R$ .

Note that, while both approaches combine classic utility from consumption with utility associated with a gain or loss, they make different predictions based upon an individual's preferences relating to gains and losses. This becomes apparent if, for example, there is an increase in risk associated with  $R$  which does not affect the decision-maker's utility with regard to  $R$ . Assuming the prospect comparisons in Equation 17, a change in risk in  $R$  should have no impact on overall expected behaviour. However, under the broader formulation of comparisons in Equation 20, an increase in the risk of  $R$  should make the decision-maker correspondingly more willing to accept risk in  $L$ , creating a type of endowment effect with respect to risk.

Koszegi & Rabin further suggested that reference points are likely to be transient in nature, shifting over time and when actual choices are made. Therefore, given the lottery  $L$  is actually chosen, at some point that lottery replaces  $R$  as the reference benchmark against which actual outcomes are judged in accordance with gains or losses relative to expectations for  $L$ . Koszegi and Rabin (2007) applied this concept more formally defining a "choice-acclimatising personal equilibrium". Essentially, if a decision-maker commits to a choice, such as lottery  $L$ , well in advance of realising the actual outcome, expectations about  $L$  will become established as the new reference point by the time the outcome becomes known. He and Strub (2019) extended this within a mental-adjustment model with loss version, finding shifts in both exogenous and endogenous reference points over time.

Approaching the issue of how such expectations are formed, the process can be slightly simplified if we assume that expectations are either exogenous or endogenous to the current choice. In the examples above, the primary reference set defined by  $R$  was exogenous to choice  $L$ . If reference points shift after choices have been made, such that  $R$  is replaced by  $L$ ,

expectations are then endogenous to choice  $L$ . From Equation 16, we see that the individual's utility would then be given by;  $U(L|L)$ . Rational behaviour would then require selection of the lottery which maximises that term.

There is ample scope to assign psychological traits with regard to the treatment of reference points. For example, disappointment aversion can apply as decision-makers are likely to develop prior expectations with regard to a lottery which may not be realised. If we apply the two alternative methods of comparison described above in relation to disappointment aversion (DA) and the Koszegi & Rabin (KR) formulation, we find the following interpretations with regard to gain-loss utility;

$$\text{DA: } U(L|L) = \sum_{i=1}^n p_i [u(x_i) + \mu(u(x_i) - \sum_{j=1}^n p_j u(x_j))] \quad \text{Equation 19}$$

$$\text{KR: } U(L|L) = \sum_{i=1}^n p_i [u(x_i) + \sum_{j=1}^n p_j \mu(u(x_i) - u(x_j))]$$

It may be noted that the KR model implies that deviations from EU are based upon subjective assessments of gains versus losses, although there is no equivalent of a value function to specify this process. Nevertheless, there is implied information regarding a prospect's ranking as, under this formulation, an outcome's rank depends upon the number of gain/loss comparisons made with the reference set. This has some obvious similarities with rank-dependent probability weighting models which transform the cumulative probability distribution.

Bhatia and Golman (2015) presented a model of reference dependence extending the concept that the choice of reference points is influenced by particular attributes of those reference points. Behaviours such as the endowment effect and status quo bias, for example,



have been explained on the basis of attention towards attributes or reference points (Ashby, Dickert & Glockner, 2012); the decision-maker then places a greater weight on key attributes when determining preferences. Therefore, while reference points are not assumed to affect perceptions regarding gains and losses, they do impact choice through a search for, and comparison of, similar attributes. As a result, changing reference points might result in changes in the weights assigned to particular attributes, perhaps modifying choices. The result is a model of attention-biased utility based upon subjective valuations of key attributes. On this basis, and assuming that attributes are considered to be mutually independent, attribute-biased utility maximisation can simply be defined in the standard way as;

$$U(x) = \sum_{i=1}^n V_i(x_i) \quad \text{Equation 20}$$

where  $V$  is the decision-makers subjective value function with  $V_i(x_i)$  then denoting the valuations of attribute  $i$  with regard to option  $x$ .

The weighting of specific attributes is accommodated by applying a non-negative, strictly increasing attention function,  $a = a(r)$  which defines the individual's attention weight given a particular reference point  $r$ . Choice is then made according to the following weighted attribute utility function;

$$U(x|r) = \sum_{i=1}^n a_i(r_i) \cdot V_i(r_i) \quad \text{Equation 21}$$

Note that no assumptions are made about the nature of the value function,  $V$ ; utility maps according the value function regardless of the reference point. In addition, there is no assumed divergence in terms of the assessment of gains or losses relative to a particular reference point. Instead, preferences are derived in accordance with a prospect's primary

attributes defined according to the subjecting weighting function. The attribute utility model is consistent with evidence that salient choice options affect an individual's attention to choice characteristics (Pachur & Scheibehenne, 2012) and hence preferences. The model is therefore capable of explaining a number of observed behavioural anomalies, including a reversal of the endowment effect for negative attributes and the strengthening of the effect for more highly weighted attributes.

Early approaches to modelling multiple reference points tended to assumed that decision makers reduced these points to a single composite (Olson, Roesse & Zanna, 1996), although the proposition has been challenged by Ordóñez, Connolly and Coughlan (2000). A typical assumption is that decision-makers applying multiple reference points impose minimum goals which they strive to attain. This has been found to elicit behaviour contradictory to Prospect Theory to the extent that risk aversion often does not appear once performance exceeds the status quo (Sullivan & Kida, 1995). Thus, while business managers have been found to exhibit risk aversion when confronted with the possibility of losses sufficient to return performance to the prior status quo, they have also been found to be risk seeking in pursuit of higher goals once the status quo had been sufficiently exceeded. Sullivan and Kida concluded that each of the multiple goals could exert some influence of behaviour simultaneously.

March and Shapira (1992) proposed a variable risk preference model such that decision-makers could switch attention between multiple reference points and goals. However, they assumed that the influence of each was mutually exclusive, implying that decision-makers adopt either a fully risk averse, preservation strategy or an active risk seeking strategy in pursuit of goals. This approach has some similarities with Lopes's "surviving and thriving" models (Lopes, 1987), although the latter focussed more on transposed decision weights rather

than reference points. The effect of multiple reference points on strategic behaviour has been considered with regard to interactive, mainly bilateral, negotiations (Neale & Bazerman, 1991; Neale, Huber & Northcraft, 1987). In relation to real estate purchases, White, Valley, Bazerman, Neale & Peck (1994) found that reservation prices tended to act as dominant reference points along with the maximum willingness to pay of the buyer. Kristensen and Garling (2000) found that initial offers could affect prior reference points, leading to modification in anchors. Within this process, a distinction is normally drawn between anchors and reference points (Kahneman, 1992). Thus, anchors are perceived to exist in terms of offer and counteroffer levels whereas reference points determine whether those offers and counteroffers are perceived as gains and losses. A reference point may therefore influence counteroffers depending on how the new anchor point is perceived. Kristensen and Garling (1997) found that proposed selling prices acted as anchor points for basing counteroffers.

A further interactive approach to multiple reference points was provided by Wang and Johnson (2009) using a model with three separate reference points; the status quo, a coalesced multiple reference point and aspirational goals. By cross-referencing outcomes with these triple “areas of outcome”, decision-makers could experience success (gain) or failure (loss). Wang and Johnson argue that each of the outcome regions (staying above the status quo, exceeding reference points or exceeding a goal) could have different decision weights associated with each. Therefore, a salesperson exceeding a base target level may derive no increase in overall utility in the event that such performance falls short of a higher goal which would have triggered a financial bonus. This conflicts with Prospect Theory’s assumption regarding relative insensitivity to small changes in outcomes since, in this example, a potentially modest increase in sales performance could potentially result in a tangibly superior outcome by triggering the bonus threshold. In such cases, great importance is likely to be

attached to achieving that small additional gain. It might therefore be assumed that such reference points will reflect critical levels of perceived utility outcomes, in which case the overall value function is likely to exhibit a greater slope in the neighbourhood of such critical reference points rather than the status quo point. Consequently, decision-makers might be expected to accept higher risk gambles in the neighbourhood of such reference points if the risky prospect offers the chance of exceeding a certain threshold.

The tri-reference model of Wang and Johnson was tested in two experiments by Koop and Johnson (2010). In the first experiment, participants were presented with a number of risky gambles within the broader context of three reference points; a minimum requirement (MR), the status quo (SQ) and a goal (G). Based upon prospective payoffs, various return outcomes were possible with respect to individual reference points. Consequently, a general outcome space could be mapped as shown in Figure 5.

Table 7. Outcomes from risky gambles relative to objective reference points ‘mapped’ into a general outcome space.

Outcome of Gamble $x$	Classification of Outcome
$x < \text{MR}$	Failure
$\text{MR} \leq x < \text{SQ}$	Loss
$\text{SQ} < x < \text{G}$	Gain
$\text{G} \leq x$	Success

A total of 12 notional risky gamble pairs were then prepared for presentation, as shown in Table 8. In each of the gamble pairs, explicit values (expressed in lira) for reference points were assigned such that  $\text{MR} = 1000$ ,  $\text{SQ} = 2000$  and  $\text{G} = 4000$ . In each case, the probability of outcomes associated with each of the risky gambles was declared to be equal, meaning that  $\Pr(a_1) = \Pr(a_2) = p = .5$ . It will be noted that each return pair,  $a_1$  and  $b_1$ , resides in the same outcome space (which then describes the common outcome relative to the identified reference

point). Therefore, in the case of Pairs 1, 5 and 9, the associated outcomes for  $a_1$  and  $b_1$  represent failure (outcomes below MR). The  $a_2$  and  $b_2$  binary outcomes were set so that they resided in adjacent outcome spaces thereby straddling a defined reference point. For example, in the case of Pair 9,  $a_2$  represents a gain ( $SQ < x(a_2) < G$ ) while  $b_2$  represents success ( $x(b_2) > G$ ), using the previously defined outcome descriptors.

Table 8. Paired risky gambles with common outcomes relative to specified reference points.

Pair Number	Gamble A		Gamble A		Common outcome ( $a_1, b_1$ )	Reference point involved ( $a_2, b_2$ )
	$a_1$	$a_2$	$b_1$	$b_2$		
1	940	960	580	1220	Failure	MR
2	1880	920	1600	1100	Loss	MR
3	3840	760	3300	1200	Gain	MR
4	4900	600	4220	1180	Success	MR
5	880	1920	620	2080	Failure	SQ
6	1720	1880	1420	2080	Loss	SQ
7	3700	1800	2820	2580	Gain	SQ
8	4600	1800	4200	2100	Success	SQ
9	760	3860	400	4120	Failure	G
10	1800	3700	1160	4240	Loss	G
11	3100	3600	2360	4240	Gain	G
12	5060	3680	4400	4240	Success	G

*Note:* The data shows pairs of gamble outcomes which are assumed to have equal probabilities (.5) of success. The gambles are notionally evaluated against pre-defined, fixed reference points representing a minimum return requirement (*MR*), the status quo (*SQ*) or a gain (*G*). Paired outcomes are then categorised on the basis of outcomes versus the various reference points such that an outcome,  $x$ , below *MR* is defined as “Failure”,  $x > SQ$  denotes “Loss”,  $x > SQ < G$  defines “Gain”,  $x > G =$  “Success”.

By varying the values of  $a_1$  and  $b_1$  while maintaining the reference points straddled by  $a_2$  and  $b_2$ , three sets of gamble pairs (4 x 3) were presented to each subject representing differing possible outcomes across the entire outcome space. In the case of each gamble, the values of  $a_1, b_1, a_2$  and  $b_2$  were chosen so that *A* always had a higher expected return than *B*, whereas *B* offered the prospect of a superior functional outcome as it allowed the possibility of exceeding a specific reference point.

In order to test the strength of any reference point effect on behaviour, two conditions were defined. In the first, *strong* (certain) condition, each of the three reference points was disclosed to participants as a single value. In the second, *weak* (uncertain) condition, SQ remained fixed while MR and G were described in terms of symmetrical probability distributions around a mean equal to the value declared in the *strong* condition. Participants were incentivised in terms of both instant monetary reward, based upon achieving certain benchmarks, and the prospect of entry into a future bonus draw with its own payoff.

Results from the experiment indicated that participants generally preferred gambles associated with achieving reference points as opposed to seeking the highest expected payoff. This was most marked under the strong condition where reference dependent gambles were chosen in preference to higher payoff gambles over 78% of the time. The same propensity for reference point dependence was also found for the weak condition, although the level of significance was lower than that found in the strong condition. The effect was, however, particularly strong around MR. Preferences for gambles around SQ were equally significant for both the strong and weak conditions. Unlike MR and G, SQ was fixed under both conditions and therefore not subject to the degree of uncertainty found within the weak condition with regard to other reference points, defined by their distribution of outcomes rather than a single value. It would appear, therefore, that the introduction of a degree of uncertainty had some impact upon strength of preferences over and above simple risk aversion around reference points.

The effect of multiple reference point dependence differentiable from simple risk aversion, expected utility and SQ dependence implied by Prospect Theory was investigated

explicitly in a second experiment. Koop and Johnson employed a similar methodology in terms of selective gambles defined precisely across a reference dependent outcome space. A number of the gambles applied in experiment 1 were restated in experiment 2 in order to create a more effective delineation of expected outcomes based upon the various possible objective functions (reference dependence, risk aversion, expected utility and prospect theory). The new values used are shown in Table 9.

Table 9. Paired risky gambles with common outcomes relative to specified reference points used in experiment 2.

Pair Number	Gamble A		Gamble A		Common outcome ( $a1, b1$ )	Reference point involved ( $a2, b2$ )
	$a1$	$a2$	$b1$	$b2$		
1	940	960	580	1220	Failure	MR
2	2000	920	1720	1100	Loss	MR
3	3650	750	3300	1200	Gain	MR
4	4050	950	4300	1100	Success	MR
5	880	1920	720	2680	Failure	SQ
6	1900	2200	1420	2580	Loss	SQ
7	3700	1800	2970	2580	Gain	SQ
8	4600	2200	4200	2700	Success	SQ
9	640	3980	720	4000	Failure	G
10	1800	3700	1360	4240	Loss	G
11	3300	3860	2560	4500	Gain	G
12	5060	3640	4400	4200	Success	G

*Note:* Modified data used in experiment 2 designed to test behaviour in relation to behavioural objectives (reference dependent, risk aversion, expected utility and prospect theory).

The values assigned to explicit reference points, denominated in lira, were; MR = 1000, SQ = 2500 and G = 4000. The common outcome and reference points were as described in the prior experiment. Similar incentives were offered to participants in terms of final entry into a draw, although in simplified form; once MR was exceeded, participants were awarded 5 entries into the final prize draw rising to 10 entries should G be exceeded.

In order to model expected utility with general risk aversion, a concave utility function of the form  $u(x) = x^{0.8}$  was applied to expected values from individual gambles. Outcomes in relation to prospect theory preferences were modelled differently based upon their gain or loss domains relative to SQ. In the domain of gains (for all  $x > SQ$ ), the utility from risky outcomes was defined by  $u(x) = x^\alpha$  while in the domain of losses (for all  $x < SQ$ ), the following expression was applied;  $u(x) = -(\lambda(SQ - x)^\beta)$ . The parameters were set in accordance with the median exponent ( $\beta$ ) of the value function and loss aversion ( $\lambda$ ) coefficients derived by Tversky and Kahneman (1992), specifically,  $\alpha = \beta = 0.8$  and  $\lambda = 2.25$ , the latter indicating pronounced loss aversion. Based upon these definitions, it is possible to evaluate each risky outcome on the basis of risk aversion<sup>4</sup>, expected value (again assigning a probability of .5 to each event within each paired gamble), expected utility and the value function central to Prospect Theory. Table 10 replicates these values, as derived by Koop and Johnson.

Overall, results once again indicated a statistically significant preference for gamble B over A. This preference was apparent around each of the reference points. Isolating results based upon divergent predictions based upon choice preference characteristic again showed behaviour consistent with reference point dependence. Thus, taking the cases in which reference point dependence predicted an outcome different from that of either risk aversion, EV, EU or PT, reference point dependence was found to have significantly greater explanatory power with regard to actual outcomes. Therefore, the inclusion of additional reference points over and above SQ was found to affect overall behaviour. The collective use of multiple reference points proved superior to a single reference point in explaining the data.

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<sup>4</sup> Risk is defined simply as the absolute numerical difference between binary outcomes within each paired gamble. The lower the dispersal of outcomes, the lower the perceived risk, with no weighting function applied.



Table 10. Risky gambles presented in experiment 2 evaluated using various objective functions.

Pair Number	Risk		Expected Value (EV)		Expected Utility (EU)		Prospect Theory (PT)	
	[a1 - a2]	[b1 - b2]	EV(A)	EV(B)	EU(A)	EU(B)	PT(A)	PT(B)
1	<b>20</b>	640	<b>950</b>	900	<b>241.09</b>	228.48	<b>-802.50</b>	-820.49
2	1080	<b>620</b>	<b>1460</b>	1410	<b>336.16</b>	329.36	<b>-569.76</b>	-601.52
3	2900	<b>2100</b>	2200	<b>2250</b>	453.61	<b>471.74</b>	-301.71	<b>-243.52</b>
4	<b>3100</b>	3200	2500	<b>2700</b>	505.08	<b>538.95</b>	-222.92	<b>-168.88</b>
5	<b>1040</b>	1960	1400	<b>1700</b>	325.03	<b>372.93</b>	-598.45	<b>-416.36</b>
6	<b>300</b>	1160	<b>2050</b>	2000	<b>445.88</b>	434.35	-295.65	<b>-283.88</b>
7	2050	<b>390</b>	<b>2750</b>	2775	<b>570.26</b>	568.12	-52.76	<b>85.30</b>
8	2400	<b>1500</b>	3400	<b>3450</b>	661.77	<b>673.90</b>	119.52	<b>226.67</b>
9	3340	<b>3280</b>	2310	<b>2360</b>	467.09	<b>477.30</b>	-292.40	<b>-274.50</b>
10	<b>1900</b>	2880	2750	<b>2800</b>	558.71	<b>559.52</b>	<b>-67.12</b>	-118.20
11	<b>560</b>	1940	<b>3580</b>	3430	<b>696.46</b>	684.77	<b>265.68</b>	231.90
12	1420	<b>200</b>	<b>4350</b>	4300	<b>812.57</b>	806.78	<b>405.89</b>	401.89

*Note:* For each gamble, the preferred option, based upon the specified objective function in each case, is denoted by emboldening in the Table. Therefore, both expected utility and Prospect Theory would predict the same choices in relation to the first five gambles shown.

Close to reference points, participants were found to be risk seeking when they had the opportunity to exceed that point but became risk averse when they faced the prospect of falling below it. Importantly, participants exhibited a more complex understanding of the reference points and modified their behaviour systematically within those boundaries, suggesting that prospect theory's status quo approach is too simplistic. Confirming Wang and Johnson's hypothesis, the three reference points were found to exert differential impacts on behaviour, with a multiple reference point more significant than status quo.

### 3.4 Discussion

The models of rational behaviour covered in this chapter considerably extend the framework of classical expected utility. PT and CPT (along with SEU and vNM-expected utility) are normative, axiomatic models which define rational behaviour. However, this description only goes sufficiently far to enable judgements to be made in relation to whether

an individual is behaving *as if* they are wholly rational in their decision-making. The models therefore consider the output of a process rather than the process itself, although there are clearly elements of the models which assume or imply process. Diminishing marginal utility and loss aversion are component of most decision-making models of this type, such behaviour captured by the concave shape of a standard utility function and made explicit in the case of domain-specific value functions. While marginal utility models sensitivity to outcomes of differing magnitudes, the probability-weighting function within CTP is a purely subjective construct which transforms objective, cumulative probability distributions into decision-weights; the curvature of this function then assumes diminishing sensitivity to probabilities over the course of the function. The rank-dependence element of CPT is a further accommodation of behavioural traits allowing even more flexible individual-specific weighting functions to be assumed, thereby explaining a wider range of observed behaviours. As with PT, decision-makers can be both risk-seeking and risk averse depending upon the prospect and domain.

The existence of reference points chosen by the decision-maker are key components of PT and CPT. Both are formulated on the basis of a single, apparently risk-free reference point, an assumption relaxed by reference-dependent and attribute models. Theories of general choice relating to reference points introduce additional elements which are essentially behavioural in nature. Under these formulations, stochastic and multiple reference points can exist, changing according to subjective decisions of the individual.

Axiomatic approaches to decision-making derive conditions under which an individual's utility is maximised. Implicit in this is the proposition that decision-makers consider all alternatives and, in the case of CPT and other rank dependent models, can order them

rationally by applying representative weighting functions which define preferences. Similarly, choice models based upon attributes or criteria assume a potentially complex process of comparison occurs in order to determine preferences. Attribute attention and multi-criteria templates may serve to focus the decision-making process yet the ability to perform such comparisons across a large number of possible options is somewhat taken for granted. The CPT editing phase assumes that prospects are filtered prior to an evaluation phase. Elements of that filtering phase are made explicit and are largely consistent with axioms of rational behaviour. The assumption, therefore, is that this is a behaviourally efficient process not subject to fundamental bias. Once again, therefore, decision-makers are assumed willing and capable to apply sufficient analysis to a range of options not matter the size of the choice set. In the next chapter, a number of largely descriptive approaches to choice reduction are discussed. The purpose of this is primarily to focus more attention on cognitive process rather than just outputs. The chapter concludes by briefly considering formal models and approaches to process-tracing as well as more models derived from neurology.

## CHAPTER 4

### HEURISTICS, PROCESS TRACING AND THE ROLE OF FEEDBACK

#### 4.1. Choice Reduction

In early work relating to choice reduction and filtering, a *lexicographic* rule was proposed whereby decision-makers were assumed to identify attributes of outcomes which are of primary importance. Individuals are then assumed to apply ranking rules to attributes which may be expressed in categorical form as; “good”, “neutral” or bad”. Accordingly, such lexicographic rules are applied to determine the option which best fits the most desirable attributes (von Neumann and Morgenstern, 1944). Only in the event of indifference between competing choices is the decision-maker assumed to consider the second most desirable attribute, and so on, until a unique solution is found.

This procedure was expressed more formally by Tversky (1969, 1972) using semiordering to impose thresholds on partial rankings. Therefore, assuming a function  $f$  on a choice set  $X$ , we can derive an ordered sequence of dimensions based upon an individual's preferences with regard to attributes,  $f = (f_1, \dots, f_n)$ . It is then assumed that there is a threshold,  $\alpha$ , which is required to distinguish between dimensions. The preference,  $f_i(x) > f_i(y)$  therefore holds only in the case;  $f_i(x) > f_i(y) + \alpha$ . Since the dimensions are ranked in terms of order of importance, the decision-maker considers them in sequence; when a dimension  $i$  is found for which  $x > y + \alpha$ , the condition rule is satisfied given the threshold and  $x$  is considered preferable (superior) to  $y$ . At this point, no later dimensions need be considered.

This process incorporates conjunctive and disjunctive rules proposed by Coombs (1951) and Dawes (1964). Under the conjunctive rule, prospects which exceed all thresholds are considered desirable while under the disjunctive rule, a prospect exceeding any of the thresholds is potentially acceptable. Jedidi and Kohli (2005) provided generalisations of conjunctive and disjunctive rules by assuming that decision rules can be based upon a minimum number of possible decision-criteria being met. Therefore, assuming that options must satisfy at least  $h$  out of  $n$  possible criteria,  $h = 1$  would describe a disjunctive rule while  $h = n$  defines a conjunctive rule. As  $h$  moves higher, more and more criteria must be fulfilled in order to be considered acceptable. Jedidi and Kohli defined this as a “subset-conjunctive rule”, accommodating situations where there is incomplete information with regard to alternatives. A probabilistic component was also added to the model, denoting a decision-maker’s perception of the likelihood of finding a particular level of an attribute acceptable. Such probabilities can then be taken to reflect the importance attached to a particular attribute.

Empirical evidence suggests that consumers do indeed engage in considerable choice reduction based upon choice attributes. For example, studies have shown that consumers typically reduce consideration of packaged goods to a subset of 3 to 4 attributes out of a possible 30 to 40 (Hauser & Wernerfelt, 1990; Urban & Hauser, 1993). Various generic attributes have also been found to affect subset choice. Therefore, variability in the perceived brand quality across choice sets has been found to influence consumer subsets, with increasing variability in quality reducing the overall number of brands considered (Belonax Jr & Javalgi, 1989). However, abundant choice of broadly similar goods can lead to weaker preferences, requiring greater cognitive effort (Chernev, 2003), although focusing upon unique, or distinguishing attributes of items on the part of sellers has been found to increase the likelihood of those items being included in consumer choice subsets (Dhar & Sherman, 1996;

Kivetz & Simonson, 2000). This implies that consumers do indeed seek differentiating characteristics, on which they place more weight, when comparing broadly similar competing items.

The lexicographic models described above suggest a framework for achieving choice reduction by matching alternatives to defined criteria. It is assumed, therefore, that not all of the available information relating to competing choices is relevant to the decision taken. The models thus have both decision and stopping rules; once sufficient conditions are met, there is no need to proceed further. The process described therefore assumes a degree of economy with regard to cognitive resource.

Heuristic decision rules similarly describe processes whereby choices are made on the basis of partial information and cues. As task complexity increases, either based upon the number of alternatives or their salient characteristics, decision-makers seek to reduce that complexity by simplifying the dimensions of the task. As with other decision-rule models, not all aspects of alternatives are assumed to be considered in a systematic manner. Instead, decision-makers are deemed to be “fast and frugal”, undertaking an evaluation by applying a type of mental process whereby judgments about current events, objects or options are made based upon past knowledge or by reference to other known cues, events, objects or options which are considered to be sufficiently similar. Tversky and Kahneman (1974) considered heuristics to be “mental shortcuts” which often triggered biases and “errors” (violations of axiomatic behaviour). They argued that each of the three main heuristics which they identified: *representativeness*, *availability* and *adjustment and anchoring*, often led to predictable biases in decision-making. These heuristics can be summarised as follows;

- *Representativeness* describes a process whereby decision-makers evaluate *A* on the basis of the characteristics of *B* which is deemed to be similar and about which something is known. Therefore, the probability of event *A* is inferred to be similar to that of *B*. Alternatively, individuals may classify *A* as similar to *B* based upon shared characteristics or may extend the characteristics of a small sample to assume they apply across a much larger sample.
- *Availability* assumes that decision-makers make evaluations based upon immediate examples, recent events, experiences or observations. The matching element of this heuristic can therefore be seen to reinforce representativeness.
- *Adjustment and anchoring* describes a process whereby decision-makers assess probabilities intuitively, taking an anchor which acts as a reference point and then adjusting until a “reasonable” representation is found.

Many other heuristics have been identified defining frugal mental processes (see Gigerenzer & Gaissmaier, 2011; Blumenthal-Barby, 2016). Some, such as *tallying* and *take-the-best* can be seen to correspond closely with sequential decision models as described earlier. Tallying, for example, is a process whereby class objects are compared one to another; one option may therefore be found to dominate another based upon a cumulation of comparative measures. For example, if asked to judge whether the UK or Germany would achieve a stronger rate of economic growth over the next twelve-months, we may consider a number of primary indicators to help us make that judgment (for example; recent GDP growth, labour productivity, unemployment, inflation and interest rates, debt levels and relative exchange rates). The decision-maker applying the tallying heuristic would then evaluate the two countries based upon each metric with the one scoring higher cumulatively then being favoured as the higher growth economy. Whereas a formal model, such as regression analysis,

would derive weights via regression coefficients for each of these variables resulting in an equation providing a numerical forecast for growth in each country, the heuristic approach assumes that the various factors are equally weighted in the tallying process (although there would appear to be no reason why the decision-maker could not mentally apply some sort of weighting procedure). This replicates the conjunctive and subset-conjunctive rules outlined above. The take-the-best heuristic more closely aligns with the disjunctive rule whereby a decision is made based upon a primary characteristic or metric. Therefore, in the case of economic growth comparisons, a decision-maker might rank the factors in terms of importance. Once a difference is found between two options, the preference is made without any further consideration.

Heuristics and choice reduction models typically involve trade-offs between cognitive effort and decision-accuracy; they are not utility maximising strategies as prescribed by the axioms of rational behaviour as those models assume full comparison of available options. Instead, choice reduction is more closely aligned with *satisficing* behaviour and *bounded rationality*. Satisficing behaviour exists when decision-makers seek acceptable outcomes considering all costs including those of information gathering, search and cognitive effort; bounded rationality exists when rational agents are constrained in their ability to formulate and solve complex problems (Simon, 1956).

In general terms, satisficing can be described as a type of decision-optimising process whereby individuals seek to maximise an objective function subject to constraints. The objective function can be defined in terms of utility which is maximised subject to constraints in the forms of cognitive and other costs. If we assume a decision-maker faces a number of choices represented by a consumption set  $\mathbb{R}_+^n$ , preferences with regard to the consumption set



can then be expressed by a utility function defined on the choice set. Assuming a general constraint  $\lambda$ , the optimising objective function can be expressed as;

$$\text{Max}_{x \in \mathbb{R}_+^n} = u(\sum_{i=1}^n xi) \quad \text{subject to; } \sum_{i=1}^n xi \leq \lambda \quad \text{Equation 22}$$

where  $\sum_{i=1}^n xi \leq \lambda$  represents the overall “resource” constraint condition of the decision-maker; constraints may be cognitive or due to a limit on the amount of effort the decision-maker is prepared to undertake and can be thought of in the same way as an income constraint in a normal consumption setting.

Maximising this function is consistent with bounded rationality. Satisficing behaviour does not require the objective function to be maximised but presumes that the decision-maker achieves at least an acceptable payoff from their choices. We can express this by assuming that individuals aspire to achieve a minimum payoff sufficiently *close to* the constrained optimum. Therefore, if the difference between the optimum payoff and aspiration level is denoted as  $\varepsilon = U^{max} - A$ , where  $A$  is the minimum required level of utility, satisficing can be defined as the choice set  $s$  which satisfies;

$$U(s) \geq U^{max} - \varepsilon$$

Satisficing heuristics and bounded rationality are examples of non-axiomatic decision-rules which diverge from the *as if* framework of the various iterations of expected utility theory. They describe an adaptive process whereby decision-makers pursue a method of information acquisition designed to match the complexity of a task. While inherently suboptimal and potentially prone to bias and error, heuristics have been found to perform well in relation to choices with binary attributes (Katsikopoulos, 2013) and have, in some cases, outperformed

formal regression models (Czerlinski, Gigerenzer, & Goldstein, 1999). By taking an algorithmic approach to decision-making, such rule-based models go some way to looking at process rather than simply output.

#### **4.2. Process Tracing**

Process tracing is a direct attempt to determine cause and effect in terms of decision-making. The methodology covers an array of techniques used to make judgmental determinations of the causes of particular behaviour based upon observations of how data and information are acquired prior to a decision being made. Whereas formal multivariate techniques, such as regression analysis, attempt to measure the importance of causal variables, deriving numerical values in the form of coefficients in the case of regression analysis, process tracing makes more general inferences about the variables which influence a particular choice (Bennett and Checkel, 2015). The approach therefore focusses upon the process which occurs between the introduction of information and the steps then taken before final decisions emerge.

Numerous methods have been used to determine how information is acquired by decision-makers, including verbal protocols and information boards. Verbal protocols are derived from explanations provided by decision-makers recorded as they perform a particular task; a technique popular in relation to problem-solving research (Lin & Yu, 2015). Information boards are designed to provide hidden information, requiring search effort from the decision-maker. Technology has naturally been applied increasingly to the task, with eye-tracing and mouse tracking providing more extensive information logging based upon the actions of the target agent.

Various process tracing methods naturally require differing levels of search effort on the part of the decision-maker; for example, physical information boards, which might present information in an envelope, require an overt act which typically takes longer to execute than mouse clicking to reveal the same information. Indeed, computerised process tracing (CPT) environments often improve the efficiency of information gathering relative to a manual-task environment, leading some to question whether the level of effort itself might affect the decision process and choices made (Russo, Johnson, and Stephens, 1989). Mouse-tracing techniques have, for example, been applied in a similar manner to information boards, with decision-makers required to move the mouse pointer to a particular part of a computer screen in order to reveal information (Norman & Schulte-Mecklenbeck, 2010). Eye-trackers have also been used, often in combination with mouse tracking, to elicit yet more precise information. For example, Franco-Watkins and Johnson (2011) employed eye-trackers to measure attention-processing; by allowing information to appear only when the decision-maker looked directly at a specific part of the computer screen and disappearing when the eye move away, the length of time information was accessed could be recorded precisely. In comparison, while a mouse pointer could determine whether the same information had been accessed, it would not be known at what stage attention to it ceased unless the mouse was simultaneously moved away. In an experiment, Franco-Watkins and Johnson found that eye-tracking data revealed a greater number of fixations of shorter duration in comparison with mouse-tracking data, the latter implying greater variability in attention duration.

While process tracing can provide evidence of causation, it does so within certain constraints. First, it should be possible to link behaviour to theories of cognition, for example, fixation and signal processing. Secondly, the approach is more case-specific than general. Process tracing views behaviour within the context provided by an environment where

decision-makers respond to cues. Should the environment or the nature of the cues change, behaviour and choices may also vary. This contrasts with many formal analytical models which test sampled data with the intention of generalising findings across states.

### **4.3. Feedback and Beliefs under Conditions of Ambiguity**

An important aspect of decision-making under conditions of uncertainty and ambiguity relates to how decision-makers form beliefs in the absence of objective probabilistic information and how those beliefs might be modified on the basis of new evidence. A widely applied framework for describing this process involves Bayesian inference. Thus, decision-makers are assumed to hold prior beliefs relating to the likelihood of particular outcomes. Probability can then be interpreted as the level of belief or confidence which the individual holds in relation to that uncertain event, a type of subjective uncertainty.

Individuals therefore form prior beliefs which are then subject to updating over time as new evidence pertinent to event likelihood becomes available. Assimilation of that new data creates *posterior* beliefs. Bayesian inference therefore contrasts with classical statistical inference as the latter assumes that probability beliefs derive from observed frequencies of random events occurring over the long-term and on the basis of repeated trials. Classical theory therefore defines uncertainty by reference to objective probability distributions conditional upon the existence of long-run frequency information whereas the Bayesian concept modifies uncertainty within a framework of evolving beliefs conditional upon new evidence.

We saw in earlier chapters that classic EU theory assumed the existence of homogenous agents forming rational expectations on the basis of full knowledge of all relevant information. This implies that rational decision-makers share the same probabilistic beliefs in relation to uncertain events. In contrast, Bayesian beliefs are heterogeneous rather than homogenous meaning that decision-makers can hold different prior beliefs as they may have differing levels of access to information or they may interpret common information differently. However, despite differences in prior beliefs, it is assumed that rational posterior beliefs undergo a process of convergence on the basis of new shared information. In general, we may then assume that an individual,  $i$ , applies a weighting function in relation to an event,  $\omega^e$ , which reflects their degree of confidence or strength of belief such that;  $0 \leq \omega_i^e \leq 1$ . When  $\omega_i^e = 0$ , the individual has no confidence that an event will occur while  $\omega_i^e = 1$  implies certainty.

Bayesian rules can be described easily if we assume that rational beliefs are founded upon base-rate probabilities modified by particular evidence relating to conditions or states. This is the equivalent of conditional probabilities in which the probability of event  $A$  depends upon the occurrence of another event,  $B$ , denoted as;

$$P(A|B) = P(A \cap B) / P(B) \text{ or } P(B)P(A|B) = P(A \cap B) \quad \text{Equation 23}$$

The probability of  $A$  occurring given that  $B$  has occurred therefore equals their joint probability relative to the probability of  $B$ . The probability of  $B$  is known as the base rate probability which then affects the likelihood of the outcome  $A$ . Since all events of  $A$  describe the complete partition sample space, we can define the following for the probability of  $B$ ;

$$P(B) = \sum_{a \in A} P(B \cap A) = \sum_{a \in A} P(B|A)P(A) \quad \text{Equation 24}$$

The Bayes' rule applied in Bayesian inference is then defined as;

$$P(A|B) = P(B|A)P(A) / \sum_{a \in A} P(B|A)P(A) \quad \text{Equation 25}$$

Models of Bayesian inference apply parameters representing weightings of beliefs.

Therefore, the term,  $\theta = P(A)$ , describes the decision-maker's probabilistic beliefs regarding the likelihood of event  $A$ . Assuming a random event with only two possible outcomes, the probability of one particular outcome can then be represented as;  $\theta \in [0, 1]$ . Beliefs about different values of  $\theta$  reflect rational expectations derived from a probability distribution resulting from observed data. This can be represented by the likelihood function,  $P(\theta | \lambda)$ ; note that  $P(\lambda | \theta)$  would give the probability of observing  $\lambda$  given a model parameter of  $\theta$ .  $P(\theta)$  is therefore the prior belief while  $P(\theta | \lambda)$  is the posterior belief. The term  $P(\lambda)$  can in theory be calculated from a probability distribution based on a summation of all values of  $\theta$  weighed according to the decision-makers beliefs with regard to each value of  $\theta$ .

Bayesian theory of subjective probabilities has been generalised in the Demster-Shafer theory (DST) of belief functions. Bayesian and DST theories both assume that decision-makers assign non-negative weights to uncertain and ambiguous events. Whereas Bayesian theory assumes that each individual event from the set of possible events is assigned a weight reflecting the decision-maker's belief with regard to the likelihood of each event, DST describes a "frame of discernment",  $\Omega$ , which includes all possible *combinations* of events. Subsets of the frame of discernment are called weighted "masses",  $\omega(x)$ . Masses satisfy the following properties;

$\omega(x) \in [0, 1]$  for all  $x \subseteq \Omega$ ;

$\sum \omega(x) = 1$  and  $\omega(\emptyset) = 0$

Masses can therefore be thought of as probability assignments; to this are added belief and plausibility functions which are defined as the upper and lower bounds of intervals. Beliefs in DST are derived from a decision-maker's interpretation of the source and represent the degree of support for a proposition. Therefore, the degree of belief for a set  $A$  is defined as the sum of all weights of a subset  $B$  which supports  $A$ . This can be shown as;

$$Bel(A) = \sum \omega(B) \text{ where } B \subseteq A$$

Plausibility is defined as the sum of beliefs which support  $A$  (*beliefs*  $\leq$  *plausibility*). Belief therefore defines the lower bound and plausibility the upper bound of a general belief interval. Therefore, if there is a belief of 0.5 and a plausibility of 0.8 for a particular proposition,  $A$ , evidence against  $A$ , and hence the belief in "not  $A$ ", is 0.2 ( $1 - 0.8$ ). The difference between the belief for  $A$  and not  $A$  (0.3) denotes the level of uncertainty based upon available evidence (Shafer, 1990).

DST implies that not every prospect can be completely encompassed by some measure of uncertainty as to its likely outcome since there may be no basis from which to determine that uncertainty. Instead, many decision tasks include elements of ignorance. Therefore, as opposed to decision-makers computing or assigning probabilities to the likelihood of an event, they assess the probability that available evidence supports the proposition of a particular outcome. DST can therefore be applied to situations where there is insufficient data to

estimate priors as required by Bayes' rule. The use of belief intervals in DST also makes it possible to assess how close the evidence comes to establishing a hypothesis.

Violations of axiomatic behaviour, such as the Ellsberg Paradox, suggest that beliefs cannot be fully encapsulated by single probability measure given the imprecision inherent in available information. Therefore, updating with new information in the presence of indeterminate uncertainty can lead to inconsistent choices. In terms of Bayesian theory, ambiguity might therefore not be accommodated fully by a single prior. DST provides a generalisation of Bayesian theory enabling ambiguity to be more flexibly interpreted within belief intervals. Further generalisations have been proposed based upon multiple priors and the concept of dynamically consistent beliefs (Epstein & Le Breton, 1993). Gilboa and Schmeidler (1989) applied a multiple-prior model with preferences updated on the basis of maximum-minimum expected utility (MEU). Thus, minimum expected utility can be defined as a function of levels of belief while maximum expected utility could be bounded by plausibility, the equivalent of DST "incalculable uncertainty". Hanany and Klibanoff (2007) explore the link between dynamic consistency and updated MEU preferences. Decision-makers are therefore deemed to update beliefs by adopting updating rules which depend upon prior choices or feasible sets defined by the problem. The rules applied are consistent with the application of Bayes' rule to the subset of measures represented by the decision-maker's preferences.



## CHAPTER 5

### BEHAVIOURAL BIAS AND ERROR

#### 5.1. Suboptimal Decision-Making and Bias

In previous chapters, we traced the development of axiomatic theories which define objectively rational behaviour. If those axioms are met, it is possible to conclude that the decision-maker is behaving *as if* they are wholly rational. Axiomatic models with free parameters, such as CPT, are capable of explaining a range of behaviours relating to decision-making under conditions of risk, uncertainty and ambiguity. We have extended the discussion to look at non-axiomatic models assuming trade-offs between cognitive effort and decision accuracy; such approaches therefore consider the costs associated with making decisions. Certain models within this category can be described by an optimising process whereby a utility-based objective function is maximised subject to perceived cost constraints. The assumption of bounded rationality is consistent with this; decision-makers are assumed to be wholly rational but operate within constraints which prevent the full evaluation of alternatives either due to cognitive limitations or incomplete data.

Other models do not necessarily assume pure utility maximisation as the objective but assume that decision-makers apply aspirations to outcomes. This is captured by satisficing behaviour under conditions of uncertainty or ambiguity. Algorithmic choice reduction models provide the framework from which this proposition can be developed. Thus, decision-makers may be assumed to pursue a systematic process of comparison based upon attributes; alternatively, they may apply a range of heuristics which transform and reduce the choice sets.

Typical criticisms of non-axiomatic models relate to the biases or errors which they may induce. For example, heuristics and other reduced-criteria decision models have an obvious potential to violate the axioms of *dominance* and *invariance*. Since all attributes of prospects are not considered, potentially superior choices may be overlooked, violating *dominance*. Similarly, the principle of *invariance* requires that the same information carry an equal weight with the decision-maker regardless of the order in which it is presented; reduced-criteria models may be unduly sensitive to the sequence ordering of information. In the case of the *representative* heuristic, numerous biases may result such as *recency bias* (overweighting recent events) or *availability bias* (overweighting the importance of information which may be more readily recalled). Decision models with stopping rules also promote *anchoring bias* as, once a satisfactory option is identified, additional cognitive processing terminates. Such biases can therefore induce choice error and suboptimal outcomes, one side of the trade-off between accuracy and effort.

Empirically, it is possible to compare the predictive performance of a reduced-set, or simple decision-rule model with that of more formal models. In doing so, it is useful to “deconstruct” sources of error, essentially distinguishing between bias and variance. Using standard methodology, we can express a functional relationship between a dependent variable  $Y$  and an independent variable  $X$  as;  $Y = f(X) + \varepsilon$  where  $\varepsilon$  is a normally distributed error term with mean = 0 (noise). Given a model  $\tilde{f}(X)$  of  $f(X)$ , the expected error for point  $x$  is then;

$$Err(x) = E[(Y - \tilde{f}(x))^2]$$

which can be decomposed as;

$$Err(x) = (E[\tilde{f}(x)] - f(x))^2 + (E[(\tilde{f}(x) - E[\tilde{f}(x)])^2]) + \sigma_\varepsilon^2 \quad \text{Equation 26}$$

Total error is therefore the sum of Bias<sup>2</sup>, variance and noise (an irreducible error which cannot be affected by changes in the model). Bias arises when the process involved in decision-making skews preferences due to a systematic over/under-weighting of particular criteria; variance relates to prediction error arising from the sensitivity of a model to different samples from the same population. Bias is independent of variance meaning that a process subject to bias may result in lower total errors than a model with high variance.

This proposition was illustrated by Wübben and Wangenheim (2008), comparing a Pareto/NBD model with a simple “hiatus” rule. Pareto/NBD (negative binomial distribution) models have been widely used in retail settings to forecast likely customer purchasing patterns over time (Schmittlein, Morrison, & Colombo, 1987; Fader, Hardie, & Lee, 2005). Following the procedure of Schmittlein *et al*, the model requires only two inputs relating to an existing customer (a customer who has made transactions in the past); how recently the last transaction occurred and how many transactions took place over a specified time interval. This can be summarised as;  $X = x, t_x, T$ , where  $x$  is the number of transactions which took place over period  $[0, T]$  and;  $t_x (0 < t_x < T)$  represents the time of the last transaction within the interval,  $[0, T]$ . From this,  $E[X(t)]$  denotes the expected number of transactions in period  $t$ ;  $P(X(t) = x)$  represents the probability of  $x$  number of transactions occurring in time period  $t$ . The expected number of transactions over the time interval  $[T, T + t]$  for an individual with a past transaction history of  $(X = x, t_x, T)$  is then given by;

$$E(Y(t) | X = x, t_x, T)$$

The model estimates three parameters; a transaction rate,  $\lambda$ , a drop-out rate,  $\mu$ , and an estimate of the “lifetime” of the customer (a point after which they become inactive),  $\pi$ .

Using data for apparel, airline and music transactions, Wübben and Wangenheim found predictive accuracy rates of 75% &, 74% and 77% respectively. This was compared with a simple “hiatus” rule in which customers who had made no transactions over a historic period, the length of which was suggested by managers, were deemed to be inactive. This simple model gave accuracies of; 83%, 77% and 77% respectively, outperforming the formal model on two out of the three data sets while matching it on the third. In essence, the Pareto/NBD approach, like many formal models, seeks to minimise bias whereas the hiatus rule is more likely to result in bias. However, the latter only estimates one parameter (the length of time before which a non-transacting customer is deemed inactive) and therefore incurs low variance. The conclusion to be drawn, therefore, is that apparently simplistic models have the potential to outperform formal models where data interactions are complex and there is a high degree of uncertainty or ambiguity (for further examples see; Wright & Stern, 2015).

## **5.2. Error versus Systematic Bias**

While algorithmic decision models, judgment and heuristics may promote choice bias, they nevertheless have the potential to outperform more complex models in some cases. However, just as formal model builders seek to reduce variance in order to improve predictive power, so rational decision-makers should seek to reduce bias where it can be identified. This proposition is perhaps most feasible if the source of bias can be appropriately described. It is therefore useful to distinguish between simple error, due perhaps to misinterpretation, misunderstanding or lack of information, and potential sources of persistent psychological bias. For example, descriptive explanations for bias inducing error include the effect of dissonance, or noise, arising from both internal (cognitive) and external decision making environments (cues, interpretation and feedback). Thus, uncertainty, lack of clarity, inaccurate

recall and mismatching have all been hypothesised as contributing factors (Griffiths, Chater, Kemp, Perfors & Tenenbaum, 2010; Lee, Amir & Ariely, 2009). Decision-makers may then perceive a higher associated risk of error encouraging the selection of options which are likely to minimise any associated costs (Haselton & Galperin, 2012). Empirically, noise has been shown to affect not only perception but also, within certain contexts, cognitive function and hence behaviour (Lupien, McEwen, Gunnar & Heim, 2009). Systematic behavioural biases have also been identified in relation to two important areas of risky decision-making; probabilistic assessment and intertemporal choice. The former is often attributed to the *representativeness* heuristic and *confirmation* bias and arises when decision-makers either understate or ignore information which should inform objective probability assessments.

### 5.2.1 Base-Rate Error

The majority of uncertain events carry a base-rate probability and an event-specific probability. For example, we might assume a base-rate probability with regard to the likelihood of it raining today derived from knowledge of the general frequency and pattern of historic rainfall. We may then modify this assessment depending upon the presence of dark clouds, the event-specific information. The *representativeness* heuristic tends to overweight the event-specific information as it compares the current event (dark clouds) with other similar events, deriving a probability from those events alone ignoring the base-rate. Bayesian probability theory describes an unbiased approach to assessing likelihood as “degrees of belief” about certain outcomes which are expressed as probabilities based upon all available, relevant information (Geisser 2017). We have previously seen that the general expression for inferred probability given a base-rate and event-specific information is then;

$$P(A_1 | B) = [P(A_1) P(B | A_1)] / [P(A_1) P(B | A_1) + P(A_2) P(B | A_2)] \quad \text{Equation 27}$$

where  $P(A_1 | B)$  denotes the probability of  $A_1$  given  $B$ ,  $P(A_2) = 1 - P(A_1)$

This can be illustrated with regard to rainfall likelihood using the following assumptions:

$$P(A_1) = 55/365 = .15 \quad (\text{it rains 55 days per year on average})$$

$$P(A_2) = 310/365 = .85 \quad (\text{the average number of dry days per year})$$

$$P(B | A_1) = .7 \quad (\text{it rains 70\% of the time when there is a dark cloud})$$

$$P(B | A_2) = .3 \quad (\text{it does not rains 30\% of the time when there is a dark cloud})$$

therefore;

$$P(A_1) P(B | A_1) = .15 \times .7 = .11$$

$$P(A_1) P(B | A_1) + P(A_2) P(B | A_2) = [.15 \times .7] + [.85 \times .3] = .36$$

$$P(A_1 | B) = .11 / .36 = .29$$

From this, we see that ignoring the base-rate probability would give an erroneous result as the decision-maker would assume  $P(B | A_1)$ , significantly overstating the actual likelihood of rainfall. Similarly, the base-rate alone would understate the true likelihood. Therefore, decision-making approaches which ignore relevant probabilistic information are likely to create bias.

### 5.2.2 Intertemporal Discounting Bias

The second significant bias regarding intertemporal choice also appears to suggest cognitive error leading to apparent inconsistencies in relation to discount rates. Early models of preferences over time simply adapted basic EU models to reflect the existence of multiple consumption streams over set time intervals (Samuelson, 1937). Therefore, given a level of endowment, the rational decision-maker contemplates various consumptions streams over their lifetime ( $c_0, \dots, c_T$ ). Applying the standard axiomatic assumptions of completeness, transitivity and continuity, a utility function then exists in relation to the time-dependent consumption flows;  $U^A(c_0, \dots, c_T)$ . Preferences relating to consumption in different time periods are assumed to be captured by a single discount rate. This results in an intertemporal utility function of the form;

$$U^A(c_0, \dots, c_T) = \sum_{t=0}^T D(t)u(c_t) \quad \text{where } D(t) = (1/1+p)^t \quad \text{Equation 28}$$

Here, the component,  $u(c_t)$  measures the individual's utility in period  $t$  while  $D(t)$  reflects the subjective discounting function applied to that future consumption, the actual discount rate being represented by  $p$ . Consequently, the Rational Man is deemed to plan his consumption efficiently over time based upon expected income, allocating in such a way that the present value of all utilities summed over each period of consumption is maximised (Koopmans, 1960). It is then clear that, given an initial level of utility derived from a particular outcome, there must be some future level of consumption such that the Rational Man would be indifferent between the two. Such a proposition made it relatively easy to determine individual discount rates from the ratio of the indifference values.

There are two useful distinctions to be drawn with regard to intertemporal choices within the classical model of discounted utility. The first relates to the process of discounting itself,

which is considered to be a rational function of uncertainty, prospective changes in tastes or, in monetary terms, expected declines in the real value of money and purchasing power over time. The second distinction concerns simple time preference, which promotes utility from consumption now over delayed utility from consumption sometime in the future. The two propositions, however, offer a degree of potential conflict which the decision-maker is assumed to resolve efficiently: Current consumption is subject to the law of diminishing marginal utility; as more of an item is consumed, so its incremental contribution to total utility declines. Conversely, deferring consumption to a later date by saving current income (or declining to borrow in order to finance future consumption, in an alternative case) incurs a time preference cost (and a future income cost in the case of borrowing). The rational decision-maker must therefore determine optimal consumption paths which resolve any such conflicts, resulting in the overall maximisation of utility; within classical theory, they are naturally assumed capable of doing so.

While the proposition of discounted utility is essentially derived from pure economic doctrine, psychological effects were also recognised as motivating factors behind intertemporal choice behaviour. Thus, the early work of von Bohn-Bawerk (1890), later formalised by Fisher (1930), suggested that humans suffered from a systematic tendency to underestimate future consumption needs and hence had a natural bias towards current consumption. This built-in bias was assumed to manifest through the discounting process which then affects the perceived “value” of the future. Similarly, Rae & Mixter (1905) proposed that intertemporal choice was governed by a number of psychological factors which either promoted or constrained a desire to accumulate. They proposed both a bequest motive, the desire to defer or confer consumption capacity for the benefit of the future, and a tendency towards self-restraint. Essentially, however, all psychological and economic factors deemed to



affect intertemporal preference were assumed to be captured by the discount rate applied to each of the future utility streams.

In effect, the seemingly straightforward model of discounted utility imposes some restrictive assumptions in order to ensure that classical axioms are maintained. Somewhat analogous to the *independence* axiom of expected utility theory, discounted utility theory assumes consumption independence whereby an individual's utility derived from consumption in any period is deemed to be independent of consumption in any other period. By extension, the marginal rate of substitution of consumption between two periods is also independent of consumption in all other periods. Thus, while the *independence* axiom of expected utility theory assumes that preferences with regard to uncertain prospects are unaffected by consequential outcomes which prospects share, so consumption independence assumes that preferences with regard to intertemporal consumption allocations are not affected by previous, identical consumption patterns.

The discounted utility model further assumes that the discount function is identical across all forms of consumption regardless of source. Similarly, the assumption of constant, continuous discounting implies that time variability is monotonically compounded (the discount rate does not change). Consequently, varying two outcomes in time by the same duration (either bringing them forward to an earlier common date or delaying them to a later common date) should have no effect on preferences. Furthermore, the application of constant discounting implies that intertemporal choices must be ranked consistently over time such that the discounted marginal utility derived from future consumption is less than or equal to the discounted utility derived from consumption one period earlier (Albrecht & Weber, 1995). Such conditions must apply across all discrete decision timeframes. The essential aspect for

the purposes of the current discussion relates to the role of uncertainty of the timing of outcomes. Under discounted utility, this uncertainty is assumed to be captured fully by the discount rate while equal, prospective changes in the timing of outcomes do not affect intertemporal choice. In this manner, choices through time are 'priced' efficiently as all factors which can affect utility are deemed to have been correctly discounted based upon all available current information.

In practice, the discounted utility model finds little empirical support. As opposed to constant compounded discount rates, observed discount rates have been found to decline over time, consistent with a process of hyperbolic discounting (Laibson, 1997; Phelps & Pollak, 1968). A hyperbolic discounting model can be described by the general form, summing cash flows and utilities over time as shown in Equation 28. However, the discount rate is no longer a constant but declines with time delay ( $t$ ), giving;  $D(t) = 1/(1 + \alpha t)$ . Hyperbolic discounting implies that near-term prospects are discounted more heavily than later prospects, the discount rate declining as a function of the delay. This was illustrated by an experiment conducted by Thaler (1981) in which participants were asked to specify the amount of money they would require; one-month, one-year and ten years in the future, in order for them to be indifferent to receiving \$15 now. The median replies were \$20; \$50 and \$100, implying annualised discount rates of 345%, 120% and 19% respectively. In addition, preferences between two delayed outcomes have also been found to reverse when presented within a more immediate timeframe. Therefore, decision-makers may express a preference for £110 in 31 days versus £100 in 30 days while simultaneously preferring £100 now to £110 tomorrow (Green, Fristoe & Myerson, 1994).

A similar temporal discounting bias has also been found depending on whether the timing of an outcome is perceived as an acceleration or delay in its receipt versus prior expectations. Therefore, in empirical tests, participants who did not expect to receive goods for several months were found to be willing to pay extra to receive the goods immediately. Conversely, when goods intended for immediate delivery were delayed for an equivalent period, participants demanded a greater sum as compensation for the time delay (Loewenstein, 1988). Empirical evidence also suggests that individuals apply different discount rates to intertemporal gains versus losses; gains being discounted more than losses of equal nominal magnitude and delay. Equally, relatively small amounts appear to be discounted more than larger amounts (Estle, Green, Myerson & Holt, 2006; Loewenstein & Prelec, 1992).

The cognitive biases with regard to probability and temporal discounting have been found to replicate with regard to subadditivity bias. Therefore, a number of studies have shown that perceptions of both value and outcome probability differ according to whether a proposition is presented in its entirety or a number of constituent parts (Benhabib, Bisin & Schotter, 2010; Fox, 1999; Fox, Rogers & Tversky, 1996; Neil Bearden, Wallsten & Fox, 2007; Read, 2001; Read & Roelofsma, 2003; Wong, 2008). In that regard, a substantial body of research has been conducted into perceived values of non-traded assets (Brookshire, Thayer, Schulze & d'Arge, 1982; Kahneman & Knetsch, 1992). These are typically termed “public goods” and might include any source of general environmental utility or entity from which utility can be derived and for which there is no objective external market price. Obvious examples might include assessments of the value of woodland or natural lakes for the purposes of cost-benefit analysis. The issue then might be to determine the potential loss in “social value” from any economic or other development which may adversely impact the integrity or enjoyment of that amenity. The same principle can be extended to environmental issues, such as the cost of

oil spills or other natural or man-made disasters. Discounting applies in such cases as, consistent with a number of financial theories relating to asset valuation, the current value assigned to public goods should, at least in part, reflect the perceived net present value (NPV) of future discounted utilities from enjoying that amenity unencumbered (Damodaran, 2012; Geltner & Mei, 1995; Lucas, 1978; Myers, 1984).

Evidence of the impact of subadditivity of probability on valuation was provided by an experiment in which participants were found to prefer a prize conditional upon the outcomes of two independent events, each with a probability of  $\frac{1}{6}$ , to an alternative proposition of an identical prize contingent on a single event, the probability of which was  $\frac{1}{3}$  (Starmer & Sugden, 1993). Read (2001), in turn, conducted experiments designed to test subadditivity versus hyperbolic discounting in intertemporal choices. Participants were offered smaller-sooner versus larger-later monetary amounts. Once a choice had been made, the other value was adjusted until a point of indifference was established. Participants were presented with choices including amounts applicable over an extended timespan as well as interim amounts at stages throughout the entire timespan. The derivation of indifference points in each case enabled applicable discount rates to be calculated for each interval. Read found clear evidence of subadditive temporal choice but no evidence of declining impatience, as implied by hyperbolic discounting. The clear implication is that intertemporal choice is subject to bias but the process appears to be more complex than simplistic discounting models assume. Decision-makers thus appear to be influenced by patterns of delay as well as the duration and frequency of intervals.

As an extension to the previous analysis, Scholten & Read (2006) developed a discounting-by-intervals (DBI) model to explain observed patterns of intertemporal

discounting. The model hypothesised that discount rates are a function of how far outcomes are from the present and also the interval between future outcomes. Therefore, the outcome associated with a smaller-sooner outcome occurring at time  $t$  ( $t_s$ ) is discounted on the basis of its own interval from the present ( $0 \rightarrow t_s$ ) whilst the larger-later amount is discounted over the interval from ( $t_s \rightarrow t_l$ ). In essence, therefore, the DBI model is structured in a manner compatible with the widely observed effects of subinterval discounting when outcomes are framed on the basis of defined, intervening stages. Based upon further empirical experiments, the DBI model again found that discount rates tend to be higher the closer an outcome is to the present; decision-makers therefore appear to attach greater importance to the prospect of receiving near-term gains. In addition, there appears to be an *interval effect* such that discount rates tend to be relatively higher when outcomes are close to one another, but sufficiently removed from the present.

### **5.2.3 Sunk Cost Fallacy**

The sunk cost effect describes an adaptive bias in decision-making stemming from previous actions, choices, commitments, investment, cues or behaviour (Arkes & Blumer, 1985, Sleesman, Conlon, McNamara & Miles, 2012). It is a bias which promotes a continuation of past behaviours and reinforcements of prior decisions to the exclusion of viable alternatives, directly violating axiomatic utility maximisation. Therefore, decisions based upon sunk costs rather than expected outcomes are necessarily objectively irrational. The fallacy implies an endowment effect, optimistic probability bias and loss aversion in relation to costs already committed.

One of the most familiar characterizations of the sunk cost effect comes in the form of physical capital investment decisions. Companies embarking upon major capital expenditures usually incur significant costs in terms of planning, consultation and time costs over and above simple monetary outlays (Somers & Nelson, 2001). These can be considered broad ‘enterprise’ costs through which many layers within the decision-making structure of a company can become ‘invested’ in the outcome of certain projects. Most capital projects inevitably involve risks and uncertainty due to gestation periods, lead times, unexpected cost overruns and potential market shifts; certain risks may be quantifiable or identifiable at inception, uncertainties, such as a sudden shift in market environment, may be less so (Dixit, 1993; Kardes, Ozturk, Cavusgil & Cavusgil, 2013). One manifestation of sunk cost bias is that decision-makers appear to become less willing to explore other uncertain options once sufficient resources have already been committed to a particular course of action (Keil, Tan, Wei, Saarinen, Tuunainen & Wassenaar, 2000; Kelly & Milkman, 2011; Staw, 1981). While there is evidence that sunk cost behaviour with respect to investments of time alone may differ from that involving tangible monetary outlays (Soman, 2001), once such financial commitments are made, the sunk cost effect appears to become robust (Garland, 1990). The sunk cost effect can be observed more generally with, for example, a reluctance on the part of consumers to cancel a pre-paid holiday despite the prospect of adverse weather or remaining with an energy or telephone service supplier despite objective evidence that a cheaper alternative is available.

Empirical evidence reveals that the sunk cost effect is sufficiently strong to elicit escalation of commitment even when the outlook for prospects has deteriorated. Thus Arkes and Blumer (1985) found that 85% of participants in an experiment were prepared to commit additional funds to a project, to which a commitment had already been made, even though a

competitor had just launched a superior offering. When no prior investment existed, only 17% of participants were prepared to commit funds to the same project. Others have found similar persistence to projects despite evidence that they are failing, suggesting that a sunk cost effect can even overcome negative feedback (Arkes & Hutzler, 2000; Bragger, Hantula, Bragger, Kirnan & Kutcher, 2003). However, while there appear to be strong psychological factors promoting escalations of commitment, switching and de-escalation do occur under certain conditions. This is particularly the case once the prospect of failure increases sufficiently to raise the likelihood of the ultimate outcome underperforming expectations by a substantial margin; the more so if an attractive alternative becomes apparent (Garland, Sandefur & Rogers, 1990; Lee, Keil & Kasi, 2012). The presence of an attractive, viable alternative therefore appears to provide an important qualification to the sunk cost effect (Stray, Moe & Dyba, 2012). In extreme cases, strategic change is forced, such as when the likelihood of bankruptcy is perceived to be high if the current strategy is maintained. Under such circumstances, abandonment, even at a debilitating cost, has been found to occur (Ross & Staw, 1993).

The sunk cost effect appears to elicit positive bias in relation to probabilistic assessments of outcomes once commitments have been made. Decision-makers thus appear to adopt a skewed optimism with regard to favoured projects. Arkes and Hutzler (2000) examined whether such a bias causes a sunk cost effect or is a consequence of it. Two experiments were conducted. In the first, some participants were presented with a scenario involving a sunk cost while the remaining participants read the same scenario but without a sunk cost element. Half of the members in each group were also informed of the objective probabilities associated with the project's outcome, thus avoiding any impact from psychological "probability inflation". Results indicated a sunk cost bias, implying that inflated probabilities are not a

necessary condition to create the effect (as probabilities were known). In the second experiment, participants provided their own probability estimates before and after a decision to invest. Participants making the decision to invest subsequently assigned significantly higher probabilities to the expected success of the project in comparison with those who declined to invest, thus implying that the inflation bias is a consequence of, or partial justification for, the decision to invest.

### 5.3 Behavioural Biases in Financial Markets

Financial markets have proved to be a rich environment within which to examine behaviour. Rational behaviour can be described by the axioms of utility maximisation encapsulated by CPT, implying that investors discount information objectively in accordance with their individual preferences and beliefs devoid of cognitive bias. Markets are thus assumed to be efficient discounters of information; differences in individual preferences being resolved through the distillation of equilibrium prices (price levels at which there is no incentive for any rational investor to buy or sell in the expectation of a risk-free gain). The result is that the prices of financial assets are theoretically “efficient” meaning that they reflect all publicly available information subject to any limitations placed upon arbitrage<sup>5</sup>. Therefore, the early Capital Asset Pricing Model (CAPM) of Sharpe (1964) derived a theoretical equilibrium price for risky assets based upon expected returns consistent with rational expectations with regard to risk. The model takes the following form;

$$E(r_i) = R_f + \beta_i (ER_m - R_f) \quad \text{Equation 29}$$

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<sup>5</sup> Arbitrage, in this context, refers to the ability of investors to make risk-free gains due to the fundamental mispricing of assets. They are assumed to either buy or sell securities until the inefficiency in pricing is fully removed. If there are limits on arbitrage, either due to its associated costs or imperfect information, mispricing can persist.



The expected return on asset  $i$  is therefore a function of the risk-free rate ( $R_f$ ), the asset's volatility relative to that of the market or benchmark ( $\beta_i$ ) and the expected excess return of the market or benchmark ( $ER_m$ ) over and above the risk-free rate (a market risk premium). Since investors expect to be compensated for the time value of money, they demand a return over and above that which is available risk-free (typically the return on secured savings or government-backed short-term paper). They also expect to be compensated for the amount of risk they assume. This is reflected in the beta term ( $\beta_i$ ) which is calculated as the covariance of the asset return with that of the market or benchmark, divided by the variance of the market or benchmark return. A beta  $> 1$  therefore denotes an asset with greater price volatility than that of the market or benchmark. In an efficient market, and consistent with the principle of stochastic dominance, the higher the beta, the greater the expected return in order to compensate for the higher risk.

Fama and French (1996) developed the basic CAPM in the form of a *three factor model*, adding explicit variables of *value* and *size* to the market risk factor of CAPM. The logic of the model derives from evidence that historical equity price data exhibits a long-term positive skew with regard to *value* and *size* factors. Thus, *value* (defined as a high *book value to price* ratio) and *smaller* stocks appear to carry positive return premia over the long-term which cannot be explained solely by CAPM beta. Therefore, the excess return of an asset can be expressed as;

$$E(r_i) = R_f + \beta_i (ER_m - R_f) + b_s (SMB) + b_h (HML) \quad \text{Equation 30}$$

where; *SMB* refers to Small (market capitalisation) Minus Big and *HML* defines High (book-to-market) Minus Low. The sensitivities of the asset to the size and value factors captured by  $b_s$  and  $b_h$ .

The CAPM and three factor models assume that significant information regarding a security's risk can be subsumed into the market-related beta measure, the covariance of the security with the general market or benchmark. This represents a major simplification of Markowitz (1954) whose derivation of efficient portfolios and quantification of portfolio risk require resolution of the full covariance matrix across all available stocks. While different in terms of computational effort, the basic assumption that covariance adequately captures risk remains.

A more general model, and an alternative to CAPM, is found in Arbitrage Pricing Theory (Ross, 1978). The APT model generalises expected return and risk measures across common risk factors. The model takes the form;

$$E(r_i) = R_f + b_{i1} RP_1 + b_{i2} RP_2 + \dots + b_{in} RP_n \quad \text{Equation 31}$$

where *RP* denotes the risk premium associated with a specific factor while  $b_i$  measure the sensitivity of the asset to each risk factor or characteristic.

It will be noted that APT does not rely upon any explicit market risk premium; instead an asset's price derives from its expected return determined in accordance with its sensitivity to specific factors with their own expected payoffs. The arbitrage element of the theory assumes that, should an asset be fundamentally mispriced, arbitrageurs will enter the market to either buy or sell the security until the equilibrium price is restored.

The models described here derive equilibrium prices for assets based upon risk characteristics. They assume that market efficiency prevails as rational investors will act quickly to eliminate any inefficient pricing, realising a risk-free return in the process. New information is incorporated into prices through risk premia coefficients for each security. This is based upon the assumption of wholly rational expectations consistent with the evolution of Bayesian probabilistic beliefs incorporating all relevant information. The challenge for models of financial market efficiency stems in part from evidence that stock price volatility is consistently far too high to be explained by information and news flow (Shiller, 1981) leading to the conclusion that, as opposed to being efficient discounters of information, investors systematically over-or underreact to news in part due to confirmation bias (De Bont & Thaler, 1985; 1987).

Numerous heuristics have been linked to biases in financial market behaviour. Thus, the *representativeness* heuristic has been associated with skewed expectations as investors have been found to associate a *good company*, assessed in relation to *reputation* and past performance, with a *good investment* (Shefrin, 2000; Solt & Statman, 1989). In the same way, certain investors are predisposed to buy past winners on the basis that historic success is a good indicator of future success (Bailey, Nofsinger & O'Neill, 2003). Such behaviour appears to explain why some investors have a tendency to chase strongly appreciating stocks whilst ignoring underperformers (DeBondt & Thaler, 1985). Evidence suggests that such behaviour is influenced both by long- and short-term performance (Dhar & Kumar, 2001). Similarly, anchoring and recency bias appear prevalent as investors tend to focus upon limited characteristics of prospects while placing greater emphasis upon recent information. These

biases are particularly apparent with less sophisticated investors who often fail to undertake adequate research either due to its lack of availability or insufficient skill (Tan & Tan, 2012).

An optimistic bias has also been identified in forecasts of financial analysts and investors when predicting future returns and outcomes (Barberis, Huang & Santos, 1999; Easterwood & Nutt, 1999; Heaton, 2002; Hackbarth, 2008; Kacperczyk & Kominek, 2002; Malmendier, Tate & Yan, 2011), although the level of optimism can decline when feedback is anticipated (Carroll, Sweeny & Shepperd, 2006; Moore & Healy, 2008). Thus, expectations have been found to decline prior to imminent, relevant announcements (McGraw, Mellers & Ritov, 2004; Narciss, Koerndle & Dresel, 2011; Shepperd, Ouellette & Fernandez, 1996; Sweeny & Shepperd, 2010; Van Dijk, Zeelenberg & Van der Pligt, 2003). In a further study, the framing of situations and risk attitudes have been found to influence levels of optimism (Balasuriya, Muradoglu & Ayton, 2013). The reverse effect has been identified by studies in which individuals have been found to overestimate the reliability of their knowledge and predictive powers resulting in the assumption of greater levels of risk (Heath & Tversky, 1991; Keppe & Weber, 1995; Krueger & Dickson, 1993; Weinstein & Klein, 1996). A number of studies have also shown that errors in risk perception can lead to the assumption of higher than expected levels of risk (Kahneman & Lovallo, 1993; MacCrimmon & Wehrung, 1990; Roszkowski & Davey, 2010; Sitkin & Weingart, 1995; Stone & Grønhaug, 1993).

Overconfidence, often stemming from recent good performance, is a further behavioural bias which, in some cases, reinforces excessive optimism. Thus, investment managers have been found to overestimate their own skill levels, a trait which has been linked with higher-than-average trading volumes (Odean, 1998). The assumption of above average skill appears in some cases to lead to a miscalculation of risk as decision-makers assume that their superior

skill mitigates any informational disadvantage which they may have, making them more prepared to accept higher risk in the expectation of greater payoff (Shefrin, 2000).

Familiarity bias has been found to influence the stock selections of certain investors, also contributing to a *home bias* (preferring to invest in domestic stocks). Familiarity bias appears to influence risk perception to the extent that some investors consider companies which are household names to be less risky than less familiar alternatives (Hiraki, Ito & Kuroki, 2003; Huberman, 2001; Seasholes & Zhu, 2010; Wang, Keller & Siegrist, 2011). The home bias effect arises as certain investors have a tendency to favour their domestic market disproportionately, in some cases missing out on potential diversification opportunities offered by international markets (French & Poterba, 1991; Graham, Harvey & Huang, 2009; Strong & Xu, 2003). Further, there is evidence that investors even favour local companies within the context of a domestic market (Coval & Moskowitz, 1999), although such effects appear to depend upon the level of investor sophistication (Karlsson & Nordén, 2007; Kimball & Shumway, 2006). The home bias may, of course, be prompted by perceived differences in levels of knowledge and information although opportunities for information gathering may not be adequately exploited (Van Nieuwerburgh & Veldkamp, 2009). In that regard, language and culture bias have been found to play a role, such that investors favour companies (and markets) which routinely report in their native language (Grinblatt & Keloharju, 2000). Consistent with this, in certain markets, foreign investors also appear to skew investments in terms of broad characteristics, including industry and sector allocations, as opposed to widely diversified market exposures. For example, non-domestic investors in Japan have been found to favour manufacturing industries, large companies and those perceived to have adopted sound accounting practices (Kang, 1997). Perhaps confirming evidence of the impact of information cost, accounting preferences and familiarity bias,

domestic US investors have also been found, in aggregate, to concentrate overseas investments more in countries whose domestic securities are dual listed in the US, regardless of relative transaction costs and historic risk-adjusted returns (Ahearne, Grier & Warnock, 2004). This appears to confirm that information costs are key determinants of collective investor preferences.

A number of studies have highlighted status quo, or sunk cost bias. Thus, in a laboratory experiment, participants were found to favour holding investments which they had notionally inherited, rather than switching to alternatives (Samuelson & Zeckhauser, 1988). The tendency to persist with existing holdings was further confirmed via a simulated stock market experiment in which subjects were found to ignore information which could have enhanced performance, preferring instead to stay with existing investment holdings (Brown & Kagel, 2009). The effect proved to be robust regardless of a stock's performance, even when low transactions costs applied. Studies of actual portfolio data have found that holdings of retail investors in brokerage accounts change very little over time (Ameriks & Zeldes, 2004) while similar results have been found for the pension accounts of US investors (Agnew, Balduzzi & Sunden, 2003). The status quo bias has also been observed in purchase decisions whereby retail investors reveal a preference for adding to existing holdings rather than diversifying or switching elsewhere (Barber, Odean & Zhu, 2009a). The effect was further confirmed by a study examining the impact of the number of alternatives on status quo bias, finding that the greater the number of feasible alternatives, the stronger the likely status quo bias (Kempf & Ruenzi, 2006). Possible explanations for this bias could lie in a recognised lack of sophistication on the part of such investors leading to a belief that they lack the knowledge and conviction to identify potential alternatives. Similarly, individual investors may feel that

they lack the time or resources to acquire the necessary information to undertake such appraisals.

### **5.3.1. The Impact of Historical Price Patterns on Expectations**

Numerous studies have sought to detect and explain patterns within markets. Thus, cycles of various forms and durations have been identified (Avouyi-Dovi & Matheron, 2005; Bolten & Weigand, 1998; Conover, Jensen, Johnson & Mercer, 2008; Granger & Morgenstern, 1963; Schwert, 1989). To the extent that investors impute signals from prior price patterns and behaviours, various biases may apply along with the potential for misinterpreting false patterns within noisy data. The impact of past trends on future expectations amongst non-expert investors was tested directly by De Bondt (1993). Using a technical analysis game, participants were presented with 6 graphs each comprising 48 monthly price series. While anonymous to the participants, the data related to closing prices of the S&P 500 Index over various periods; three representing bull markets and three bear markets. The graphs were presented in random order but with values changed so that the source could not be identified. In addition, no actual dates were used although the graphical data contained monthly markers. The participants were then asked to predict future prices 7 and 13 months into the future. In addition, they were asked to provide a range around their estimates, in the form of a high and a low point, such that there would be a 1 in 10 chance that the actual price would be higher and a 1 in 10 chance that it would be lower than their projected ranges. Results indicated that, on average, participants were more optimistic in bull markets as opposed to bear markets. However, while predicting a continuation of general trend in rising markets, forecasts tended to be somewhat muted in comparison with past, actual trends. In bull markets, 50.6% of participants were found to be strong trend followers,

posting clearly higher expected future price estimates, while only 11.1% were deemed contrarian (expecting future declines in prices). In bear markets, participants were roughly equally divided in terms of future expectations in terms of gains and losses. In all cases, participants tended to hedge their forecasts by skewing their probability distribution data (as defined by their confidence ranges) in the opposite direction to that of the forecast. Thus, for strongly rising projected trends, confidence ranges would exhibit a negative skew around the forecast point, and vice versa for negative projections.

A number of studies have addressed the issue of expectations based upon patterns or biases within broader categories of historical data. Thus, investors have been found to interpret strong historical earnings and sales momentum as evidence that a stock is currently cheap (Lakonishok, Shleifer & Vishny, 1994). In the event that the stock price appreciates subsequently, investors then have a tendency to assume confirmation, thereby increasing optimism further (Einhorn & Hogarth, 1978; Koehler, 1991; Nickerson, 1998). A virtuous loop can then be created in the event of continued positive earnings momentum with investors more likely to anticipate future positive earnings surprises (Carhart, 1997; Hendricks, Patel & Zeckhauser, 1993). Such positive feedback has been found to inform buy decisions significantly more than negative feedback (Shafir, 1993).



**CHAPTER 6**  
**EMPIRICAL TEST OF UTILITY MAXIMISING BEHAVIOUR**  
**WITHIN COMPETITIVE AUCTIONS**

**6.1. Introduction**

Auction markets represent potentially rich environments within which to analyse behaviour. Classical auction theory rests on an assumption of *Revenue Equivalence* whereby rational sellers should expect the same average profits from all standard auctions, implying that buyers should also be indifferent between auctions of equivalent items (Vickery, 1962). Efficient auctions then exist when final sale prices converge with objective values as the number of bidders increases. This is the case even if individuals only have partial information with regard to objective values as the actions of rational bidders are assumed to aggregate all available relevant information in the final price (Milgrom, 1979). Rational bidders are therefore assumed to act in order to maximise their own consumer surplus, defined as the difference between their private market value ( $v$ ) for an item and the price ( $p$ ) which they have to pay to acquire it.

Early theory often described auction environments comprising a fixed number of risk-neutral bidders each acting on the basis of independent information. Whereas Myerson (1981) showed that optimal auctions which maximise the seller's revenues could occur even in the absence of symmetry of information between buyers and sellers, Maskin and Riley (1984) found that the specific format of an auction could affect bidder behaviour, in some cases determining outcomes. As a result, first-price sealed bid auctions were found to be generally more profitable to a seller than standard auctions. Milgrom and Weber (1982) replaced the

assumption of independent information with the concept of “affiliated” information, which exists when one bidder has more optimistic information about an item’s value. They suggest that such situations increase the average profitability of standard auctions as affiliated information tends to increase the optimism, and hence willingness to pay, of other bidders.

Prior to the Internet age, auctions tended to be considered as isolated, standalone events. The advent of online auctions, through sites such as eBay, has dramatically changed this landscape to the extent that multiple, close-to-identical items are regularly offered. Thus, online marketplaces are characterised by substantial competition between sellers, buyers and close-to-identical items. In theory, such high levels of competition between buyers and sellers and low cost of access to relevant information should lead to efficient outcomes as implied by revenue equivalence. Selling prices for identical items should therefore converge to “fair value” since rational bidders would compete for items currently priced below that equilibrium level.

However, as opposed to being indifferent between auctions offering apparently identical items, numerous factors have been found to affect bidder preferences and resulting behaviour. Thus seller reputation has been identified as a significant factor affecting potential buyer interest (Bajari & Hortacsu, 2004). Similarly, auction duration (Haruvy, Popkowski Leszcyc, 2008), the presence of reserve prices (Katkar & Reiley, 2006), lot sizes and bid increments (Rothkopf & Harstad, 1994; Bapna, Chang & Gupta, 2009) have all been found to affect the relative attractiveness of particular auctions with bidders typically favouring more liquid auctions with smaller fixed bidding increments. The presence of reserve prices has been found to act as a relative deterrent as highest bidders cannot be sure of securing an item in the event that the (undeclared) reserve price is not met.

The particular structure of an auction can also affect its relative appeal. Thus, the presence of a “buy-it-now” (BIN) option, whereby buyers can purchase the item at a declared price, ending the particular auction, has been found to be significant. BIN allows bidders to adopt an exit strategy based upon the posted BIN price rather than pursue the potentially riskier strategy of bidding to the conclusion of the competitive auction. Such auctions have been found to improve seller outcomes when sellers already have high reputation scores (Anderson, Friedman, Milam & Singh, 2008; Hardesty & Suter 2013; Standifird, Roelofs, & Durham, 2005; Wang, Montgomery & Srinivasan, 2008; Yoo, Ho, & Tam, 2006). BIN options therefore appear to create differentiation between close-to-identical products which then improves outcomes for certain categories of seller (Ackerberg, Hirano & Shahriar, 2006). Auctions with this feature have also been analysed in relation to consumer traits with some evidence of impulse buying and risk aversion (Angst, Agarwal & Kuruzovich, 2008). Other behavioural biases, such as herding, have also been identified (Ariely & Simonson, 2003; Bockstedt, Goh & Ng 2013); in aggregate, bidders appear to be attracted to auctions which are more active in terms of bidding activity, even though this would indicate higher potential competition (Dholakia, Basuroy & Soltysinski, 2002; Simonsohn, & Ariely, 2008). Similar evidence has been found for quasi-endowment and opponent effects leading to over-bidding (Heyman, Orhun & Ariely, 2004).

In terms of the efficiency of outcomes, Sun (2005) found evidence of substantial price dispersion across 3,164 sequential eBay auctions of close-to-identical items, violating the assumption of revenue equivalence. The degree of price dispersion is somewhat surprising given the relatively low cost of information acquisition within online auctions. Bidders therefore appear to place substantial weight on non-item related characteristics which differentiate between auctions and adopt very different strategies as evidenced by level of

search effort. In the case of concurrent auctions, evidence of cross-bidding and active switching between auctions in order to seek the lowest price has been found (Anwar, McMillan & Zheng, 2006; Rand & Jank, 2013; Liang, 2014), although not all participants appear to switch between competing auctions, the resulting inertia contributing substantially to differentials in final price outcomes. A further feature, peculiar to fixed-duration online auctions, is the prevalence of very late bidding, a process known as 'sniping' (Barbaro & Bracht, 2004; Borle, Boatwright & Kadane, 2006; Ockenfels & Roth, 2006). Sniping represents an attempt to enter a sequentially higher bid sufficiently close to the end time of the auction to prevent a complete response from other bidders. The strategy is not, however, entirely risk-free in terms of item capture as, should a competitor manage to place an even later bid, the original sniper may themselves run out of time to counterbid.

While studies have examined bidding outcomes in contemporaneous online auctions, the behaviour of auction participants in terms of tracking overlapping auctions has been less well explored. Haruvy and Popkowski Leszcyc (2010) did investigate switching between contemporaneous auctions of identical goods from a perspective of search cost using data from eBay. Designed to examine the factors which contributed to price dispersion, the cost of searching was found to be a significant factor in explaining inertia amongst bidders. Lowering search costs, by offering financial incentives (the removal of shipping or postage costs), was then found to reduce inertia leading to greater switching activity and lowering price dispersion as a result. The study used actual bids in the different auctions to represent the degree of switching; it therefore did not capture data relating to switches between auctions which did not result in actual bids.

An agent-based approach examining switching and bidding behaviour was employed by Rand and Jank (2013). Actual bidding data was again taken from eBay relating to auctions in which equivalent, brand new Canon SD1000 cameras were being offered by different sellers. In each case, the cameras were offered without accessories, making the items close-to-identical. The data covered 1155 auctions producing 19,007 bidding records. A total of 5849 bidders participated in the auctions with 1554 bidding in more than one auction. Based upon the empirical data, the authors constructed an agent-based model with switching and bidding rules in order to reproduce the price evolutions seen in the empirical data. The model assumed that bidders behaved rationally in terms of item capture, and therefore sought to achieve the lowest possible winning bid across available auctions. Since there were multiple overlapping auctions at any point in time, certain preferences were assumed based upon bid increment and relative ending times of auctions (lower bid increments and earlier ending times being preferred). It was found that greater competition and price convergence could be achieved across competing auctions when active switching was encouraged through identification of the lowest priced auctions. By identifying current low-priced auctions with near ending times, new bidders were attracted to those auctions while existing potential bidders remained on those auction platforms increasing competition and raising average closing bids. The authors therefore concluded that lack of information is a prime driver of observed price disparity across multiple simultaneous or overlapping auctions of close-to-identical items. Even though physical search costs in online platforms are low, there is a tangible time cost involved which a number of bidders appear reluctant to incur resulting in inertia. Similar to the findings of Haruvy and Popkowski Leszczy *op cit*, when the search costs are reduced, more active switching can result leading to greater competition and cross-auction price convergence.

While empirical studies of switching between simultaneous or overlapping auctions make extensive use of the voluminous data which is available from online commerce sites such as eBay, switching activity is largely inferred from actual bid patterns. These approaches therefore fail to include data for auction switching which does not result in bidding activity. The study below makes an attempt to fill this gap by testing switching activity independent of actual bidding using a simulated platform with two simultaneous, short, fixed-duration auctions of identical items. Mouse tracking is used to capture the relevant actions of participants, thereby recording all acts of switching from one auction to another. As a result, the study generates more complete data on switching propensity in comparison with studies which rely solely on recorded bid data.

## **6.2. A Study of the Effect of Information on Switching Propensity in Simultaneous Auctions of Identical Items**

The study was undertaken to investigate the effects of information flow, including real-time changes in prevailing bids, “live” bid differentials between auctions and in-auction bidding activity, on switching frequency in the case of directly overlapping, fixed duration auctions of identical items. The study therefore examines the degree of active tracking across pairs of auctions independent of actual bidding or bidding strategy. Simulating two auctions of identical items with equivalent start and ending times eliminates some of the factors found to explain auction preferences in studies of price dispersion, such as timing differences and duration. In addition, the simulated environment provides “close-to-zero” physical costs of switching between auctions. However, given that the two simultaneous auctions are competitive, there is a potential opportunity cost associated with not switching as bidders may then overpay in one auction relative to the other.

### 6.2.1. Participants

A total of 180 participants, allocated equally and at random across 6 conditions, took part in a simulated, very short-term competitive online auction experiment of five minutes duration. The short duration was chosen in order to keep participants engaged in the task. By gender, 98 of the participants were males with 82 females. Ages ranged from 18 years to 62 years ( $M = 29.69$ ,  $SD = 8.96$ ), as shown in Table 11. Each participant was assigned a unique user ID relating to the experimental condition to which they had been allocated. Participants had no prior knowledge of the item to be offered through the simulated auctions or any details regarding any of the conditions. No significant difference between ages was found based upon gender; using the two-sample t-test for unequal variances,  $t(178) = 1.92$ ,  $p = 0.057$ .

Table 11. Breakdown of participants by age and gender

	Male	Female	Total
Number	98	82	180
Average Age	28.53	31.09	29.69
Max Age	62	53	62
Min Age	18	18	18
SD (Age)	8.96	9.28	8.96

Participants were informed that they were not participating in a real auction and that they were not entering into any form of actual or implied contract as a result of entering notional bids. It was further disclosed that there would be no associated monetary commitment as a result of taking part in the auction and that no item would actually be received in the event of entering the highest closing bid. It must be acknowledged that this practical constraint represents an obvious limitation; in the absence of monetary cost and incentives, certain aspects of behaviour may be distorted or, in some cases, not even present. In essence,

participants could bid without consequence. Thus, while all participants were asked to engage in the auctions as if they were genuine bidders, taking into account their own valuation of the item concerned and any other factors they considered relevant, there can be no guarantee that this outcome was achieved. This aspect will be discussed further when the results of the study are considered, although some evidence relating to hypothetical versus real rewards can be considered briefly here.

The use of a simulated environment in the current study largely reflects its aims. Empirical studies of auction behaviour, such as those mentioned earlier in the chapter, typically use actual data collected by online site operators such as eBay. While this data is extensive and deep, it relates overwhelmingly to actual bidding activity. This proves useful in terms of testing models of bidder choice, such as those used by Haruvy and Popkowski Leszcyc (2010). However, there is no equivalent data which is universal in terms of coverage relating to aspects of search behaviour and auction switching on their own. Therefore, to the extent that it is considered, auction switching must be inferred from bid evidence. The simulated environment provides a structure in which switching activity alone can be identified and captured independent of any other actions taken by participants in the study. While actual data relating only to general search behaviour from live auctions involving actual monetary commitments and potential rewards in the form of item capture is clearly preferable, acquiring such data would require an enormous undertaking which may not be considered economic. To some extent, evidence from other areas might prove useful. For example, studies of Internet browsing and search queries have been undertaken in order to evaluate consumer behaviour. Thus, data logging searches for items on e-commerce sites have been undertaken (Eastman & Jansen, 2003). However, while significant data is



obtained, the context and motivation for an individual's search behaviour is typically not evident from that data (Jansen & Spink, 2005).

A more direct comparison of the effect of a lack of actual monetary commitment is provided by studies of hypothetical versus real rewards. A number of such studies have been undertaken in relation to risky choices within laboratory settings. Thus, Irwin, McClelland and Sculze (1992) undertook a laboratory experiment testing the impact of hypothetical versus real rewards on bidding behaviour relating to insurance. Based upon a notional gamble incurring losses with declared probabilities, participants were required to bid for insurance in order to protect themselves from those potential losses. In the event that they were successful in obtaining insurance, they lost nothing in the event of a loss (but paid the insurance premium). If no loss accrued, all participants received a fixed amount added to their starting cash balances. Certain participants were told from the outset that all monetary amounts were hypothetical while others actually gambled with real money, potentially receiving cash at the end of the experiment. Results indicated that participants in the hypothetical return condition entered a slightly greater dispersion of bids, with more high and low bids than was the case in the real reward category. A greater willingness to accept potential losses (bidding zero for insurance cover) was evident in the hypothetical reward group. Those in the real reward condition exhibited greater risk aversion, more regularly seeking insurance in order to preserve their capital. The authors concluded that the difference in behaviour was due to "decreased concern about the task" when returns were hypothetical. Conversely, in a study of discounting associated with delays using real and hypothetical rewards, Madden *et al* (2003) found no significant effect of reward type on discount rates.

### 6.2.2. Materials & Procedure

The simulated online auction platform was created and coded using Microsoft Visual Basic and compiled as a stand-alone executable application. The application interacted with the internal clock of the host machine in order to generate precise timings for the length of the simulated auctions, the triggering of pre-programed events (“competitor” computer bids) and the capturing of the timing of user interaction through mouse clicks. The visual display resembled that of a typical eBay-type auction with navigation buttons available to trigger switching between auctions and the generation of history views (a list of all bids to date in the auction by bid level together with the timing of the bids and the “identity” of the bidder). It was therefore a very easy process to move from one auction to the other, a process which involved little cost in terms of time. To this extent, switching costs are considered to be close-to-zero with neither auction being disadvantaged versus the other in that regard. Each auction used identical formatting with the same buttons displayed for switching, viewing histories and entering bids. Each page informed the participants that an additional auction of an identical item with an equivalent ending time was taking place. Participants could then switch from one auction to the other at the press of a button. Such switching was cost free and virtually instantaneous, implying no associated time cost (Appendix A).

The key events of auction switching, generation of history views and bidding sequences were all initiated and triggered via clicking on appropriately labelled button controls clearly displayed on the screens. All mouse click events associated with these buttons were logged in real time, in accordance with the computer’s internal clock, with the corresponding data stored in dynamic data files. The data recorded in this way included the exact time of each

mouse click along with the nature of the function executed by the mouse click and the auction page from which the action was triggered. At the conclusion of the auctions, all of the interactive data gathered during the course of the auction experiment and stored in the dynamic data files was automatically downloaded into a master file, named in accordance with the participant's unique user id, and saved to the computer's hard disk for subsequent analysis.

### **6.2.3. Method**

Participants were presented with two simultaneous "online" auctions for an identical item (a car with an indicated "on the road" value of £5995). The auctions were conducted using custom-built software. Participants could interact with the software in order to compare the items being offered, track "real-time" bids and to place their own (notional) bids. In order to create a degree of realism, the software contained pre-programmed bids for each auction; there was no direct competition between participants in terms of bidding as the software was entirely standalone. Pre-programmed bids would be triggered at set times measured in relation to the starting time of the auction. All sequential pre-programmed bids complied with fixed bid increments for that auction, which remained static throughout. Pre-programmed bids became more frequent towards the end of each auction in order to reflect greater competition between bidders, consistent with observed data in online auctions (Shmueli & Jank, 2005). While the frequency of pre-programmed bids changed over the course of the auction, the intervals between bids were irregular, ensuring that the next bid could not reasonably be anticipated in terms of timing.

All auctions ran for exactly five minutes. The start time was triggered by a participant clicking a button to commence the auction. This then set the ending time exactly five minutes after the triggered start time, measured according to the computer’s internal clock. The final highest, pre-programmed computer bid for each auction was £3100. Opening pre-programmed computer bids were not identical across all auctions with fast auctions starting at lower levels than slow auctions in order to ensure that closing, pre-programmed bids converge with the desired end value. The structure of the auction pairs was varied based upon programmed bidding intensity, with one pair varying bid increment. In total, six conditions were then created, involving differing combinations of auction styles. These are summarised in Table 12.

Table 12. Description of experimental conditions, including auction type and summarised programmed parameters .

<i>(Auction Type)</i>	Auction Pairings by Type											
	Condition 1		Condition 2		Condition 3		Condition 4		Condition 5		Condition 6	
	Slow	Fast	Slow	Slow	Fast	Fast	Fast	Slow	Slow	Slow	Fast	Fast
Computer Bids (Number)	23	46	23	23	46	46	46	23	23	23	46	46
Bid Increment	50	50	50	50	50	50	50	50	100	50	50	50
Opening Bid	2000	850	2000	2000	850	850	850	2000	900	2000	850	850
Ending (Computer) Bid	3100	3100	3100	3100	3100	3100	3100	3100	3100	3100	3100	3100
First Computer Bid (Secs from Start)	2.1	1.5	2.1	4.9	1.5	0.8	1.5	2.1	6.6	2.1	2.4	1.5
Final Computer Bid (Secs from Start)	299.2	299.6	299.2	298.9	299.6	299.0	299.6	299.2	299.8	299.2	299.7	299.6

*Note:* The study defined six conditions using combinations of “Fast” and “Slow” auctions. Fast auctions had 46 potential computer bids while slow auctions had 23 such pre-programmed bids. The number of possible computer bids allowed the various individual auctions to converge to an identical final computer bid value. Opening bids were fixed and identical across all Conditions other than Condition 5. Closing computed bids were identical for all auctions.

The conditions reflected varying paired combinations of single auctions categorised by overall bidding intensity (“fast” or “slow”), determined by the total number of pre-programmed bids over the life of each auction. Fast auctions had a total number of 46 pre-programmed computer bids while slow auctions had 23 possible pre-programmed computer bids. This meant that fast auctions would appear more active and therefore more competitive than slow auctions, enabling the impact of general bidding intensity on switching frequency to be examined. Across the six conditions, slow auctions were compared together as were fast auctions. Fast auctions were also compared with slow auctions (note that the same individual auction schedule could appear in more than one condition).

While opening bid levels of fast auctions were well below the opening bids in slow auctions, the gap between bids in the two auctions narrowed progressively as the auctions progressed. In the closing stages, the auctions reach general parity and therefore became genuinely competitive with each other at this point. From the point of general parity to the end of the auction, the identity of the auction with the lowest bid at differing points of time changes. Within slow and fast auction combinations, this would represent the first period in which the bid level of the fast auction could exceed that of the slow auction. In the case of two auctions with the same bid frequency (*fast-fast* or *slow-slow*), the identity of the auction with the lowest prevailing bid at any point in time would vary earlier in the process. The comparable pre-programmed bids for each Condition pair are shown in Figure 5 while the full data is available in Appendix B. In most of the conditions, bid increments were fixed at £50, the exception being condition 5, comprising two slow auctions, where different fixed bid increments (£50 and £100) were applied to the auctions respectively. This enabled any bias in relation to bid increment to be tested. Condition 6 comprised two fast auctions, as with

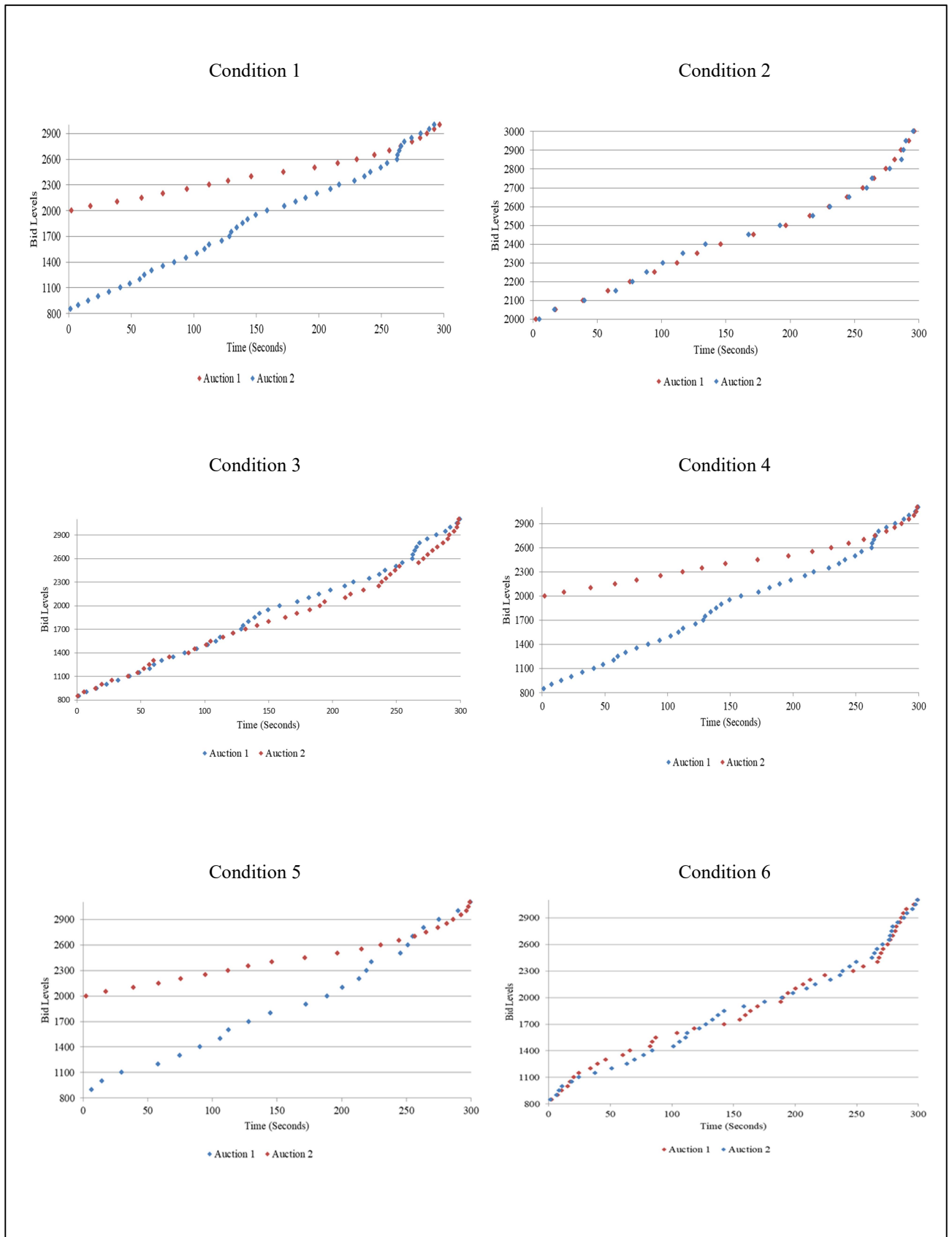


Figure 5: Path of pre-programmed bids for the two auctions by Condition .

condition 3, although the amount of interchange between auctions in terms of cheaper relative bids at any point in time was greater under condition 6 than condition 3.

Participants were pre-assigned randomly to each Condition, with the same number of participants in each category. Participants were shown how to use the software and were free to ask questions on any aspect of its functionality at any time. Following initial online instructions, each participant was required to “log in” using a unique User ID. By default, participants were then presented with the first auction page. At this point, the simulated auctions were “live”. Standard information contained the start time of the auction being viewed, set automatically from the computer’s internal clock, the ending time (start time plus 300 seconds, or 5 minutes) and time left to expiry of the auction, updating in real time. Each auction proceeded on the basis of a pre-set initial bid, progressing in predetermined, fixed increments. Participants were informed of the level of bid required to become the highest bidder at each point in time but were not able to enter their own jump or proxy bids. Participants were free to bid in either auction as often as they wished, so long as they were not already the current highest bidder in that particular auction. In the event that participants did not bid, the programme would trigger its own pre-programmed bid sequence according to the elapsed time in the auction. The auction pages would then update automatically with the latest data.

Participants entered bids via a button which then triggered a prompt for confirmation. The prompt provided summary details of the level at which the bid would be made and requested the user to confirm the intention to bid. At this stage, a bid could either be confirmed or cancelled. Once confirmed, and in the event that no equivalent or higher intervening bid had been received (triggered by the computer), the user would then become

the highest bidder at that point in time. In the event that an equivalent or higher intervening bid had been received prior to the user confirming acceptance of the bid, participants were informed that they had been outbid and were then free to initiate a new bid at a higher level up until the point at which the auction ended. Participants could access the full bidding history of either auction at any time by clicking a “View Bid History” button. This option, available throughout the duration of the auctions, took participants out of the existing auction page into a new “view”. Bidding history contained information on the timing and levels of prior bids and the “identity” of the bidder at each stage. This information did not update in real time. Therefore, new bids entered whilst this page was open would not be displayed. Participants could return to the “live” auction by closing the History View window. Since the aim was to examine overall bidder behaviour rather than simply to analyse actual bids made, the software was designed to capture all relevant mouse clicks by participants as well as the actions associated with those clicks. All switches between auctions, history views and initiated and confirmed / cancelled bids were therefore logged with the time of each click captured. Upon the conclusion of the experiment, the software generated a fixed format file automatically which contained all of the interactive data for that participant.

### **6.3. The Model**

The study used a number of static and dynamic variables which could affect switching propensity and auction choice. The static variables are fixed bid increments, which are set at the start of each auction and remain constant over the course of the auction, while the dynamic variables are the elements which change during the course of each auction: the latest bid with the associated ID of the bidder; bid histories to date; differentials between prevailing bids in each auction and changes in bidding intensity (the frequency with which competitive



bids occur within a given timeframe). The variables are therefore characteristics of the auctions rather than fundamental differences between the items offered. While the study makes assumptions about utility maximising behaviour, it does not seek to measure utility directly. There is, however, an implied link between search effort, as evidenced by switching frequency, and utility as switching enables bidders to identify the auction which offers the greatest potential consumer surplus at any point in time.

The study generated binary data reflecting the actions of participants captured by mouse clicks. Therefore, a mouse click triggering a switch between auctions would result in a “1” being added to the data field relating to auction switches. The exact time that the action was triggered was also recorded. Combining this data with the pre-programmed data enabled all mouse-based clicks to be placed within the context of broader auction data. For example, since the exact time of an auction switch was captured, the prevailing bids in each auction at that time could be determined, as could all other associated data (highest bidder.ID, the number of bids to date and bidding frequency to date).

The data generated by the study was analysed using log-linear regression in which search effort is the dependent variable (DV). Search effort is measured by the number of switches between auctions undertaken by participants over fixed, non-overlapping periods. Each auction lasted for a relatively short period of time (5 minutes). Switching frequencies were therefore measured over discrete 10 second periods thereby enabling a common basis for comparing switching activity over the course of each auction (this is similar to the empirical approach to timespan data adopted by Lee, Ziolo, Han, and Powell, 2018). The independent variables (IVs) were as follows;

*Bid Intensity*; measured by the number of observable competitor bids over the previous discrete 10-second period. By recoding all mouse clicks triggered by the participant, a record is maintained of which auction was displayed at various times. The number of observable bids would therefore comprise bids from both auctions if a participant switched between auctions during the previous discrete 10-second period;

*Max Diff*; the maximum differential between prevailing bids in the two concurrent auctions measured over the previous discrete 10-second period

*Category*; a categorical variable, taking values 1 to 6, representing each of the *conditions*.

### **6.3.1 Hypotheses**

The study is designed to investigate the effect of relevant information relating to price divergence, level of competitiveness and auction structure on bidder's auction switching propensity. It is assumed that bidders behave rationally and therefore seek to maximise their utility in the form of consumer surplus. As a result, bidders are expected to track relative bid levels between auctions in order to establish which one of the pair offers the higher payoff (the possibility of acquiring the item at the lowest possible price). Therefore, while the primary focus of the study is to establish potential determinants of switching behaviour, the assumption of rational behaviour also implies that participants will bid in a manner consistent with maximising their potential payoffs. Consequently, the majority of actual participant bids should be placed in the auction offering the lowest level of valid bid at the time the bid is placed.

The key hypotheses can then be described formally as follows:

1. The frequency with which competitive bids occur (bidder intensity) is expected to affect the frequency of switching. The frequency of competitive bids is a proxy for the degree of competition for the item, increasing the prevailing highest bid in the process. Since rational bidders seek to maximise their utility, as measured by consumer surplus, they are expected to switch to the other auction in order to identify which of the two auctions offers the greatest potential payoff at that point in time. A positive coefficient is therefore expected for this variable.
2. The relative bid differential between the two auctions is expected to affect switching propensity. As the level of prevailing bids in the two auctions approaches parity, so bidders are expected to switch more regularly, again reflecting the desire to maximise potential payoff. A negative coefficient was therefore expected for relative bid differential, implying that switching propensity should increase as relative bid differentials narrow (Dholakia, Basuroy & Soltysinski, 2002; Haruvy, Leszczyc, Carare, Cox, Greenleaf, Jank & Rothkopf, 2008; Kayhan, McCart & Bhattacharjee, 2010).
3. Switching propensity is also expected to be influenced by experimental condition. The different conditions provide combinations of auctions with differing characteristics. Paired auctions with broadly similar relative bidding intensities (*slow-slow* or *fast-fast*) are expected to show greater underlying levels of switching than combinations with slow and fast auctions. This simply reflects the fact that the bid differential within slow-fast combinations is significant at the outset of the auction, only converging in the latter stages. As a result, there is less need for rational bidders to check current bid levels in the slow auction until the fast auction is seen to substantially close the initial gap in bid levels.

### 6.3.2. General Description of the Data Generated by the Study

Across all conditions and all 180 participants, the collected data shows that 4181 clicks were recorded, the overwhelming majority of which (3252, 77.78%) represented switching between auctions. A total of 842 bids were entered of which 123 “failed” due to higher intervening (computer) bids. Thus, 719 bids prevailed as the highest current bid at that point in time (85.39% of total bids made). The majority of participants made multiple bids with 119 participants making three or more bids and 70 making five or more bids. History views (viewing auction bid histories to date) accounted for just 87 (2.08%) of the total number of clicks, and were triggered by 54 of the 180 participants (3.0%). While all participants switched between auctions, 10 failed to enter a single bid during the course of the auctions. Sniping-type bids within the final second of the auctions were made by 19 participants (11.18% of active bidders) while 38 participants (22.35%) placed bids within the final two seconds. 95.13% of all successful bids were placed at the lowest available level at the time of each bid.

Switching activity was not evenly distributed in terms of time interval across the various auction pairs. Based upon average switching frequencies across all auctions, early levels of switching activity tended to subside slightly after the first minute of the auctions. However, activity picked up progressively after approximately the 150 second mark, accelerating appreciably in the latter stages of the auctions, reaching a peak some 20 seconds before the end of the respective auctions, on average (Figure 6). There are some differences between auction pairs described by the various Conditions. The auction pairs represented by Conditions 1 and 4 (both *slow-fast* combinations) showed the greatest change in overall

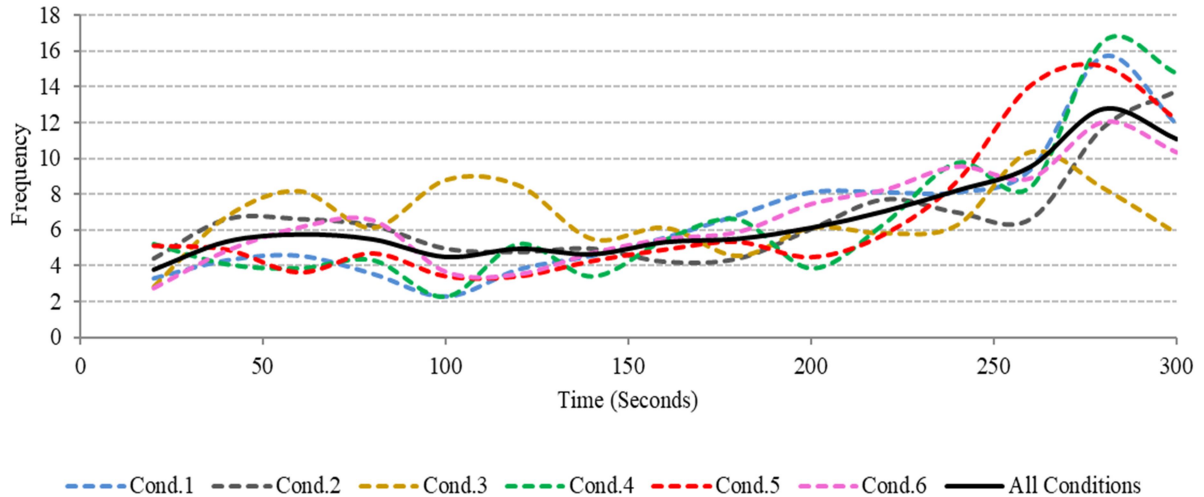


Figure 6. Frequency Distribution of Inter-Auction Switching Behaviour: Aggregated by Condition.

switching activity over the course of the auctions. The relatively lower levels of switching activity in the early stages of the auctions is probably explained by the wide initial bid differential between the auctions, as noted earlier. However, once the auctions became genuinely competitive with regard to prevailing bid levels, switching activity increased significantly. Participants therefore appeared to react more strongly to the changing competitive nature of these auction pairs. Conditions 2 (*slow-slow*), 3 (*fast-fast*) and 6 (*fast-fast* with active bid cross-over) represent the auction pairs with similar bidding intensities across the respective pairs. Somewhat different switching behaviour is apparent for these Conditions.

Participants within Condition 2 continued to switch actively right to the end of the auction, collectively showing the highest cumulative switching frequency within the last 10-seconds of the auctions. Participants in that category also switched most actively at the start of the auction periods. It is not clear why this was the case although it is possible that the relatively slow nature of the auction being viewed prompted participants to check if bid levels

in the alternative auction were proceeding at the same general slow pace or relatively moving out of line. Switching activity tailed off relatively sharply in approximately the last 40 seconds for participants in Condition 3 while participants within Condition 6 tended to track the pattern of average switching behaviour across all participants, with switching activity peaking approximately 20 seconds before the end of the auctions. Participants assigned to Condition 5 (auctions pairs with different bid increments) appeared to be affected by the different bid increments between the auctions, starting to increase switching activity as the prevailing bid differential between the two auctions narrowed to two bid increments. The general pattern of switching behaviour is broadly consistent with expectations, tending to increase significantly as the auctions approached their climax. It is at this stage that active bidders are likely to be most engaged in attempting to capture the item. The slight decline in aggregate switching activity in the final few seconds of most auctions is then consistent with the proposition that bidders had, by that stage, identified their target auction and wished to concentrate on entering a valid bid before the auctions ended.

### **6.3.3. Statistical Test Results**

The relationship between switching frequency, bid intensity and relative bid differentials was tested using log-linear regression. The broad effect of Condition was also tested using a categorical variable. The results of the regression analysis are shown in Table 13. The goodness of fit statistics compare an independent (constant-only) model, in which the linear combination of explanatory variables reduces to zero, with the full model, including the effect of the IVs. The Cox and Snell and Nagelkerke  $R^2$  measures indicate that the full model had significant explanatory power. As a result, the null hypothesis (that the coefficients of the IVs are zero) can be rejected. The Akaike Information Criterion (AIC) and Schwarz's

Bayesian Information Criterion (BIC) test the quality of the model by measuring the amount of information lost as a result of parameter estimation. The measures therefore examine the goodness of fit adjusted for any overfitting resulting from the inclusion of additional variables. Lower values for AIC and BIC for the full model versus the constant-only model confirm that the IVs have some explanatory power and the full model improves the independent model.

The confidence tests assess the extent to which the full model maximizes the value of the likelihood function which makes the observed data most likely given the model's parameters. Higher values for each of the measures suggest a greater degree of confidence. Each of the test statistics: -2 Log -Likelihood ( $\chi^2(3, N=180) = 124.43, p < .0001$ ); Score ( $\chi^2(3, N=180) = 84.88, p < .0001$ ) and Wald ( $\chi^2(3, N=180) = 192.50, p < .0001$ ) is highly significant, denoting that the IVs, in combination, significantly enhance the explanatory power of the full model.

The regression model coefficient for Bid Intensity is positive ( $b = .069, \text{Wald } \chi^2(1, N = 180) = 14.04, p < .0001$ ), implying a direct relationship between the intensity of recent bidding activity and the frequency of auction switching. Thus, as auctions appear to become more active in terms of bidding levels, the greater the likelihood of participants switching between auctions, consistent with the proposition that rational bidders seek to maximise consumer surplus by identifying the current auction with the higher potential payoff. The coefficient for Max Diff (the maximum difference between auction bids over a fixed interval) is negative ( $b = -.001, \text{Wald } \chi^2(1, N = 180) = 130.93, p < .0001$ ), indicating a tendency for switching activity to increase as the differential between the two auction bid levels narrows. The categorical variable for condition was also found to be significant ( $b = .07, \text{Wald } \chi^2(1, N$

= 180) = 13.84,  $p < .0001$ ), suggesting that the various auction pairings tended to impact participant behaviour with regard to switching activity.

Table 13. Results analysis of switching frequency

Summary goodness of fit statistics

Statistic	Independent (Constant Only)	Full Model
Observations	180	180
Degrees of Freedom	179	176
-2 Log(Likelihood)	1297.60	1173.18
R <sup>2</sup> (Cox and Snell)	0.00	0.50
R <sup>2</sup> (Nagelkerke)	0.00	0.50
Akaike Information Criterion	1301.60	1183.18
Schwarz's Bayesian Criterion	1307.99	1199.14

Confidence tests:

Statistic	DF	Chi-square	Pr > Chi <sup>2</sup>
-2 Log(Likelihood)	3	124.43	< 0.0001
Score	3	84.88	< 0.0001
Wald	3	192.50	< 0.0001

Model parameters for the components

Source	Value	SE	Wald Chi-Square	Pr > Chi <sup>2</sup>
Intercept	2.710	0.090	1007.01	< 0.0001
Bid Intensity	0.069	0.020	14.04	0.0000
MaxDiff	-0.001	0.000	130.93	< 0.0001
Condition	0.070	0.020	13.83	0.0000



The results support the hypotheses relating to the impact of short-term recent bidding intensity and current price-differentials between auctions on switching propensity; each of these predictor variables is significant. Since price outcomes are determined by the level of competition, as measured by relative bidding intensity, the model implies that rational bidders can be expected to switch between auctions when bid frequency in the currently viewed auction increases as they seek comparable information about the second auction. Therefore, bidders appear to be sensitive to changes in the dynamics of an observed auction when they are aware of the existence of another, simultaneous auction offering a close-to-identical item. This is consistent with the proposition that bidders adopt a primary goal of seeking to maximise their consumer surplus by tracking simultaneous auctions in a manner consistent with enabling them to potentially acquire the item at the lowest possible price. These findings are consistent with the conclusions drawn from the empirical work of Haruvy and Popkowski Leszcyc (2010) and Rand and Jank (2013) with regard to information and participation costs. In the case of the simulated auctions described here, search costs were minimal, enabling active bidders to easily identify the auction with the greater current potential payoff.

Different experimental conditions based upon the various categories of auction pairs were also expected to result in significant differences in overall switching propensity. It will be recalled that the conditions paired auctions in various ways based upon their characteristics defined by the number of pre-programmed computer bids possible in each. Auctions were therefore classified either as “fast” or “slow”. Fast auctions had a maximum total number of 46 computer-generated bids over the 5-minute period for which the auction ran versus a maximum number of 23 bids in slow auctions. Since all auctions were programmed to end at the same bid level, opening bids differed between fast and slow auctions in order to allow for

the greater number of bids within fast auctions, each one increasing the prevailing highest bid by the same bid increment. Therefore, the greatest disparity between bid levels in fast and slow auctions would occur at the start of the auctions, the gap closing as the auction progressed. Due to the acceleration in bidding activity towards the end of each auction, relative bid differentials between the two auctions would narrow increasingly quickly, being eliminated as the auctions reached their final stages. At this point, the auctions were considered to be highly competitive in nature; maximum bid differentials between them were no greater than one bid increment, making relative final outcomes difficult to determine. For convenience in relation to the following comments, the six conditions are summarized again below (Table 14);

Table 14. Summary of auction pairs by category

Category	First Auction Identifier	Second Auction Identifier
1	Slow 1	Fast 1
2	Slow 1	Slow 2
3	Fast 1	Fast 2
4	Fast 1	Slow 1
5	Slow (Bid Increment: £100)	Slow 1
6	Fast 3 (With Active Switching)	Fast 4 (With Active Switching)

The regression analysis indicated that the categorical variable for condition had significant explanatory power ( $b = .070$ , Wald  $X^2(1, N = 180) = 13.83, p < .0001$ ). It is not, however, possible to determine where the differences between the various categories occur.

In order to obtain greater clarity, a one-way ANOVA was performed with post hoc tests for condition. The results are shown in Table 15.

Table 15. ANOVA of the Impact of Condition on Switching Behaviour.

Descriptive Statistics				
Condition	Mean	Std Dev.	Std Err	N
1	12.90	8.18	1.49	30
2	17.73	7.51	1.37	30
3	21.13	6.74	1.23	30
4	14.40	9.27	1.69	30
5	15.37	9.75	1.78	30
6	25.30	11.69	2.13	30

Analysis of Variance (Switching Frequency)					
Source	Type III SS	Df	Mean Sq.	F	Prob.
Model	3265.69	5	653.14	8.06	< .0001
Error	14102.50	174	81.05		
Total	17368.19	179			

Post hoc Tukey tests for Condition					
Cross-Condition Comparison		Mean Diff.	SE	q	Prob.
1	2	-4.83	1.64	2.94	0.30
	3	-8.23	1.64	5.01	0.01 ***
	4	-1.50	1.64	0.91	0.99
	5	-2.47	1.64	1.50	0.90
	6	-12.40	1.64	7.54	0.00 ***
2	3	-3.40	1.64	2.07	0.69
	4	3.33	1.64	2.03	0.71
	5	2.37	1.64	1.44	0.91
	6	-7.57	1.64	4.60	0.01 ***
3	4	6.73	1.64	4.10	0.04 **
	5	5.77	1.64	3.51	0.13
	6	-4.17	1.64	2.53	0.47
4	5	-0.97	1.64	0.59	1.00
	6	-10.90	1.64	6.63	0.00 ***
5	6	-9.93	1.64	6.04	0.00 ***

Notes : The F-score confirms a significant overall impact of condition upon switching behaviour. The post hoc (Tukey) analysis provides additional information on the significance of switching propensity on the basis of condition. The Table shows the critical ( $q$ ) values based upon significance ( $\alpha$ ) at the 5% level with

the associated probability that the mean differences observed across conditions occurred by chance. Significance levels are denoted by: \*\*\*,  $p < .01$ ; \*\*,  $p < .05$

From the summary descriptive statistics, it is apparent that fast auctions paired together (conditions 3 and 6) generate the highest average number of switches; the average number of switches in condition 3 was 21.13 ( $SD = 6.74$ ) and 25.30 ( $SD = 11.69$ ) for condition 6. Pairs of fast with slow auctions (conditions 1 and 4) generated the lowest average number of switches; 12.90 ( $SD = 8.18$ ) in the case of condition 1 and 14.40 ( $SD = 9.27$ ) for condition 4. The higher average number of switches associated with fast-fast auctions in comparison with pairs involving at least one slow auction is consistent with the proposition that bid frequency or intensity impacts switching behaviour; in the event that there are two simultaneous auctions with the same relatively high level of bid intensity, rational bidders are likely to engage in greater levels of switching in order to identify the auction offering the lowest relative bid level. The somewhat higher dispersion found for condition 6 versus condition 3 suggests that some bidders in the former condition were affected by the more frequent changes in terms of which auction in the pair offered the lower bid level at various stages of the auctions.

In the case of fast and slow auction pairs, a significant disparity in bid levels (greater than 2 incremental bids) persists for a large part of the total auction time. These auctions only become competitive in their latter stages once the differential in bid levels between the two auctions closes to  $\pm$  one bid increment; around this point, rational bidders need to engage in greater switching in order to establish the auction offering the lowest current bid level. The two slow auction pairings (conditions 2 and 5) appear to generate marginally higher average switching activity than fast-slow pairs but below the levels seen for fast-fast pairings. This can be explained by the lack of bidding intensity in slow auctions relative to fast

auctions. Nevertheless, relatively narrow differences in prevailing relative bid levels in the case of condition 2 appear to provide some incentive for bidders to switch in order to track the two auctions. Bid increments themselves only appears to have only a marginal impact on switching frequency, as evidenced by condition 5. Consequently, the average switching frequency for condition 2 is found to be 17.71 ( $SD = 7.51$ ) versus 15.37 ( $SD = 9.75$ ) for condition 5

The analysis of variance shows that there was a statistically significant difference between categories ( $F(5, 174) = 8.06, p < .0001$ ); the null hypothesis that condition means are equal is therefore rejected. The post hoc (Tukey) analysis provides more detailed information on statistically significant differences between mean switching activity based upon category or condition. Pairwise comparisons show significant differences ( $p < .0001$ ) in switching propensities between condition 1 (slow-fast) and conditions 3 (fast-fast) and 6 (fast-fast with active switching), in line with the observations noted above. This is mirrored by condition 4 (fast-slow) versus the same fast-fast conditions, 3 and 6;  $p < .05$  in the case of condition 4 and 3 comparison and  $p < .0001$  in the case of conditions 4 and 6. Of the slow-slow auction pairs, significantly lower average switching frequencies were found for conditions 2 (slow-slow) and 5 (slow-slow with different bid increments) compared with condition 6 (fast-fast with active switching). However, no significant differences are found in relation to the other fast-fast pair (condition 3); it is not clear why this should be the case. No statistically significant differences in mean switching frequencies are found in relation to the slow-fast pairings (conditions 1 and 4) and either of the slow-slow pairings (conditions 2 and 5).

Overall, these findings reinforce the earlier interpretation of results particularly as they relate to the effect of bidding intensity on switching behaviour. Therefore, bidder behaviour in terms of tracking effort is likely to be a function of the specific characteristics of close-to-contemporaneous auctions. In the event that these auctions for identical items are competitive, in the sense that bidding is active and prevailing bid differentials between the auctions are considered to be sufficiently narrow, such information being easily accessible to potential bidders, then switching activity and participation are likely to increase. Again interpreting these results within the context of the findings of the studies conducted by Haruvy and Popkowski Leszcyc (2010) and Rand and Jank (2013) discussed earlier would then suggest that multiple auction environments which are close-to-frictionless in terms of information and switching costs should tend towards price convergence for identical items as the number of bidders increases. However, in the event that there is limited access to relevant information, or the costs of its acquisition are considered by potential bidders to be too high, then inertia may result, thereby enabling price dispersion to persist.

#### **6.4. A Consideration of Auctions with Posted Exit Prices**

Competitive auctions, of the type considered above, are essentially variable price formats through which selling prices are determined as a result of interaction between interested buyers. The advent of online platforms means that multiple sellers offering similar items effectively form a single marketplace in which buyers, sellers and items compete. Rational sellers are assumed to seek to maximise their sales revenues while rational bidders attempt to maximise their consumer surplus by acquiring items at the lowest possible price, equal to or less than their private value for that item. In the absence of any frictions in the form of

participation costs or lack of information, pricing outcomes should be efficient as price differentials between auctions of identical items would see bidders switch to the auction with the lowest prevailing bid, entering their own bids until such price differentials were eliminated. This essentially describes markets where perfect competition prevails such that buyers neither under- nor overpay and sellers maximise their revenues.

While online marketplaces such as eBay were originally dominated by standard auctions of the type described above, more recently, the variable pricing formats of many online auctions have changed as a result of the inclusion of fixed-pricing in the form of buy-it-now (BIN) options<sup>6</sup>. BIN options allow bidders to end an auction by agreeing to purchase the item at the fixed, posted price. BIN options are therefore the equivalent of known exit strategies at certain prices as opposed to participating in standard auctions where final price outcomes are uncertain. However, within the context of competitive auction markets, BIN options appear to be puzzling choices for sellers as, unlike reserve prices which are designed to achieve minimum level of revenue, BIN options would appear to have the effect of placing an upper limit on maximum selling prices, thereby potentially reducing revenues. Not surprisingly, considerable research has focused on the implications of fixed-price offerings for both buyers and sellers.

Using an analytical model involving a consideration of broad participation costs, Wang, Montgomery and Kannan (2008) found that BIN options could actually result in higher seller profits when customer participation costs, including opportunity costs and the uncertainty of acquiring the item at an acceptable price, were high. BIN options were also found to attract more buyers who might otherwise have ignored the auction. Similarly, Budish and Takeyama

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<sup>6</sup> eBay introduced BIN options in 2000. The majority of items on that site are now fixed price offerings.

(2001) have shown that BIN options can increase seller revenues when buyers are risk-averse as such buyers tend to view the BIN option as a form of insurance. Anderson, Friedman and Singh (2008) undertook empirical analysis using extensive auction data relating to eBay transactions of homogeneous goods in auctions of relatively short duration. The data included auctions with BIN options as well as those with no fixed-priced offering. The analysis controlled for numerous characteristics which may affect the appeal of an auction, such as seller reputation, type of seller, product characteristics (new and used items were included in the data) and auction characteristics (including low start prices, auction length and product description). Results indicated that seller reputation did not consistently boost final selling prices, although it appeared to increase participation levels (increasing the number of bids rather than their level). Across the entire sample, the presence of BIN options did not have a significant impact in terms of increasing final selling prices and buyers appeared to be largely indifferent to the presence of a BIN option. However, it was found that buyers tended to reject BIN offers when the seller attempted to extract a premium but tended to accept the offer when there was little or no premium over alternative auctions. In contrast, a study by Yoo, Ho and Tam (2006) found that the presence of BIN options increased price outcomes as a result of an “information effect” associated with BIN postings. Thus, BIN prices were deemed to provide bidders with a better estimate of likely final prices. In instances where BIN options were not triggered, auctions with this option performed as well as standard auctions with no fixed price offering. Therefore, it was concluded that the information effect of BIN prices improved overall market efficiency, the positive effect outweighing any negative effect from the presence of BIN options themselves. Chan, Kadiyali and Park (2006) examined the effect of BIN prices on willingness-to-pay (WTP), finding that setting “optimistic” BIN prices actually increased WTP, reducing it when BIN prices were set comparatively low. Furthermore, with regard to the overall data, there appeared to be



evidence that sellers often set BIN prices sub-optimally, apparently misinterpreting competitive aspects of auctions for similar items. They found a greater tendency to underprice rather than overprice BIN, thereby negatively affecting potential revenues.

In order to consider the impact of BIN options within the simulated framework described above, a further auction pair was examined in which a BIN option was available in one of the auctions; all other characteristics of the auctions making up the pair stayed the same as before (item, duration, programmed bids, and so on). The data was analysed using logistic regression (described in more detail in Chapter 8). The output, dependent variable was BIN selection by the participant, a binary variable taking the value 1 if selected and 0 if not. Certain assumptions were made concerning likely bidder behaviour in response to a BIN option. It was again assumed that bidders would be wholly rational and seek to acquire the item at the lowest possible price. On that basis, it was expected that prevailing auction prices would help to inform the buying decision. To the extent that bidders expected auction prices to exceed the BIN levels, the likelihood of that event happening should increase. Thus, whereas in the initial study, bidders might be primarily concerned with the comparative paths of each auction, in the presence of a BIN option, they can be expected to make some sort of forecast about likely future prices. In the case of actual BIN auctions with multiple bidders, it might be expected that individual bidders may even form views about the likelihood of other active bidders triggering the BIN option; that aspect was not relevant in this simulation.

Predictor variables were based upon the specific characteristics of the auctions (two fast auctions were paired, the equivalent of condition 3 in the study described above). In order to capture the aspect of bidders' assessments of likely auction ending prices in the event that the

BIN option was not triggered, simple forecasts were computed at each point in time using all available price versus time data to date, with the parameters of the forecast then used to estimate the ending prices for the two auctions. A simple linear forecast was calculated in the form;  $a + bx$  where  $a = \bar{y} - b\bar{x}$  and  $b = (\Sigma(x - \bar{x})(y - \bar{y})) / \Sigma(x - \bar{x})^2$ ,  $\bar{x}$  and  $\bar{y}$  denoting the sample means. Since forecasts were calculated on the basis of all available information-to-date, updated forecasts were recalculated whenever new pricing information within the auctions became available. Forecasts were therefore both time and bid dependent. A major simplifying assumption was then that all bidders applied identical forecasts. The full list of predictor variables is as follows;

*F.Gap*; defined as the difference between the highest forecast ending price across the two auctions and the prevailing BIN price;

*Cum Switches*; the cumulative number of switches undertaken by individual bidders at each point in time;

*Cum Placed Bids*; the total number of bids placed by individual bidders at each point in time;

*Cum Bid Intensity*; the total number of proxy “competitor” (computer-generated) bids at each point in time.

The full set of predictor variable data was derived from relevant values of each parameter measured each second; a minimum elapsed time of 25 seconds was required before any forecasts were calculated. The data applicable for each participant depended upon whether

or not they chose to execute the BIN option. In the event that this option was activated, data for that bidder would only reflect available information up to the time that the BIN option was triggered. In the event that the BIN option was not triggered, full data was compiled for the period over which the BIN option remained available. The overall approach is somewhat similar to that employed by Anderson, Friedman and Singh (2008) to the extent that an attempt has been made to distinguish between auctions based upon characteristics and changing dynamics of the auctions. However, the dataset is clearly much more limited and other characteristics of the auctions, such as duration, participation costs and item are deliberately equalised between the two auctions.

The hypotheses relate to the effect of key predictor variables on the output variable (exercising the BIN option).

1. The likelihood of exercising the BIN option will increase as the differential between the forecast ending level of the competitive auctions approaches the BIN price. The BIN option provides a risk-free exit strategy in comparison with the uncertain outcome which results from remaining in the auction. Therefore, rational bidders are likely to evaluate the BIN price relative to expectations regarding closing prices within a standard auction format.
2. The likelihood of exercising the BIN option will increase as the frequency of bids intensifies within the paired auctions. Bidders are likely to attach degrees of uncertainty to their forecasts of final auction outcomes as one of the key determinants, the frequency of competitive bids, is also an unknown. Therefore, evidence that bidding intensity is increasing may tend to increase the uncertainty around bidder forecasts.

### **6.4.1. Participants**

A total of 60 participants, allocated equally and at random across two conditions, took part in the study. As with the first study, a pair of simultaneous auctions of identical items was run with each auction ending at the same time. The auctions were both “fast”, identical to condition 3 of the initial study. The first auction in the pair contained a BIN offer. By gender, 31 of the participants were male with 29 females taking part. Ages ranged from 21 years to 62 years ( $M = 33.40$ ,  $SD = 9.36$ ). Each participant was assigned a unique user id and had no prior knowledge of the item to be offered through the simulated auctions. The auctions proceeded in a manner identical to that described in the first experiment.

### **6.4.2. Design**

The same simulated online auction software platform was used, the only difference being the availability of a “Buy It Now” button in the first auction. The same interactive user data was logged in real time following the same procedures as before with the timing of any BIN execution also being logged and stored in the dynamic data files. At the end of the simulated auctions, all of the captured interactive data was again downloaded to a master file named in accordance with the specific user id and saved to the hard drive of the computer for subsequent analysis.

### **6.4.3. Method**

This experiment introduced a BIN option into the first auction (Appendix F). The BIN option was available from the start of the auction and offered participants the chance to purchase the item at a declared, fixed price. Selecting the BIN option ended the competitive nature of the auction and provided certainty with regard to acquiring the item concerned. The initial BIN price was set at £2500, well above the opening bid levels of each auction. As the auctions progressed, the BIN price increased periodically by increments of £150. A total of three increments were applied so that the BIN price increased from its initial level to a final level of £2950. Participants were thus exposed to opportunity costs and risks. Not only did they have to assess the BIN option against the anticipated outcome of the competitive auctions if this option were not triggered, they also had to speculate on whether, and for how long, the BIN price would remain available and unchanged. The BIN options expired before the end of the auction with participants receiving on-screen notification of this impending expiry 20 seconds before the event, with an on-screen 'clock' counting down the seconds to expiry. Thus, participants had to make a final decision on whether to accept this fixed price purchase or carry on with the competitive auction in the normal way.

#### **6.4.4. General Overview of the Results**

In aggregate across all participants, a total of 801 clicks were generated. The average number of clicks per participant (26.7) was similar to that of condition 3 in the first study (27.47). Note, however, that the average duration of auctions with a BIN option included in the pair was 236.18 seconds as BIN selections ended the relevant auction at that time. Switching activity accounted for 649 clicks (81.02% of total clicks), a somewhat higher percentage than the 77.31% found to condition 3

Excluding BIN selections, 114 user bids were initiated 18 of which failed due to higher intervening computer bids (12.50% of total submitted bids). Including any BIN options, 9 participants made 3 or more bids with 5 making 5 or more. Overall, 19 participants selected the BIN option (63.33% of participants), 10 making that decision within the last 20 seconds of the BIN options availability. This was, of course, the timeframe for which an on-screen message informed participants that the BIN option was coming to an end. The majority of participants therefore selected the BIN option at its highest level. Overall, no participants selected BIN at its introductory level of £2500; two triggered it at £2650 with the remaining five buying at £2800. Of the participants choosing the BIN option, only 5 made active auction bids prior to selecting BIN. There was a total of 38 History Views (summoning a table showing the bid history of the currently watched auction to date), 25 of which were triggered by participants who subsequently selected the BIN option.

#### **6.4.5. Statistical Test Results**

Logistic regression was used to test the likelihood of BIN selection given predictor variables which captured certain dynamic characteristics of the auctions. The results are shown in Table 16. As before, the goodness of fit statistics compare an independent (constant-only) model, in which the linear combination of explanatory variables reduces to zero, with the full model, including the effect of the IVs.

The Cox and Snell and Nagelkerke  $R^2$  measures indicate that the full model had significant, but modest explanatory power. As a result, the null hypothesis (that the coefficients of the IVs are zero) can be rejected. The Akaike Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (BIC) shows mixed results, AIC indicting some

improvement in terms of model quality while BIC suggesting that better specification may be possible.

Table 16. Results analysis of BIN likelihood

Summary of goodness of fit statistics

Statistic	Independent (Constant only)	Full Model
Observations	6376	6376
Sum of weights	6376.00	6376.00
DF	6375	6371
-2 Log(Likelihood)	247.32	213.87
R <sup>2</sup> (Cox and Snell)	0.00	0.13
R <sup>2</sup> (Nagelkerke)	0.00	0.14
AIC	249.32	223.87
SBC	256.08	257.67

Confidence tests

Statistic	DF	Chi-square	Pr > Chi <sup>2</sup>
-2 Log(Likelihood)	4	33.45	< 0.0001
Score	4	39.15	< 0.0001
Wald	4	21.74	0.000

Model parameters for the components

Source	Value	Standard error	Wald Chi- Square	Pr > Chi <sup>2</sup>
Intercept	-9.31	2.82	10.94	0.001
F.Gap	0.00	0.01	0.04	0.846
Cum Switches	-0.05	0.05	1.16	0.282
Cum Placed Bids	0.32	0.15	4.38	0.036
Cum Bid Intensity	0.17	0.08	5.12	0.024

The confidence tests assess the extent to which the full model maximizes the value of the likelihood function which makes the observed data most likely given the model's parameters. Higher values for each of the measures suggest a greater degree of confidence. Each of the test statistics: -2 Log -Likelihood ( $\chi^2(4, N=6376) = 33.59$ ,  $p < .0001$ ); Score ( $\chi^2(4, N=6376) = 39.26$ ,  $p < .0001$ ) and Wald ( $\chi^2(4, N=6376) = 21.75$ ,  $p < .0001$ ) is highly

significant, denoting that the IVs, in combination, significantly enhance the explanatory power of the full model.

The regression model coefficients indicate the extent to which the predictor variables increase the likelihood of BIN being triggered. Individual coefficients can then be interpreted as log-transformed probabilities measuring the expected change in the log odds of the output variable taking the value of 1 (the reference event, defined as the BIN option being triggered) given a unit increase in one predictor variable holding all other variables constant. Positive coefficients attached to a predictive variable therefore indicate that the probability of the reference event increases as the value of the predictive variable rises.

Two of the predictor variables are significant at the 5% level; *Cum Placed Bids* and *Cum Bid Intensity*; the former measures the cumulative (non-BIN) bids placed by a participant while the latter measures the cumulative frequency of all bids (including computer-generated, notion “competitor” bids). Consequently, there is no evidence to support the first hypothesis that the relationship between the prevailing BIN price and the measure used to forecast auction ending values has any impact on the likelihood of the BIN option being triggered. However, the results do support the hypothesis that changes in bidding intensity affect the likelihood of BIN being triggered. Switching behaviour again appears to carry no particular significance with regard to BIN preference and is, presumably, more a functional element of the complete bidding strategy. Overall, therefore, while the measure used here to forecast outcomes of the standard auctions did nothing to increase the likelihood of the BIN option being triggered, evidence that changes in bidding intensity are significant holds the possibility that rational bidders indeed make some evaluation of likely outcome when deciding upon strategy. It may also be relevant to note again, therefore, that the majority of BIN events



were triggered in the period just before the removal of that option, thereby taking away the risk-free exit and acquisition strategy.

## **6.5. Discussion**

Two studies were conducted using a simulated platform to gather data relating to behaviours of bidders in notional auctions. Mouse tracking was used to record all significant interactions of study participants with the software. The simulated environment, structured in this way, made it possible to examine important aspects such as switching behaviour independent of any particular bidding strategy which might be adopted. This then made it possible to examine other factors which influence that switching.

There are inevitably a number of limitations associated with the studies. Primary amongst them is the fact that these were not real auctions, no bids had any monetary consequence and no items could be acquired as a result of a successful bidding campaign. As a result, any conclusions must be treated with a degree of caution. While participants were asked to behave as if the auctions were real, there can be no guarantee that this outcome was achieved and it remains possible that individuals may have behaved very differently in this study from the way they might ordinarily behave in a “live” auction with consequence. Similarly, the simulated environment used pre-programmed computer bids to add a degree of authenticity. As a result, there was no genuine competition between bidders.

In addition, the studies were somewhat limited in scope. Running pairs of short-duration auctions which overlap exactly in terms of time period would be something of a special case. For popular items sold on major online platforms, the period of overlap may not prove too restrictive; there are, after all, likely to be several live simultaneous auctions with

approximately the same ending times. However, the restriction of allowing participants to consider just two possible auctions is somewhat limiting. Other studies have shown that, while multiple overlapping auctions of close-to-identical items appear to create an integrated marketplace with high levels of competition between sellers, buyers and items themselves, a necessary condition for efficient outcomes for buyers and sellers, numerous frictions arise which in fact result in a degree of inefficiency. Much of this appears to be associated with participation costs which appear to create a degree of inertia. Multiple contemporaneous auctions appear to require widespread information dissemination in order for them to be efficient as it is that which enables rational bidders to identify relatively “cheap” auctions. The current studies had no such constraints; participation costs were extremely low, largely restricted to the minimal time cost associated with switching from one auction to the other. Indeed, it could well be that the opportunity cost to a rational bidder seeking to acquire the item at the lowest available price is greater than any information search costs of the type mentioned here.

There are further aspects of fixed duration auctions which can complicate certain types of analysis. In essence, the outcome of a standard auction with no fixed-price exit option is determined by a single event; the placing of the final, winning bid. Strictly, there is no requirement for any bidder to participate in any way before that point is reached. It is therefore possible that many aspects of a bidder’s behaviour might remain undisclosed and undetected. The studies reported here did, of course, try to partly address this aspect via mouse tracking. Indeed, by capturing events other than bidding, the data did show that there was a degree of interaction throughout the auctions. This could itself have been aided by the very short duration of the auctions adding to the focus of participants.

Notwithstanding, a number of findings derived from the studies described above support evidence from other empirical studies. In particular, the studies presented here are supportive of the proposition that bidders behave as if they are utility maximisers, qualified to the extent that this is found to be the case when participation costs are sufficiently low. Therefore, bidders appear to seek and demand certain information in order to inform their decisions and strategies. Consequently, while auction markets may be considered to be fairly well-defined environments in terms of understanding processes and parameters, developing effective strategies is perhaps a somewhat more complicated task. Therefore, a typical bidder in an auction may be considered to face certain probabilities and uncertainties which vary over time. To the extent that they operate across multiple auctions, they face an array of such probabilities. The efficiency of the decision-making process is then likely to be at least a partial function of the availability of and access to critical, relevant and timely information, echoing Rand and Jank (2013).

In the next chapter, we examine aspects of behaviour in relation to financial markets. This is the prelude to the final empirical study presented in Chapter 8.

## CHAPTER 7

### THE BEHAVIOURAL ANATOMY OF FINANCIAL MARKETS

#### 7.1. Efficient Markets and Observed Market Price Behaviour

Standard theories of market efficiency, such as CAPM and APT, assume that all decision-makers hold rational expectations with regard to asset prices. Therefore, in the strongest form of the Efficient Markets Hypothesis (EMH), all available relevant information is deemed to be known by all decision-makers and reflected fully in current prices as a result (Shiller, 2003). Rational investors are therefore considered to be highly efficient discounters of information, agreeing upon its value in terms of price impact. This is a necessary condition for equilibrium as it ensures that all securities are perceived as being “fairly priced” by all agents at all times (Malkiel, 2005). Consequently, there is no possibility for speculative gain based upon knowledge alone and specific trading strategies can be profitable only on the basis of luck.

We can easily express the basic principles of market efficiency as follows: Assume the rate of return for any asset is defined as;  $R = (P_{t+1} - P_t + C_{t,t+1}) / P_t$ , where  $P_{t+1} - P_t$  represents the capital gain over the period from  $t$  to  $t + 1$  and  $C_{t,t+1}$  is any cash flow (for example, dividends) received over the same period. At time  $t$ , investors hold rational expectations regarding future prices and cash flows, giving;  $R^e = (P^e_{t+1} - P_t + C^e) / P_t$ . Such expectations reflect unbiased, optimal forecasts incorporating all available information, including relevant fundamentals affecting an asset’s return components. Allowing expected cash flows to distil into forward prices implies;  $P^e_{t+1} = P^F_{t+1} \rightarrow R^e = R^F$ . This holds if agents are homogenous in their views and are aware of the rational views of all other agents. From this, we are able to

define market equilibrium,  $R^{\mu}$ , as the state in which all assets are efficiently priced such that;  
 $R^F = R^{\mu}$ .

Arbitrage conditions exist when  $R^F \neq R^{\mu}$ . Therefore, when;  $R^F > R^{\mu} \rightarrow P_t \uparrow \rightarrow R^F \downarrow$ , the forecast rate of return over and above the equilibrium rate leads to an increase in the price of the asset triggering a reduction in the expected rate of return until convergence is achieved. The reverse applies in the case of forecast rates of return below the prevailing equilibrium;  $R^F < R^{\mu} \rightarrow P_t \downarrow \rightarrow R^F \uparrow$ . In this manner, potential profit opportunities are removed entirely, assuming zero-cost adjustment mechanisms. “Noise” traders have the ability to create short-term price distortions away from equilibrium thereby providing rational agents with further opportunities for arbitrage. As the term implies, noise traders hold erroneous beliefs, trading on the basis of random signals or data which have no information value in terms of price formation; they are therefore irrational components of the market. Since noise traders systematically incur losses rather than gains as rational traders intervene to correct their errors, they are expected to be eliminated from the market over time.

Should the EMH be an accurate description of market dynamics, actual trading volumes would be extremely low as prices adjust to changes in rational expectations conditioned in most cases by new information. It is this discounting mechanism, along with homogeneity of views, which instantly determines prices meaning that there is no requirement for new equilibria to be established as a result of physical transactions; the instant adjustment therefore eliminates any actual opportunity for arbitrage as no trading volume takes place. Note that within this framework, prices are determined solely by forward-looking expectations; the past is therefore no guide to the future. As a result, all asset prices must follow a *random walk* devoid of serial correlation since new information is, by definition,

unknown to any market participant beforehand (Fama, Fisher, Jensen & Roll, 1969; Schwert, 2003).

In fact, observed behaviour of asset prices in financial markets is very different from that implied by EMH. As opposed to low volume and constrained price volatility, recorded trading volumes and price volatility are both unusually high given the level of information flow (Shiller, 1981). In addition, the distribution of asset prices reveals the presence of *fat tails* with large leptokurtosis as opposed to the normal return distribution assumed by models of market efficiency. A fat-tailed distribution has a much greater likelihood of extreme events than would be the case under a normal distribution; for example, while a three-standard deviation event only has a 0.3% chance of occurring on the basis of a normal distribution, the magnitude of fat tails is much more unpredictable and can accommodate extreme events ( $> 3SD$ ) which occur empirically with much greater frequency than would be expected under EMH, as evidenced by numerous market bubbles and crashes (Veldkamp, 2005).

Contrary to the assumption of random walks, serial correlation has also been observed in stock price data, implying that past price behaviour may carry some information about future short-term returns (Lo & MacKinlay, 2011). Such anomalies have been found in relation to both short- and long-term returns, with longer time frames exhibiting negative serial correlation, or mean reversion, the reverse of the positive serial correlations (continuation of trend) found with regard to shorter-term movements. Such anomalies appear to be significant; Chaudhuri and Wu (2003), for example, found that between 25% and 40% of the variability in long-term returns could be predicted on the basis of mean reverting behaviour. Strong evidence of mean reversion after significant declines has also been found (Bali, Demirtas &

Levy, 2008), with markets tending to recover to their longer-term trends after significant declines. This suggests that investors may both under- and over-react to events or news, challenging the proposition that all information is efficiently reflected in prices (Dhar & Kumar, 2001).

Further observed price-related anomalies have been found in relation to volatility clustering and correlations between trading volume and general price volatility. Volatility clustering implies that large changes in prices tend to be followed by further large changes while small changes tend to be followed by other small changes (Mandelbrot, 1971). Therefore, absolute returns (and their squares) have been found to display significant positive, slowly decaying autocorrelation over timeframes ranging from a few minutes to several weeks, therefore;  $\text{corr}(|r_t|, |r_{t+\tau}|) > 0$ . Trading volume has also been found to be positively correlated with the level of price volatility implying “long memory” behaviour (Lobato & Velasco, 2000).

The proposition of rapid assimilation and full discounting of new information into stock prices has also been tested empirically. In that regard, certain stock-specific traits have been found to influence short-term returns, with some factors persisting over time. Taylor (1999) undertook a study examining return trends, persistence and sensitivity using data on over 1,000 UK-listed equities. The study investigated a number of stock-specific factors derived from detailed earnings forecast data provided by independent equity analysts at brokerage firms as well as fundamental valuation data (such as price-to-earnings ratios, price-to-book values and dividend yields). The study found evidence of the assimilation of earnings revision data into stock prices to the extent that positive and negative revisions to mean estimates resulted in systematic impacts on stock prices. Consistent with loss and risk aversion, the

negative effect of downward revisions of earnings estimates was found to be cumulatively greater than the positive effect from equivalent upward revisions. The intensity of analyst revision activity (percentage of analysts revising their estimates upwards or downwards over the immediately prior period) was also found to affect stock prices in the direction of the revisions, implying that investors were also influenced by the sheer number of analysts making revisions independent of the magnitude of any changes in estimates. These findings support the proposition that investors react to and assimilate relevant news. However, the study found that, while positive and negative earnings ‘surprises’ (defined as reported earnings outcomes more than two standard deviations away from prior average forecasts) tended to have a fairly immediate impact on stock prices in the direction of the surprise, prompting a jump in short-term price volatility, the full effect of more constrained earnings revisions appeared to be realised over a longer period with no evidence of a significant immediate bias. This is consistent with the findings of others regarding under-reaction to new data (Ball & Bartov, 1996; Brennan, Chordia & Subrahmanyam, 1998; Fama & French, 1996), suggesting that ongoing news is absorbed into stock prices incrementally over time, perhaps as more investors become aware of the latest changes in expectations of others. With regard to stock valuation metrics, Taylor found evidence of longer-term cyclicity in terms of returns to such factors. This is consistent with the anomalous “value effect” observed by others (Fama & French, 1996).

## **7.2. “Noise” Trading, Limits to Arbitrage and Non-Homogenous Expectations**

Deviations of observed market behaviour from that implied by models of market efficiency have led to the development of alternative behavioural models relaxing some of the core assumptions of EMH. We have already noted that, under EMH, noise traders should



quickly be eliminated from the market as they hold erroneous beliefs which can be exploited by rational agents. This proposition rests upon the key assumptions of homogeneity with regard to rational beliefs and cost-free arbitrage. In practice, limits to arbitrage can exist for a number of reasons (Gromb & Vayanos, 2010). For example, since actual trading involves costs, this may place a limit on the ability or willingness of rational agents to arbitrage away all pricing inefficiencies. In addition, should heterogeneity, rather than homogeneity, of views prevail, pricing disequilibria may persist, meaning that arbitrage is no longer a risk-free proposition.

Heterogeneity of views was assumed by Arthur *et al* (1996) in order to develop a model in which expectations were derived from inductive rather than deductive reasoning (as assumed by EMH). This can be explained by expressing an equilibrium price at time  $t$  for a security with stochastic return components (dividends and price) incorporating all available information,  $I_t$ , as;

$$P_t = \beta \sum_i w_{i,t} (E_i [D_{t+1} | I_t] + E_i [P_{t+1} | I_t]) \quad \text{Equation 32}$$

where  $\sum_i w_{i,t}$  defines the weighted average of agents' expectations discounted by the common factor,  $\beta = 1/(1 + r)$ .

The stochastic element of expectations is represented by the conditional variance of combined expectations,  $\sigma_{j,t}^2$ , given  $I_t$ . The weighting function,  $w_{i,t} = (1/\sigma_{j,t}^2) / \sum_k 1/\sigma_{k,t}^2$  can then be thought of as the degree of "confidence" assigned to agent  $i$ 's forecast relative to those of all other agents. In the event of homogeneity, expectations for future dividends and prices are identical, reducing the weighting function to  $1/N$ . Since arbitrage resolves all pricing

inefficiencies and dividend forecasts are by definition unbiased, Diba and Grossman (1988) show that Equation 31 can be reduced to;

$$P_t = \beta^k \sum_{k=1}^{\infty} E [D_{t+k} | I_t] \quad \text{Equation 33}$$

The current price thus derives from the unbiased estimates with regard to dividends while fluctuations in price occur as a result of changes in information. The deductive element of EMH stems from the fact that rational behaviour with homogeneity of views and knowledge of the rational views of all other agents implies that decision-makers can, in theory, logically calculate equilibrium prices. Therefore, under EMH, market efficiency can be wholly resolved even subject to limits to arbitrage based upon cost alone.

Where agents hold heterogeneous views, there can be a range of expectations with regard to the key variables which determine prices and also differing views relating to the impact of information on prices. In such an environment, arbitrage opportunities may exist but may not be acted upon due to perceptions of risk over and above costs. Once limits to arbitrage are assumed, the existence of noise traders allows the possibility that pricing inefficiencies may persist allowing trading based upon such anomalies to be deemed rational. Rational traders might then trade in a particular manner on the basis of the irrational, yet non-random, behaviour of others. Therefore, should noise traders act on the assumption that a trend in a particular stock will persist, rational agents may exploit this by themselves investing in the same security, thereby participating in, and extending, that trend. Rational traders therefore still exploit inefficiencies created by others but may not always do so in a manner which promotes market efficiency. Since there is no longer a mechanism for ensuring the timely

reversion of securities prices to “fair value”, the potential exists to generate excess risk-adjusted returns (Schleifer & Summers, 1990).

Under conditions of heterogeneity, the treatment of information relevant to price outcomes is no longer common to all agents but may reflect past prices, cash flows, past earnings, volumes, volatility and a wide range of fundamental indicators. Heterogeneity therefore allows for the possibility that each agent applies different weightings to some or all of the information when deriving their private expectations. Consequently, there is no coordinated model of expectations, nor is it possible for one agent to discern the expectations of others. This results in a lack of consensus with regard to  $P_{t+1}$ . From the perspective of a single agent, an assessment of current fair value is therefore a function not only of his own expectations but also his expectations with regard to the views of all other agents. Therefore, agent  $i$ 's expectations relating to  $P_{t+1}$  would be formed as follows;

$$E_i[P_{t+1} | I_t] = \beta E_i [ \sum_j \{ w_{i,t+1} (E_j [D_{t+2} | I_t] + E_j [P_{t+2} | I_t]) \} | I_t ] \quad \text{Equation 34}$$

Since current expectations with regard to  $P_{t+1}$  must include a forward looking element to  $P_{t+2}$  when heterogeneity prevails, agents must make yet more assumptions about the views and weightings of other agents. This process is wholly inductive as there is no logical way to weight the unknown subjective views of others in order to derive a coherent rational price expectation.

### 7.3. Financial Market Simulation & Agent-Based Models

The role which psychology plays in shaping asset prices in markets has been explored using agent-based model simulations. These approaches allow for interaction between agents to be examined using different rules-based systems. Heterogeneity of views is a central assumption of such models which, in the context of financial markets, have frequently sought to explain *stylised facts*, or observed behavioural anomalies, to the extent that they can be replicated by agent interaction within simulated markets.

While numerous agent-based models have been proposed, most share common features (for a comprehensive review, see LeBaron, 2006). Central to all models are agent preferences since it is through such preferences that potential interactions are defined. In theory, any behavioural biases can be assigned to categories of agents; it is also possible to allow agents to switch between strategies based upon cues. Hurdles or triggers for activity must be specified within the models which then determine buying and selling activity, the net effect of which is reflected in price changes. Agents are therefore assumed to have in-built propensities to trade often based upon a weighted interpretation of feedback in the form of information. In order to enable the simulated models to operate with a degree of realism, methods of clearing or matching trades must be accommodated. For example, Day and Huang (1990) generated buy and sell orders subsequent to the declaration of prices issued by market-makers. Trades on each side (purchases and sales) were then aggregated; when there was excess demand, the underlying asset price increased, falling in the case of excess supply. The magnitude of price changes in such models is often proportional to the level of excess demand or supply, for example;

$$P_{t+1} = P_t + \lambda(B(P_t) - S(P_t))$$

Equation 35

Perhaps the most realistic price discovery mechanism was provided by Farmer, Patelli and Zovko (2005) whose model allowed agents to post bids and offers which could then be matched with all other agents in the system allowing a greater chance of clearing trades at prevailing prices.

A number of agent-based models have been structured on the basis of a small number of strategies which agents use to trade risky assets. For example, Alfi, Cristelli, Pietronero & Zaccaria (2009) considered a model comprising  $N$  interacting agents divided into *fundamentalist* and *chartist* investors. The former were assumed to evaluate opportunities on the basis of fundamental “fair value”, broadly consistent with the typical behaviour of long-term, large institutional investors in actual financial markets, while the latter were short-term traders searching for emerging trends in price patterns; such agents being wholly reliant upon manipulations of historical price data. The two categories of agents essentially exert market influence in opposite directions. *Fundamentalist* agents are deemed to provide a stabilising influence within the market as they seek to increase overall efficiency by correcting price divergences away from fair value. Formally, these traders seek to move all risky assets towards their perception of a fair fundamental price ( $p_f$ ). The behaviour of *fundamentalist* traders can then be described by the following stochastic equation, which assumes an underlying random walk modified by the strength of action by agents;

$$p(t + 1) = p(t) + \lambda(p_f - p(t)) + \sigma\varepsilon(t)$$

Equation 36

where;  $\lambda$  measures the strength of action exerted on  $p(t)$  by *fundamentalist* agent;  $\varepsilon$  is residual noise with amplitude  $\sigma$ .

*Chartist* traders use only price information to derive expectations. In this case, they are assumed to pursue a simple strategy based upon trends which are detected and defined by the degree to which a current price ( $p_t$ ) diverges from a moving average computed using a range of previously observed prices. These traders employ the common expectation that future prices will continue to move further away from the moving average, whether in a positive or negative direction. The following stochastic equation can be used to describe *chartist* behaviour, again expressed as a random walk modified by a “force” centred upon the difference between  $p_t$  and the moving average;

$$p(t+1) = p(t) + \frac{b}{(M-1)} F(p(t) - p_M(t)) + \sigma \varepsilon(t) \quad \text{Equation 37}$$

where:  $p_M(t) = \frac{1}{M} \sum_{i=t}^M p(i)$  defines the moving average calculated over the time interval  $M$ ;  $b$  is a measure of the strength of the “force”.

As opposed to the stabilising influence of *fundamentalist* traders, the actions of *chartists* are essentially destabilising to the extent that they tend to move prices away from fundamental fair value in a persistent manner contributing to greater disequilibrium. Such behaviour has the potential to create extended directional moves in prices perhaps, leading to bubbles and crashes in the extremes.

Simulated market evolution under this model is sensitive to the relative number of *fundamentalist* ( $N_f$ ) to *chartists* ( $N_c$ ) agents as that determines their collective impacts subject to propensities to trade. Alfi *et al* allowed agents to switch from one category to the other during the simulation adding a further degree of realism. The probability of change was based upon two factors: a herding element and a “consideration” of price. The first component is designed to capture observed behaviour whereby some agents appear to adopt

the strategies and actions of others; it was assumed that the likelihood of this effect would depend upon the relative number of agents in the two categories with agents more likely to switch to a larger category. The second component assessed switching probability on the basis of the price behaviour. The likelihood of switching from a *chartist* agent to a *fundamentalist* agent was assumed to be proportional to  $p(t) - p_F$ , while the likelihood of switching the other way, to become *chartist*, was proportional to;  $p(t) - p_M^{(t)}$ . In order to more accurately replicate actual market behaviour, switching was controlled in such a way that asymmetry between the agent categories was maintained, ensuring  $N_f > N_c$ .

Alfi *et al* ran a number of simulations varying  $N$  from a single agent to multiple heterogeneous agents. They found consistent evidence of fat tails and volatility clustering. In general, periods of high or low volatility reflected the relative weighting of agent styles with greater *chartist* influence triggering higher volatility. There was also evidence of positive autocorrelation with decay. Varying  $b$ , the strength of *chartist* action, amplified fat tails and the likelihood of bubbles and crashes when the impact of *chartist* agents increased. Similarly, volume/volatility correlations were found to increase coincident with transition rates towards *chartists*.

Many agent-based models have shown that they can generate anomalies of the type observed in actual markets, similar to the results of Alfi *et al* described above (see Hommes, 2006). However, in most cases, agents are assumed to behave as if they have no price impact as a result of their own actions and that these actions are taken independently of all other agents. In contrast, Chakrabarti and Roll (1999) developed an information acquisition model in which agents observe the actions of other large traders, adjusting their own beliefs accordingly. Thus, agents receive information signals each period and also feedback relating

to the trades of all other agents. This data is assumed to be combined and used to develop expectations of future returns to securities in accordance with Bayesian updating of probabilities.

Based upon a series of simulations with varying parameters, Chakrabarti and Roll found that price volatility increases with signal diversity but that the resultant price behaviour provides a better forecast of future value. Thus, while greater trading activity increases price volatility, the information content of prices appears to be improved. However, under certain simulated conditions, the process of learning through which agents modify their own beliefs based upon observed behaviour of others has a tendency to reduce volatility while at the same time increasing the accuracy of market price as a forecast of value. The greater overall efficiency of asset prices therefore reduces trading volumes due to smaller disparities between asset prices and fair value.

#### **7.4. Feedback, Information and Search Effort**

The evidence provided by agent-based models assuming heterogeneous agents subject to limits to arbitrage suggests that such frameworks can provide a useful basis from which to explore behavioural anomalies in financial markets. With noise trading able to affect asset prices in a systematic and persistent manner, the absence of an automatic short-term adjustment process restoring price equilibrium means that rational agents can adopt a range of strategies designed to exploit anomalies with a potential of generating excess risk-adjusted returns. In addition, by creating or contributing to general price disequilibria, noise traders themselves have the opportunity to realise gains as a result of their own concerted actions, as



opposed to the almost certain losses their irrational behaviour should realise in markets described solely by informational efficiency.

Noise traders are defined within efficient markets theories as agents who make decisions based upon data which should have no objective information value in relation to price discovery; they therefore operate on the basis of erroneous beliefs. Fully efficient markets require that current prices reflect all relevant information; it is from this that conditions for equilibrium can be derived. Therefore, past prices, old news, historical volatility, volume or any other such data should be of no value in predicting future prices as they are all fully discounted in current prices. As seen in the discussion of agent-based models, noise traders are usually described as agents whose actions are conditioned, in some cases solely, by signals derived from technical analysis (the study of historical price patterns). In all cases, expectations of such traders are predicated upon the assumption that prices exhibit serial correlation as opposed to being wholly random. Historic prices therefore represent essential feedback from which signals are taken which then trigger actions. Positive feedback traders bet upon a continuation of a trend while negative feedback traders take positions contrary to a trend on the assumption of mean reversion; the two therefore exert differing influences in the market, the latter tending to stabilise, while the former destabilise prices away from any equilibrium. Positive feedback trading is typically associated with overvaluation of stocks and, in extreme cases, bubbles (DeLong *et al*, 1990a). Both types of trading contribute to short-term serial correlation in stock prices when the influence of feedback agents is sufficiently high.

Fundamental investors are assumed to trade on the basis of information; data which is relevant to price discovery. The core assumption is that such traders derive a perception of

fair value with a belief that, over the long-term, prices will converge towards such fair values. Since agents are heterogeneous in their views, information acquisition and the subjective weighting of its importance can vary across agents. In addition, the presence of persistent noise trading affecting outcomes means that, while fundamental traders may apply anchors in terms of fair values, they may not act on evidence of price divergence due to the potential for trends to persist.

We have seen that models of market efficiency assume that all information is known to agents with no explicit consideration of search effort. For their part, agent-based models recognise differences in the type of information routinely used by traders, for example, *chartists* versus *fundamentalists*, but are also generally silent on the factors which may determine levels of search effort associated with its capture. For example, simulated models which allow switching between styles of heterogeneous agents generally assume that there are no limits to switching imposed by information costs or deficits.

Information search effort is often considered within the context of a trade-off between benefits and costs; benefits are expected to accrue in the form of better decisions while the costs relate to the time and cognitive effort which must be devoted to acquire relevant information. Therefore, in the consumer field, greater information search is often perceived as a risk-reducing strategy to the extent that a better-informed decision should reduce the risk of error (Kivetz & Simonson, 2000). However, despite generally easier access to information, particularly given the widespread availability of technology, it should not be assumed that more is always better than less. For example, within complex environments in which knowledgeable agents operate. Payne, Bettman and Johnson (1988), found that decision-makers often adapt to the environment from which information is acquired.

Therefore, in environments with extensive real-time data and news flow, agents appear to develop the belief that rapid decisions hold the key to success. In practice, more frequent feedback has been found to lead to worse performance due to placing greater emphasis upon recent data with a tendency to overreact to random noise (Gilovich, Vallone & Tversky, 1985).

In an extension of this proposition, Lurie & Swaminathan, (2009) empirically tested the effects of timeliness and frequency of information in rapidly changing decision-making environments. They found that decision-makers who received more frequent feedback generated lower performance than those with less frequent feedback, but only in high-variance environments. Recipients of more frequent feedback did however tend to place greater weight on recent information. Escalating costs of data acquisition in high-frequency feedback domains did nothing to temper information search behaviour, and therefore failed to lead to improved performance. Based upon process tracing methods, the frequency of feedback was found to not only affect the amount of information processed but also the manner in which it was processed. Thus, when the frequency was high, decision-makers were far less likely to place recent information in context, again overweighting the significance of the most recent information received. When feedback frequency was reduced, decision-makers were more likely to consider information from prior periods. In addition, decision-makers in high frequency feedback environments were found to be less selective in their processing of information.

While numerous studies have considered the supply of information, Vlastakis and Markellos (2012) examined both the supply and demand for information within financial markets as it related to the largest stocks traded on the NYSE. They found a significant

relationship between the search for information and risk aversion, particularly associating the increased effort with residual uncertainty regarding an event or its potential outcome; information search effort therefore increased systematically as investors sought to reduce uncertainty. Greater volatility was also explained on the basis of investor concern over the impact of new information on stock prices. The same relationship was found during periods of general market “distress”. A further analysis of demand for information was conducted by Dimpfl and Jank (2016) using data relating to retail investor internet searches for four major global indices (Dow Jones, FTSE, CAC and DAX). High levels of search activity were found to follow unusually high market volatility while heightened search activity also proved to be a reliable forecaster of future market volatility (bi-directional Granger causality). The findings provide further evidence of “long memory” characteristics of markets and support agent-based model findings relating to noise trading and volatility. In addition, they suggest that retail investors can significantly amplify noise trading effects which may even be predicted based upon information search activity. However, no direct examination of retail investor trading data following periods of high internet search activity was undertaken to substantiate this point. The research is consistent with the results of a study by Bank, Larch and Georg (2011) who found a causal relationship between increased internet search activity and higher trading activity and stock liquidity in Germany.

#### **7.4.1. Trading Propensity**

We have noted that one of the observed anomalies in financial markets is overall trading volume which appears to be excessive given the level of information flow and variability in investment style characteristics (Barber & Odean, 2000). Theoretical models of market efficiency imply low trading volume environments as prices rapidly absorb new information,

often in a manner not requiring physical trading to occur. In such environments, noise traders are needed to generate actual trading volume as rational agents take advantage of the mispricing brought about by irrational traders (Pfleiderer, 1984). As noted earlier in this chapter, markets with heterogeneous agents and limits to arbitrage are expected to generate higher trading volumes in comparison with EMH as rational investors again exploit the actions noise traders. However, in this case, rational investors may seek to exploit trends engineered by noise investors as opposed to simply seeking to restore equilibrium fair value prices as such arbitrage is no longer risk free. Within such markets, trading volume may therefore be influenced by the relative importance of noise traders versus fundamental agents.

Various behavioural biases have been suggested as explanations for excessive trading propensity. Thus, Odean (1999) has argued that the *disposition effect* and overconfidence affect trading levels, the former describing a process whereby investors prefer to sell positions with cumulative gains rather than losses. Investor's perceptions of their levels of competence have also been used to explain trading frequency (Graham, Harvey & Huang, 2009) with those assuming greater competence tending to trade more. A further important area of consideration relates to trading propensity as a function of investor style.

In the discussion relating to agent-based models, we noted that heterogeneous agents were assumed to apply different investment styles. In broad terms, noise traders are typically assumed to be short-term in nature, often relying upon an analysis of prior price behaviour in order to derive trading signals (*chartist*). Counterbalancing this are fundamental investors, often assumed to be institutional in nature, who analyse investment prospects on the basis of a concept of long-term value. From this, we would therefore expect each category of investor to respond to different signals. While *chartists* seek to exploit patterns in prices,

*fundamentalists* derive signals in part from price changes, as they affect relative value, and also new information. The actions of each type of agent can, of course, help to generate actionable signals for the other due to the transmission effect of changes in price.

Breaking down investment styles in this manner is very general in nature. In practice, there are many sub-categories which are useful descriptors of these broad classifications as they may affect investor behaviour differently. Therefore, within the *chartist* category, some investors may seek to identify emerging momentum in prices (the start of a trend which is likely to be sustained) while others may be contrarian, seeking “extended” stocks whose prices have diverged significantly from some threshold, often defined by a moving average of prices. Contrarian investors therefore expect trends to be reversed, a process often known as *mean reversion*. Many other strategies are possible based upon the identification of particular price patterns or characteristics (for a detailed overview, see Edwards, Magee & Bassetti, 2018). Within the *fundamentalist* category, a distinction might be drawn between growth and value investors or size preferences (small cap versus large cap, for example). These classifications may be further broken down based upon any specific primary sector focus or parameters, for example: technology stocks; cyclicals; high yield; private market value; low beta. The resulting portfolios can then be analysed objectively based upon “factor” models which describe stock-specific characteristics (Lee & Stefek, 2008).

There is empirical evidence that investor behaviour is conditioned by investor style. Froot and Teo (2008) found strong evidence that institutional investors adhere closely to their defined styles, reallocating assets within a particular style rather than diversifying into other styles. Style trading also has implications for underlying propensities to trade (Nagel, 2005). Adherence to a particular investment style requires investors to rebalance their portfolios to

the extent that the underlying characteristics of their holdings drift over time. Typically, styles where characteristics are relatively volatile should see more signals as a result of that volatility, potentially triggering greater trading volume; the same might apply to very narrowly defined styles. A distinction can again be drawn between short-term trading and long-term investing. The former implies that traders act upon signals which change materially over a relatively short timeframe. Conversely, the characteristics of stocks held by long-term investors are likely to be inherently more stable, meaning that new information affecting signal thresholds is likely to be less frequent. It might be noted, however, that the majority of trading styles take account of price movements to varying degrees. Therefore, even long-term value investors are likely to measure value at least in part by reference to price using such metrics as: price-to-earnings ratios; dividend yields (dividend per share as a percentage of price); market value (share price multiplied by shares outstanding) to book value per share, and so on. Clearly, in most cases the price component of these ratios will change much more frequently than the other components. However, under relatively stable market conditions, short-term changes in price alone are unlikely to dramatically change valuation perspectives. *Chartists* are expected to be more affected by short-term price changes since they are essentially trading on the basis of perceived patterns independent of general price levels. Short term price volatility, or even its absence, may even form part of the signal generation for *chartist* investors.

Blackburn, Goetzmann & Ukhov (2014) found evidence that investors do indeed vary their trading propensities depending upon style. Thus, value investors, who were generally contrarian in their asset allocations, tended to rebalance portfolios less frequently than growth investors who more actively pursued momentum as a strategy (defined according to positive rates of change in underlying factors such as earnings per share). Significantly, investors who

invested in both styles were found to adapt their trading behaviour according to each style component. The same adaptation to context was found in relation to risk appetite; therefore, investors with assets allocated to both equities and fixed income may exhibit risk-seeking behaviour with regard to equities while being risk averse in relation to the fixed income portfolio (Shiller, 1999). Within equity portfolios, investors have also been found to vary trading behaviour according to the nature of stocks held. Thus, investors displaying the disposition effect in relation to stocks bought on speculative grounds may not exhibit the same behaviour in relation to stocks purchased for yield (Grinblatt & Matti, 2001). While confirming the same relationship between style and trading propensity, Keim and Madhavan (1994) also found that the mechanism of trading varied according to style, with differing implications for price impact. With regard to investment style, systematic relationships between past excess return and trading decisions were found for both value and momentum style managers; value managers only buying stocks following a decline in price. A number of momentum investors were found to buy stocks which had, on average, experienced relatively strong price performance ( $> 6\%$ ) in the week prior to the trades. Other longer-term style managers showed no obvious effect with regard to prior price outcomes. Heterogeneity was found with styles. Thus, some *chartist* managers clearly followed contrarian rules, yet did not appear to be guided by positive excess returns. In some cases, asymmetries were found between buy and sell decisions. For example, some institutional investors who regularly bought stocks pursuant to price declines did not follow an equivalent rule when they sold. The reasons for such asymmetries were not clear. With regard to trade execution, large purchases by long-term investors were routinely spread over a longer time period, presumably to minimise market impact. However, sales of a similar size were executed more quickly, suggesting that the market impact of buying activity is considered to be greater than that of selling. However, when trading was motivated by information which might have a short



duration, such as is the case with *chartists* trading on short-term price signals, market orders were used extensively to execute the trades in full, regardless of any associated market impact.

Grinblatt and Keloharju (2001) analysed extensive data relating to individual and institutional investors in Finland in order to determine motivations for trading. Using multivariate analysis, the interactive effect of causal variables was studied with Logit regression then being employed to analyse buy and sell decisions. The disposition effect and tax-loss selling were found to be major determinants of propensity to sell with a strong tendency to retain holdings with cumulative losses in excess of 30%. Stocks exhibiting strong recent gains and with prices at or close to monthly highs were more likely to be sold. Negative past returns were found to affect trading propensity relatively more than positive past returns while a stock trading at a short-term (monthly) high or low tended to increase the likelihood of trading decisions in the case of contrarian or momentum styles. Within these broad findings, significant differences were found based upon the sophistication of the investor. Thus, contrarian behaviour was found to be most prevalent for private investors and non-profit organisations. Finance and insurance companies, along with other domestic groups considered to be more sophisticated, showed significantly less contrarian behaviour in response to short-term price changes. Foreign investors tended to contrast with the majority of domestic investors to the extent that they overwhelmingly tended to pursue momentum strategies.

### 7.4.2. The Herding Effect

The process of herding describes the aggregation of similar and correlated behaviour by a sufficient number of investors to create an observable trend in securities prices. The phenomenon has been identified in relation to significant events, such as market spikes or crashes where investors overwhelmingly execute trades in the same direction, creating direct evidence of cause and effect (Fenzl & Pelzmann, 2012; Persaud, 2000; Prechter 2001). Herding does not only manifest itself in relatively short-term moves in prices, however, as it can also be seen in longer term trends (Nofsinger & Sias, 1999; Shapira, Berman & Ben-Jacob 2014; Summers, 1986). It also appears to be a robust characteristic of market behaviour independent of the level of sophistication of market participants.

Empirical evidence for highly correlated behaviour at the individual trade level for retail (non-sophisticated) investors is provided by a study by Barber, Odean & Zhu (2009a). This study, examining actual trading data for US retail investors over a period from 1991 to 1999, found significant evidence of coordinated trading across individual investors. Thus, in any particular month, the majority of individual investors were found to buy and sell the same stocks. Serial correlation was also found to the extent that, in aggregate, if individual investors were net buyers of a particular stock in one month, they were likely to also be net buyers of the same stock in the following period. From the overall data, four broad characteristics of individual trading were identified: First, investors revealed a preference for buying stocks with strong historic returns. The same bias was evident with regard to sales, although the effect was more pronounced based upon short-term (one or two quarters) rather than long-term (up to 12 quarters) price behaviour. Thirdly, buying was found to be more concentrated in fewer stocks relative to selling activity. Finally, retail investors were found to

be significant net buyers of stocks with unusually high trading volume. In a further extensive study of tick data derived from small trades executed on behalf of retail investors, it was found that, over short periods, noise trading was responsible for pushing stock prices too high (Barber, Odean & Zhu, 2009b). The reverse was found in terms of selling impact. In both cases, the effects were subsequently found to reverse with temporarily low prices tending to recover whilst excessively high prices tended to drift back. In essence, these studies of retail investors suggest that past outcomes and trends are considered to be relevant to the future, implying that expectations of serial correlation or continuing directional trends in security returns are quite significant. Where this condition exists, a sunk cost effect also tends to manifest itself with decision-makers likely to add to positions where recent feedback (price performance) is positive. Such investors therefore appear to be heavily dependent upon feedback which then informs current decisions which are not then simply forward-looking expectations independent of what has gone before.

The observed coordinated behaviour of individual investors in terms of buying and selling activity could be caused by numerous factors. Quite possibly, retail investors might be exposed to similar, relatively limited sources of information, including buy and sell recommendations of certain brokers' analysts, wealth advisors or 'tip sheets' upon which these investors may be unusually reliant. In addition, the development of online resources and forums may again lead to a general homogeneity of view as the same information and opinions are widely disseminated and are likely to fill a high percentage of the available attention span of non-full-time investors (Sprenger, Tumasjan, Sandner & Welp, 2014). Beyond such information and data sources, retail investors may also develop their own style biases and preferences, such as growth versus value. Again, these may be influenced by any peer interaction via forums or other core sources of information. Retail investors may also

feel less constrained by benchmark comparisons such that their performance versus a recognised index such as the S&P 500 becomes less important in the short term. As a result, they may be more prone to select investments purely on a stock-by-stock basis, maybe influenced by apparently attractive ‘stories’ (Barber & Odean, 2008; West, 1998). The clustering of trading activity in the proximity of new recommendations may indicate a perception of a cost associated with delay (an assumption that it is better to ‘get in early’). This implies an expectation of a herding effect which rewards investors who respond immediately to new recommendations ahead of the majority of other investors who may delay. Such behaviour is likely to be encouraged by those selling their recommendations or who benefit from the subsequent placing of trades (brokerage firms); it may also be learned behaviour on the part of investors to the extent that they see or experience short-term movements in share prices subsequent to new recommendations. The representativeness heuristic, status quo effects and familiarity bias may therefore explain this process and also help to explain the observed tendency to buy past winners. Attention bias may also be a significant factor as stocks which have generated high historic returns tend to be more newsworthy and attract additional coverage and comment in the financial media. Such strong attention effects have indeed been identified empirically (Barber & Odean, 2008). In essence, therefore, non-professional investors may have access to restricted sources of information upon which they rely. As a consequence, they may be less likely, and may feel less able, to investigate alternatives outside of these sources. The presumed lack of sophistication and expert knowledge of non-professional investors suggests that they base decisions upon inputs received from those who are considered to possess some relevant expertise (Kaustia & Knüpfner, 2012). This expertise may be perceived based upon reputation, claims or past experience.

Herding has also been found to be prevalent in the more sophisticated institutional arena with studies confirming an association between net changes in institutional share ownership and stock returns over the same period (Grinblatt & Titman, 1989; Grinblatt, Titman & Wermers, 1995; Hsieh, 2013; Jones, Lee & Weis, 1999; Sias & Starks, 1997; Walter & Moritz Weber, 2006; Wermers, 1999, 2000; Wylie, 2005). Information cascades, whereby investors infer information content on the basis of the trades of others, have been proposed as a basis for this behaviour; essentially, some investors may believe that perceived competitors have an information edge in a particular area and therefore mimic their behaviour (Alevy, Haigh & List, 2007; Banerjee, 1992; Ghashghaie, Breymann, Peinke, Talkner & Dodge, 1996; Watts, 2002). This effect has again been suggested as a contributory factor in market bubbles and crashes (Orléan, 1989; Topol, 1991; Lux, 1995). Institutional investors have also been found to interpret information in a similar manner, in effect adopting the same signals and responses (Froot, Scharfstein & Stein, 1992; Gleason, Mathur & Peterson, 2004; Oehler & Chao, 2000). Within the mutual fund industry, a word-of-mouth process has been hypothesised to the extent that fund managers appear more likely to buy, sell or hold a stock if other managers in the same locality are executing similar trades (Hong, Kubik & Stein, 2005). While the authors attribute this to word-of-mouth dissemination, either through investment managers intermingling or via information leakage from other agents, it could also be caused by reliance upon the same primary data sources, such as regional brokerage houses. Thus, to the extent that the same managers have a tendency to pay attention to the output of local brokerage research and analyst opinion, there is clearly scope for unintended mimicking behaviour, similar to that seen in the retail environment. Major brokerage houses, in turn, have a tendency to pass their most timely information on to their largest clients first, with some evidence that this creates a positive performance bias for larger clients in comparison with smaller clients, who receive the same price sensitive information later (Fong, Gallagher,

Gardner & Swan, 2004). This may encourage smaller managers to pay closer attention to recent trades by their larger counterparts. Similarly, in the mutual fund arena, Fong *et al* (*op cit*) found positive evidence of leadership and follower relationships whereby trades undertaken by the best performing managers appeared to be scrutinised more closely by others, leading to subsequent mimicking behaviour.

The potential for negative reputation outcomes provides a further possible behavioural explanation for herding. Thus, certain investors may wish to avoid being seen as contrarian (not following the herd or consensus), particularly if those decisions become associated with poor performance (Boyson, 2010; Choi & Sias, 2009; Scharfstein, & Stein, 1990). Pomorski (2006), in a similar investigation, found that mutual funds with the worst recent historical performance tended to mimic the best funds more than average return funds. Also, small and newer funds tended to mimic more than established funds.

Herding behaviour in the institutional arena may also to be a consequence of style bias. Thus, managers who employ a particular, well-defined investment style create biases in terms of the types of securities they select; managers of similar styles therefore have a tendency to select from the same sub-group of stocks (Falkenstein, 1996; Lobao & Serra, 2002; Mohamed, Bellando, Ringuedé & Vaubourg, 2011; Oehler & Wendt, 2009). Fund size may also lead to further filtering of perceived available investment opportunities as the largest funds are less able to allocate meaningful amounts of their capital to smaller capitalisation issues due to liquidity constraints, again reducing the overall size of perceived investment choices (Kadapakkam, Kumar & Riddick, 1998; Pollet & Wilson, 2008). Fund size may also be a major factor in explaining short-term serial correlations in trades. Thus, in order to minimise total market impact, large funds may prefer to spread their purchases and sales over

a number of trading days, resulting in repeats of the same trades in the same stocks (Sais, 2004). Finally, of particular relevance to markets exhibiting excess, fads, such as the internet bubble, have been proposed as an explanation for herding behaviour, with strong relative performance from identifiable groups of stocks forcing greater following effects due to gathering performance disparities between bubble stocks and the rest of the market (Barberis & Shleifer, 2003; Sharma, Easterwood & Kumar, 2006; Singh, 2013; Walden & Browne, 2008). This would imply that, under certain conditions, professional investors may become substantially reactionary, placing trades primarily on the basis of current trends.

## CHAPTER 8

### THE EFFECT OF STYLE, FEEDBACK & CONTEXT ON INFORMATION DEMAND AND TRADING PROPENSITY, AN EMPIRICAL STUDY

#### 8.1. Introduction

The following case study examined the effect of short-term feedback and market context on the demand for, and use of, information within a dynamic, complex environment with high levels of uncertainty. The empirical data was derived from a novel simulated financial market environment employing actual historical prices and fundamental data together with mouse tracking in order to capture and record the actions of participants.

As well as considering evidence for well-established behavioural characteristics relating to risk and loss-aversion, the study tested the findings of Lurie & Swaminathan, (2009) with regard to apparent emphasis upon recent feedback and scope of utilisation of new information. Similar to the studies conducted by Vlastakis and Markellos (2012) and Dimpfl and Jank (2016), demand for information was also examined testing the effects of volatility and market “distress”. Trading propensity was explored within the context of sell-versus-hold and sell-versus-buy decisions using methodology similar to that employed by Grinblatt and Keloharju (2001). The degree of overall adherence to self-declared investment styles was also examined (Shiller, 1990).

Mouse tracking elements of the study add significantly to the detail and findings regarding overall behaviour as all mouse-based interactions with the underlying data were recorded. Over the course of the experiment, participants were exposed to considerable



feedback and potential information flow. Feedback stemmed from the regular display of portfolio and stock performance information with an Index benchmark offering a potential reference point against which performance could be evaluated. A substantial amount of information relating to potential investments was also available to participants; this could be accessed in a straightforward manner using the software platform provided. Nevertheless, participants were required to take well-defined actions in order to retrieve the various types of information available, enabling each item of information demanded to be logged. The information itself was dynamic, updating according to the latest data available for each period. Therefore, participants had the opportunity to evaluate “new” information on a regular basis.

## **8.2. The Study**

A total of 57 individuals took part in a case study of portfolio management behaviour. The simulated stock market environment was designed to produce a high degree of realism, using actual historical data drawn from a 12 month period running from early May, 2010 to the end of April, 2011. Index data was taken from the US S&P 100 Index to represent the market as a whole while the daily closing prices of 98 stocks comprised the investable universe for participants.

To place the selected data range within a broader context, the S&P 100 Index fell by 38.49% over the course of 2008, due largely to the widespread market dislocation resulting from systemic risks in the broader financial system. The Index declined further in the initial part of 2009, reaching a low of 676.53 on 9<sup>th</sup> March 2009. This represented a total fall of 25.10% from the end of 2008. Subsequent to the 2009 low, the Index underwent a rapid

recovery to 3<sup>rd</sup> May 2010 (the start date for Phase 1). To illustrate the broader context visually, the closing levels of the Index from 10<sup>th</sup> May, 2007 to 28<sup>th</sup> April, 2011 are shown in Figure 7.

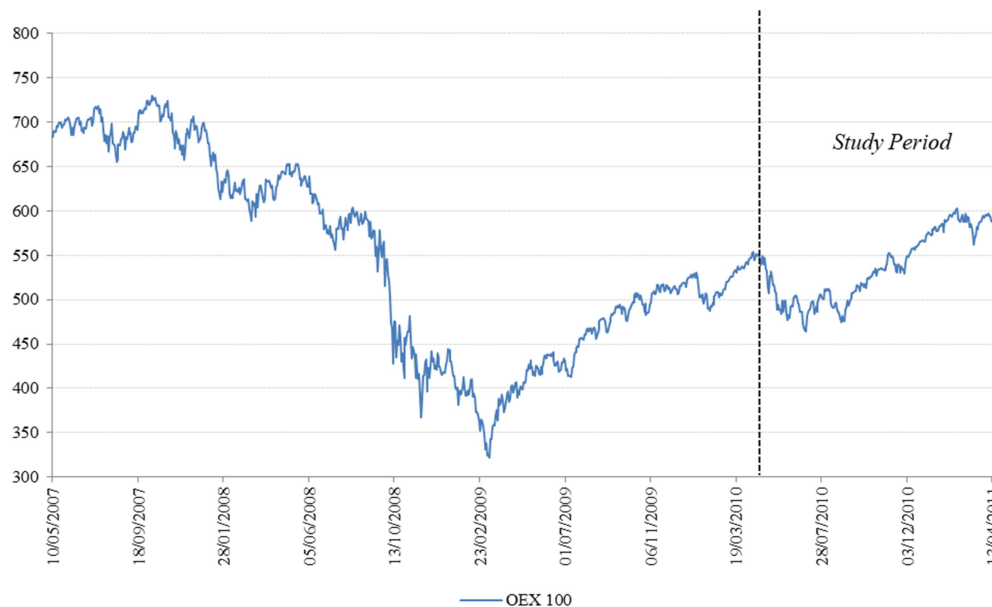


Figure 7. S&P 100 Index, daily closes (May, 2007 to April, 2011)

The stocks chosen for inclusion in the study were all members of the S&P 100 Index for the period concerned and were all classified as being large capitalisation in terms of market value (defined as being in excess of \$10bn). The smallest company in the sample had a market capitalisation at the time of approximately \$18bn, ranking 243<sup>rd</sup> out of all of the corresponding NYSE constituents. The largest company had a market capitalisation of approximately \$500bn, ranking first within the S&P in terms of size. The 98 companies covered 11 standard, recognisable sector classifications (e.g. healthcare, technology, financials, energy). Each of the stocks was highly liquid with average daily volumes ranging from just under 1m shares to over 60m shares traded.

In order to help minimise the chance of memory association from participants, all of the information was rendered anonymous. Thus, while participants were informed that actual market prices were being used, the stock exchange and period from which they were taken remained undisclosed. All prices were expressed in generic terms with no currency assigned. In addition, actual company names were replaced by fictitious, pre-assigned names. The same applied to the Index itself. Participants were simply told that all of the stocks available to them as possible investments were large capitalisation issues and highly liquid. Any perceptions of size or liquidity biases should therefore have been eliminated.

While all data relating to the companies and the Index was rendered anonymous as to source, relevant industry/sector classifications were maintained. In order to create an environment with a level of richness in terms of information content, actual fundamental data applicable to the time period covered was also incorporated and made available to participants. Participants were informed that they would each be managing their own notional portfolio, simulated over a period of time. In order to provide an incentive, participants were told that the individual achieving the highest portfolio return at the end of the experiment would receive £100 worth of gift vouchers from a major store.

### **8.2.1. Participants**

Participants for the study were recruited from a number of seminars and workshops offered by a UK-based investment management company. These events were held over a number of days and featured presentations on a range of relevant investment-related topics. The seminars and workshops were offered entirely free, although attendees were asked to take part in the study in return; such participation being entirely voluntary. In the event, all

attendees agreed to participate and were tested jointly. The participants all declared an “above average” interest in financial markets with a number having direct experience in the industry. In terms of profile, 52 out of the 57 participants (91.22%) held first or higher degrees (various subjects) and/or held various professional qualifications. Independent advisors (IFAs) and wealth planners accounted for one-third of the total number (19 out of 57); 6 participants were enrolled on an MBA course at a local business school while 2 participants were currently employed as investment managers. The remaining participants were all active private investors managing their own money or had prior experience in stockbroking or portfolio management at some stage in their careers. The participants comprised 48 males (84.21% of the total) and 9 females; ages ranged from 24 to 64 years ( $M = 44.25$ ,  $SD = 9.53$ ). The breakdown of participants by role, age and gender is shown in Table 17. There is no significant difference in age between genders; using the two-tailed sample test for unequal variances,  $t(55) = -1.63$ ,  $p = 0.11$ . No significant differences were also found for years of relevant investment experience based upon gender;  $t(55) = 1.67$ ,  $p = 0.44$ .

Table 17. Breakdown of participants by experience, gender, average age, investment experience and qualification.

Role / Main Experience	Gender	Breakdown by		Average Age	Average Years of Investment Experience	Average Starting Age (in Role)	Number with Higher Degrees / Professional Qualifications
		Gender					
IFA	Male	16		44.69	17.31	27.38	15
	Female	3		43.00	21.33	21.67	3
	Total	19		44.42	17.95	26.47	18
Investment Manager	Male	5		50.80	22.80	28.00	5
	Female	0		NA	NA	NA	NA
	Total	5		50.80	22.80	28.00	5
MBA	Male	2		37.00	16.50	20.50	2
	Female	4		35.25	10.25	25.00	4
	Total	6		35.83	12.33	23.50	6
Private Investor	Male	22		43.00	15.86	27.23	19
	Female	2		43.09	15.00	28.00	2
	Total	24		43.08	15.79	27.29	21
Stockbroking	Male	3		58.33	27.33	31.00	2
	Female	0		NA	NA	NA	NA
	Total	3		58.33	27.33	31.00	2
Overall Totals	Male	48		45.13	17.81	27.31	43
	Female	9		39.56	15.00	24.56	9
	Total	57		44.24	17.368	26.88	52

The breakdown of participants by self-reported investment is style shown in Table 18.

Table 18. Breakdown of participants by self-declared investment style.

Self-Declared Style	Number of participants
Growth	12
Judgmental	20
Technical	13
Value	12

### **8.3. Materials and Methods**

Participants were provided with custom-built software loaded onto a number of networked desktop computers. The software contained all of the tools necessary for each participant to manage their own notional portfolios within the context of the environment provided. At the outset, and as the study progressed, participants could evaluate potential investments, buy and sell securities and change portfolio holdings as they chose, subject to the general constraint that no shorting (selling a stock not already held) or leverage (borrowing to finance a purchase) was allowed. There was no time limit imposed in relation to creating individual portfolios, although individual transactions, once executed, were binding. Once satisfied with their portfolios, participants could ‘step forward’ to the next valuation period. At that point, the latest portfolio valuation would be displayed and participants could see a summary of their performance since the prior valuation date together with the performance of each stock in their portfolio and the return of the market index. In addition, at each stage, participants could see the cumulative performance of their own

portfolio and that of the market Index up to that point. Participants therefore had two key pieces of performance information available to them at each stage; absolute portfolio returns, in monetary and percentage terms, and relative returns (portfolio return compared with the market Index return over the same period).

Since the aim of the study was to extract as much information regarding user interaction as possible, the software was designed to capture all keyboard actions and mouse clicks, with the derived data being logged and saved in files stored on each computer. Each participant began the experiment with a notional 5,000,000 (generic currency unit) of cash which they were free to allocate across the universe of available investments in any way they chose, subject to the general constraints regarding leverage and shorting. Participants were introduced to the full functionality of the software over a 45 minute period during which they could practice, test and ask questions relating to the software.

Unique identification codes were assigned to each participant. Participants were also presented with a single sheet of paper on which they were asked to select their generic investment style from a printed list of four broadly defined categories; value, growth, technical or judgmental. In addition to tick boxes applicable to each category, a brief description of the general characteristics of each style was also provided on the sheet. For example, it was suggested that value investors may favour stocks offering high dividend yields, low price to earnings ratios (PE), low price to book value ratios (PBV), or low price to cash flow ratios (PCF). Alternatively, they might assess stocks on a relative basis by comparing, for example, an individual stock's P/E ratio against that of the market average, or by comparing combinations of factors, such as a high relative return on equity (ROE) versus low relative PE. Growth investors were described as being more likely to place greater

emphasis upon positive, dynamic rates of change of key underlying variables, such as the year-over-year rate of change in earnings per share (EPS). Technical investors were categorised as those who based stock selection predominantly upon historic price patterns, deriving signals in whichever manner primarily from that source. The final, judgmental category was used to capture all other explicit or co-mingled styles. Thus, investors who considered numerous indicators and factors, without necessarily assigning a dominant role to any particular source, would fall into this category. Similarly, hybrid styles, such as a combination of growth and technical analysis, might also be so assigned.

For the purposes of the experiment, self-reported styles were used as descriptive elements designed to reflect the participants' own perceptions of any particular investment (heuristic) bias which they may have. There was no presumption that participants would necessarily adhere to such styles throughout the course of the experiment, although it was assumed that participants would tend to adopt such style biases and methods at least in the initial stages of the portfolio building and management processes. Adherence to particular styles was, in any case, analysed from the detailed portfolio data in order to establish any factor biases in the portfolios created and the consistency of those biases over the course of the experiment. On that's basis, style biases can be inferred and compared with the self-declared style attributes.

Participants were able to access a substantial amount of fundamental and price-related data as well as manage their portfolios via the software platform. On opening the software programme, participants entered the main "Dashboard" area where they input their unique User IDs. This allowed access to the various data and portfolio management tools. The Dashboard view is shown in Figure 8.

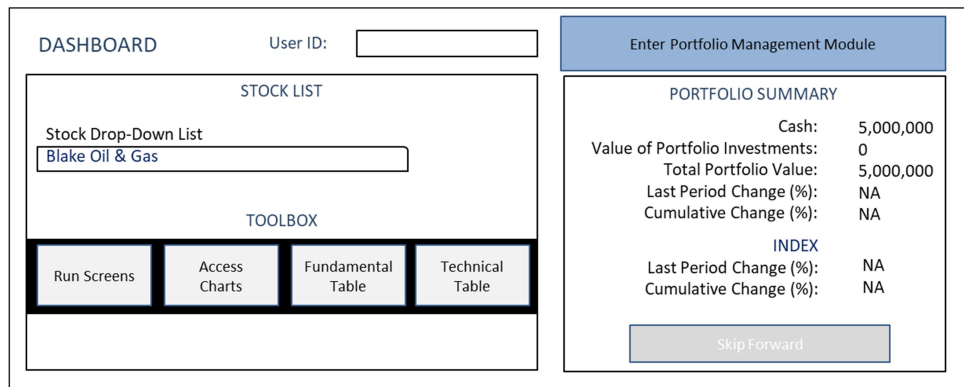


Figure 8. “Dashboard” screen for navigate the software.

*Note:* Upon entering the software, participants input their User IDs into the box shown on the Dashboard above. They were then able to access the main modules of the programme. The drop-down list enabled them to quickly summon information on a single security. Alternatively, participants could choose to enter one of the data modules available through the Toolbox. Should they wish, participants could move directly to the Portfolio Management Module. It was possible to return to the Dashboard at any time during the experiment, although all of the information accessed through that route could also be accessed from any of the other modules at any time.

Fundamental and technical (price-based) data was presented in the form of tables while historical price charts were also readily available for each security and the market Index. Simple menus available in each section of the programme made it easy to move freely from one table to another as required. By default, fundamental (company-specific) and technical indicator tables were displayed according to assigned company name, sorted alphabetically (please see Appendix H for more information on the tables and their contents).

Additional information could be accessed via the tables by double-clicking on the name of a chosen security. This gave participants the option of generating a historical price chart for the selected security which then gave access to additional tools for plotting and creating various technical indicators in graphical form. It was therefore possible to use the software to perform analysis as defined by the user as opposed to being limited to pre-set metrics and defaults. Participants also had the option of changing default table views by sorting either of the tables based upon any of the data columns. For example, participants could choose to sort the fundamental data table on the basis of the P/E column. This would then sort the



information in the table from high-to-low based upon P/E. All of the underlying data in the tables updated over the course of the study to reflect the latest prices and other data which would have been available at each point in time; all ratios and other computed data were instantly recalculated where necessary.

The primary purpose of the tables was to enable participants to perform various types of analysis to support their decision-making. In order to make the software as user-friendly as possible, once a potential investment had been identified, participants could add the security to their existing portfolio easily from either table simply by right-clicking the security code. This inserted the selected security, along with the latest price, into the portfolio form ready for shares to be purchased as required. There was no obligation for participants to conduct any transaction simply by transferring securities in this way; entering actual transactions could only be accomplished via the portfolio form as a result of activating the formal procedure for enacting such transactions. Participants were not required to view any of the available data at any stage in order to create, maintain or modify their portfolios. If desired, they could simply “step forward” to the next valuation period without conducting any search or review activity and without making any changes to holdings. Consequently, it is safe to conclude that viewing data was an active decision on the part of participants.

Each data and information request was logged by the software, identifying the time of the request derived from the computer’s internal clock, the identity of the security for which information was requested and the type of data polled (for example, fundamental or chart data). Participants could also request information on the market Index as a whole (market Index data was included at the bottom of each table).The actual portfolio construction process

was designed to be both easy and flexible. A dedicated portfolio form could be accessed from the main Dashboard or directly from any of the activated table or chart forms. Figure 9 shows a typical portfolio view.

While security names and prices could be added to the portfolio form either directly from a drop-down box or via the tables, transactions could only be undertaken from within the portfolio form. To change the number of shares held, participants clicked in the appropriate row under the “Shares” column (double-clicking to buy shares, right clicking to sell). These actions triggered a dialogue box enabling participants to enter the required number of shares

### Portfolio Management Form

Stock List:

		<b>Total Portfolio Value:</b>		5,203,946.00
		<b>Available Cash:</b>		179,192.93
Security	Code	Price	Shares	Value
Aplian Healthcare	DCS1002	22.02	12,500	275,250.00
Bosphur Financial	DCS1005	44.82	8,600	385,452.00
Catsom Industrials	DCS1008	68.54	6,000	411,240.01
Decorum Materials	DCS1018	48.54	7,500	364,050.01
Ecom Pharmaceutical	DCS1021	21.74	14,300	310,882.00
Grafton Oil	DCS1031	62.08	5,400	335,232.01
Halter Goulding	DCS1034	12.72	15,800	200,976.00
Hayden Dynas	DCS1037	19.28	13,600	262,208.01
KP Systems	DCS1044	25.93	12,000	311,160.00
Lear Computing	DCS1050	259.04	1,000	259,040.01
SafeGuard Technologies	DCS1076	44.43	9,000	399,870.00
Starlight	DCS1085	25.88	13,100	339,027.99
TG Systems	DCS1089	35.99	9,500	341,905.02
True Vision	DCS1092	23.02	15,700	361,414.01
Yoga Exploration	DCS1099	17.54	15,000	263,100.01

Figure 9. Example of portfolio holdings list.

*Note:* By default, the portfolio view listed stocks alphabetically, showing the latest price, the number of shares held and the value of each holding. The total portfolio value was shown along with any residual cash. Participants could initiate transactions by selecting the appropriate row and clicking the corresponding “Shares” column. A new potential holding could be added by selecting a new stock from the drop-down box labelled “Stock List”.

to buy or sell. Once entered and confirmed, a summary screen would appear displaying details of the proposed transaction, including the value of the trade, net of transactions costs. At this stage, participants were required to accept the trade in order to execute the transaction, or cancel (see Figure 10). In the event that a proposed trade was cancelled, no monetary amounts or number of shares held changed. However, once a trade was accepted, it became binding. At that point, the new number of shares would be displayed in the appropriate cell, the aggregate value of the total holding being updated. The available cash balance also adjusted automatically, as did the total value of the portfolio (the aggregate portfolio value changing due to the impact of transactions costs). Since no intraday prices were used, all transactions were executed at previous end-of-day closing prices. In order to increase realism, notional spreads along with commissions were applied to both purchases and sales such that total transaction costs amounted to 0.30% of the underlying value of each transaction. Therefore, any participant investing the total amount of available cash (5,000,000) at the earliest opportunity would experience an immediate drop in the ending value of their portfolio of 0.30%.

Checks were incorporated into the software to prevent “illegal” trades. Thus, as no shorting was allowed in this experiment, share balances could not be negative (it was therefore not possible to sell more shares than the number already held). Similarly, since no leverage could be employed, cash balances could not fall below zero. In the event that proposed trades violated either of those conditions, an error message would appear informing participants of the “violation”. The proposed trade was then reset (cancelled) and all open dialog boxes were closed with no changes to the portfolio or cash balance being made; if they wished, participants were then free to enter a new trade which would not violate the constraints. Within the available constraints, participants could select any number of stocks

they wished, trading in any size. In the event that participants wished to change a previous trade, they could do so only on the basis of a new transaction, thereby incurring additional transaction costs. No time limits were imposed on the portfolio building process; participants could therefore take as long as they wished to create the desired portfolio. Once any changes to portfolios had been completed, participants moved forward to the next valuation date by selecting the “Advance to Next Period” option on the form. This took participants to the next “valuation day”. At that stage, the latest portfolio would be displayed updated to reflect closing prices and holding values for the next period.

Portfolio Management Form

Stock List:

Security	Code	Price	Shares	Total Portfolio Value	Available Cash
Aplian Healthcare	DCS1002	22.02	12,500		
Bosphur Financial	DCS1005	44.82	8,600		
Catsom Industrials	DCS1008	68.54	6,000		
Decorum Materials	DCS1018	48.54	7,500		
Ecom Pharmaceutical	DCS1021	21.74	14,300		
Grafton Oil	DCS1031	62.08	5,400		
Halter Goulding	DCS1034	12.72	15,800	200,776.00	
Hayden Dynas	DCS1037	19.28	13,600	262,208.01	
KP Systems	DCS1044	25.93	12,000	311,160.00	
Lear Computing	DCS1050	259.04	1,000	259,040.01	
SafeGuard Technologies	DCS1076	44.43	9,000	399,870.00	
Starlight	DCS1085	25.88	13,100	339,027.99	
TG Systems	DCS1089	35.99	9,500	341,905.02	
True Vision	DCS1092	23.02	15,700	361,414.01	
Yoga Exploration	DCS1099	17.54	15,000	263,100.01	

**Bosphur Financial**

SELL (number of shares)

Price : 44.82

Total Value of Transaction: 89,613.11

Figure 10. Example of a transaction dialogue box.

*Notes:* The above example is that of a participant activating a sale of Bosphur Financial. In an initial dialogue box, participants simply entered the number of shares to be sold. This information triggered a second dialog box (shown above) containing a summary of the trade, including the total value of the proposed transaction. Participants therefore had one final opportunity to cancel the proposed trade, or else proceed by clicking the “Accept” button.

Participants received summary feedback regarding the performance of their portfolios and individual holdings throughout the course of the study. At each valuation date, the aggregate performance of the portfolio for the latest period would be displayed along with the cumulative performance since inception. As a comparison, the equivalent performance data for the market Index was also provided. Participants could therefore see at a glance whether the performance of their portfolio exceeded or lagged that of the market Index. Participants also received performance information about individual holdings within the portfolios, including latest period returns as well as cumulative gains or losses in relation to book costs. At no stage was the performance of other portfolios managed by other participants disclosed. Therefore, while participants could easily see how they were performing versus the Index benchmark, they had no information on how their performance compared with that of their peers. While performance feedback was provided automatically to each participant throughout the experiment, all other information relating to fundamental or price data required participants to take actions in order to retrieve it thereby enabling their level of demand for information to be assessed.

In order to ensure that the study was not unduly time consuming (approximately three hours had been allocated for the entire experiment), portfolio valuation periods were set to occur at intervals of 5 trading days. Therefore, having completed the initial portfolio and stepping forward, the next valuation would reflect price changes over the subsequent 5 trading days. Consequently, over the course of the entire experiment, participants would create, review, evaluate and manage a total of 51 portfolios from start to finish.

#### 8.4. Description of the Source Data

As noted earlier, actual historical data was used for the study. The period chosen, 3<sup>rd</sup> May 2010 to 28<sup>th</sup> April 2011, was selected due to its specific characteristics. Within this time period, there are two distinct phases: In the first part of the period, the general market Index experienced a significant decline, with the S&P 100 (the index chosen for the experiment) falling from a starting level of 1202.26 on 3<sup>rd</sup> May 2010, to a low of 1022.58 on 2<sup>nd</sup> July 2010, representing a fall of 14.94%. From that low position, the Index recovered in an irregular manner to reach 1148.67 on 24<sup>th</sup> September 2010, representing an increase of 12.33% from the low point. Over the period, from 3<sup>rd</sup> May 2010 to 24<sup>th</sup> September 2010, the market as a whole therefore declined by 4.46% based upon closing prices. For the purposes of the study, this time period is described as *Phase 1*.

The second intra-period, described as *Phase 2*, spanned the period from 24<sup>th</sup> September, 2010 (the end of *Phase 1*) to 11<sup>th</sup> March, 2011. This period is marginally longer than the period defining *Phase 1* (116 versus 101 trading days). Over the course of *Phase 2*, the market Index displayed a generally rising trend, ending some 13.55% above the opening level. The closing price of 11<sup>th</sup> March, 2011 was therefore 8.49% above the level of the Index level as of 3<sup>rd</sup> May, 2010 (marking the start of the experiment). In summary, *Phase 1* was a period of above average volatility with sharp declines in Index levels over the initial period, which saw significant cumulative declines. Towards the end of the *Phase 1* period, there was some reversal of this downtrend as the market as a whole experienced cumulative gains. *Phase 2* involved a continuation of that recovering trend with reduced levels of volatility relative to that present during *Phase 1*.

The study is designed to examine the impact of real-time feedback on the demand for information, as evidenced by search activity conducted by participants. Much of the feedback to participants relates to the performance of their investments, both at the portfolio and individual stock level. It is expected that participants will evaluate this feedback according to some reference points; in this case, the primary reference points are likely to be absolute returns, either gains or losses, and performance relative to the Index benchmark, the latter providing a context for the overall investment environment. The distribution of individual stock returns might therefore play a significant role in determining both relative and absolute performance of participant portfolios. This was examined in two ways; the first simply looked at the basic distributions of the data; the second used Monte Carlo analysis to generate a series of random portfolio whose returns could be calculated.

Over the entire period for which data was applied to the experiment, the capital return (excluding dividend yield) for the S&P 100 Index was 13.16%. The Index is weighted according to the market capitalisation (share price multiplied by the number of shares outstanding) of its individual constituents. The level of the Index is therefore calculated in the following manner;

$$I_x = \sum_i P_i \cdot Q_i / Divisor \quad \text{Equation 38}$$

where  $P_i \cdot Q_i$  is the market capitalisation of stock  $i$  based upon the current price,  $P_i$ , and the free shares outstanding  $Q_i$ . The divisor is an adjustment factor to allow for stocks leaving and joining the Index over time, ensuring continuity in its price level.

The capitalisation weighting of the Index means that the largest companies, in terms of their market value, exert more of an influence on the price outcome due to their higher weighting relative to smaller capitalisation issues. Examining each of the 98 stocks included in the experiment, the median return was found to be 13.90% over the entire period. The median return is thus similar to that of the market capitalisation weighted return. However, the equally-weighted mean return was found to be significantly higher than the market capitalisation weighted Index return and the median stock return (17.09%), reflecting the overall impact of outliers. The summary of the findings is presented in Table 19.

Table 19. Summary of returns for 98 stocks used in experiment.

Highest Individual Stock Return	74.44%
Lowest Individual Stock Return	-36.98%
Mean Stock Return	17.09%
Median Stock Return	13.16%
Standard Deviation	21.14%

In order to assess the impact of the skewed stock return data distribution on overall portfolio performance, Monte Carlo simulations were conducted based upon randomly generated, static (non-trading) portfolios. A total of 250,000 such random portfolios were created based upon arbitrary criteria. Specifically, the number of assumed holdings for each random portfolio was itself subject to a process of random number generation, and could range, in whole numbers, between 8 and 20 holdings. Once the number of planned holdings had been determined in this manner, the identity of each security to be included in the new portfolio was again selected at random from the available universe of 98 stocks<sup>7</sup>. For each

<sup>7</sup> Therefore, the first random number generated the number of holdings which would appear in that particular portfolio. The second set of random numbers to be generated identified the specific stocks which would make up



portfolio, the initial cash was then allocated equal between holdings, ensuring a fully invested portfolio which was then held without modification throughout the entire time period corresponding to the experimental data range. Therefore, once created, no trading was assumed to take place. Consequently, the total return for each portfolio over the entire period  $0$  to  $t$  was simply calculated based upon the weighted cumulative return of all included holdings, derived from;

$$P_{ret} = ((\sum_i P_{i,t} \cdot Q_{i,0} / \sum_i P_{i,0} \cdot Q_{i,0}) - 1) * 100 \quad \text{Equation 39}$$

The simulation of random portfolios confirms a return skew relative to the Index, as shown in Table 20.

Table 20. Monte Carlo random portfolio simulation (250,000 portfolios).

Maximum Portfolio Return	42.45%
Minimum Portfolio Return	-16.29%
Mean Return	16.02%
Median Return	16.20%
Standard Deviation	6.03%
95% Confidence Interval	
Mean + 1.96.sd	27.84%
Mean - 1.96.sd	4.21%

*Note:* The data in the Table defines the overall return distribution derived from the Monte Carlo simulation. The distribution provides a basis against which to judge participant performance in terms of skill (as measured by the likelihood of a particular return outcome occurring by chance).

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that required number of holdings. For this purpose, stocks were ranked alphabetically by name with the random number identifying the stock at that corresponding position in the list. The total number of unique random numbers generated at the second stage obviously equalled the initial value of the random number determining the total number of portfolio holdings; in other words, the same security could not be randomly selected more than once and included more than once in the portfolio.

Based upon these parameters, a randomly generated, fully invested, non-trading portfolio was found to have a greater than 68.23% chance of outperforming the Index over the period. (In other words, a random investor applying no investment skill would be expected to outperform the Index benchmark 68.23% of the time.) This is a structural bias within the data which could have some impact on the results generated by the study, particularly if participants place emphasis upon their relative performance (portfolio return versus that of the Index), taking positive outperformance as positive affirmation of their current strategy. It is equally possible that some investors may mistake underlying data bias, resulting in excess portfolio return, for skill.

## **8.5. Hypotheses**

The study is designed to examine the level of participant demand for information primarily as a function of short-term feedback and certain “environmental” factors which provide context. The key hypotheses can be described formally as follows:

1. Participants would exhibit systematically different behaviour in the domain of losses versus gains. Consistent with existing literature, the level of demand for information was expected to be influenced by the nature of short-term feedback in the form of portfolio and individual holding returns (Bank, Larch & Georg, 2011). Specifically, negative returns were expected to trigger greater information search relative to gains, consistent with risk aversion and response to uncertainty (Vlastakis & Markellos, 2012). An inverse relationship was therefore expected between portfolio return metrics and demand for information.

2. Demand for information would vary according to the level of overall volatility. The standard deviation of Index returns over rolling 21-day periods was used as a proxy for general volatility. Higher levels of market volatility were expected to trigger additional search and exploration activity, again due to risk-aversion and the desire to reduce uncertainty. A direct relationship is therefore expected between the two variables.
  
3. Significant differences in demand for information were expected based upon investment style. Technical style participants, who derive information primarily from price behaviour and patterns, were expected to engage in greater levels of information search activity relative to other, more fundamentally based, styles. Technical trading is inherently more short-term in nature than many fundamental styles and is therefore more likely to be responsive to short-term price movements. Value investing, by contrast, is likely to be significantly less susceptible to the influence of short-term market price changes. The search for price patterns and structures suggests that Technical style investors are likely to be presented with potentially actionable signals more frequently than fundamental style investors, particularly within more volatile periods. The basic differences between styles were therefore expected to be magnified in periods of heightened volatility.

### 8.5.1. Description of Variables and Testing Procedures

The overall effect of real-time feedback on information demand was investigated using a series of regression models. The dependent variable (DV) in each case was frequency data representing the number of search actions per participant for each portfolio period, measured by the number of relevant mouse clicks generated by participants,  $f(DS)$ . Relevant mouse clicks included all requests for stock-specific fundamental or price data. A visual representation of  $f(DS)$  is provided by Figure 11.

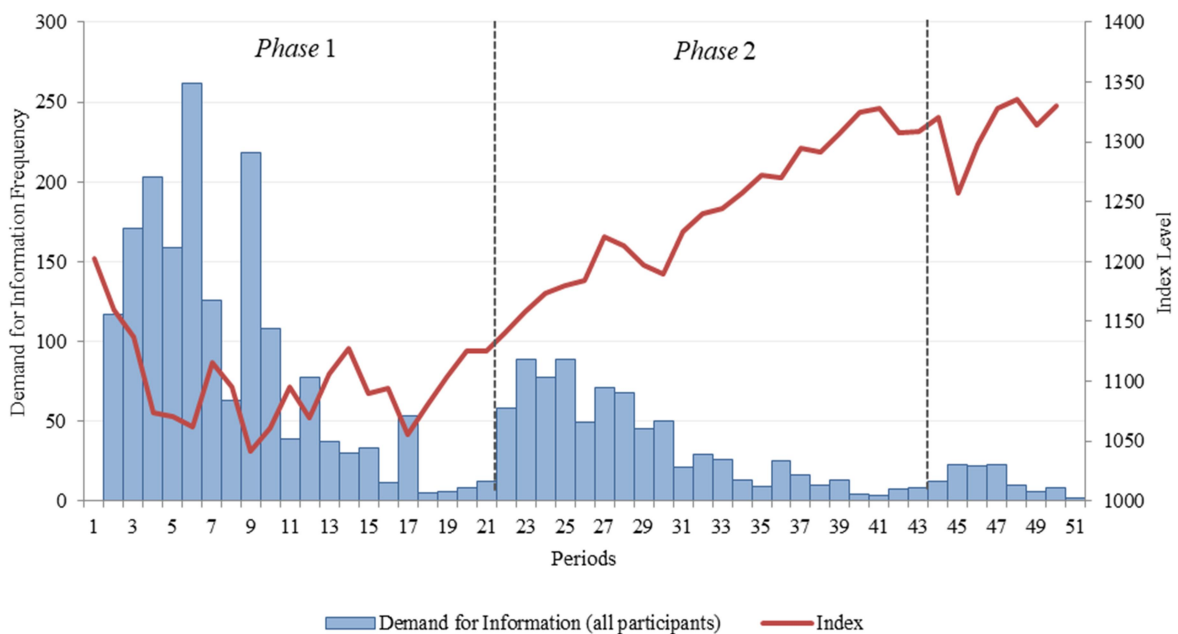


Figure 11. Frequency data of  $f(DS)$  for all participants overlaid with the market Index

Casual inspection of the data suggests a clear pattern with regard to the distribution of  $f(DS)$  with aggregate demand for information appearing to be correlated with the short-term directional bias of the market and changes in trend.

Formal analysis of the data was conducted using log-linear (Poisson) regression, reflecting the frequency data DV, tested for overdispersion. Four models were tested; each model had common independent variables in the form of most recent period returns, cumulative returns to date and short-term market volatility. Additional dummy variables were then added to capture any effects from style and phase. Table 21 displays the detail of the models tested.

Table 21. Dependent and independent variables (IVs) of the models

	DV	IV <sub>1</sub>	IV <sub>2</sub>	IV <sub>3</sub>	IV <sub>4</sub>	IV <sub>4B</sub>	IV <sub>5</sub>
Model 1	$f(DS)$	$P_{perf\%}$	$C_{perf\%}$	$Mvol$			
Model 2	$f(DS)$	$P_{perf\%}$	$C_{perf\%}$		$Istyle$		
Model 3	$f(DS)$	$P_{perf\%}$	$C_{perf\%}$			$Sstyle$	
Model 4	$f(DS)$	$P_{perf\%}$	$C_{perf\%}$				$P_{1,2}$

where:

$f(DS)$ , the DV, is the sum of all relevant mouse clicks triggering the retrieval of

fundamental or price data for individual stocks in each period;

IV<sub>1</sub> ( $P_{Perf\%}$ ) is the most recent period portfolio performance in percentage terms;

IV<sub>2</sub> ( $C_{Perf\%}$ ) represents cumulative performance of the portfolio to date in percentage terms, updated for each period;

IV<sub>3</sub> ( $Mvol$ ) is a proxy variable for volatility, computed as the rolling standard deviation of daily market Index returns over the prior month<sup>8</sup>.

<sup>8</sup> Volatility, computed by standard deviation, measures the short-term variability in broad market Index returns and is therefore a proxy for the state of the prevailing investment environment. To the extent that participants are inherently risk averse, increases in volatility may imply heightened levels of absolute risk in the overall investment environment and may, therefore, suggest greater potential for future losses.

IV<sub>4A</sub> (*Istyle*) is a dummy variable representing broad investment style. Participants are categorised as either *Technical (chartists)* or *Fundamental (fundamentalist)* investors. The assignment to each classification was made according to the self-described investment styles of participants and took the default values of 0 for *Fundamental* investors and 1 for *Technical* investors. The *Fundamental* classification therefore encompasses the *Value*, *Growth* and *Judgmental* sub-classifications of self-declared styles.

IV<sub>4B</sub> (*Sstyle*) is a categorical variable based upon the four sub-styles self-declared by participants. The categorical variable took values from 1 to 4 based upon these sub-styles corresponding with: *Growth*; *Judgmental*; *Technical*; *Value*.

IV<sub>5</sub> ( $P_{1,2}$ ) is a separate categorical variable denoting the various “phases” within the time period of the study as described earlier. The categorical variable took values of 1, 2, and 0; the first two corresponding to *Phase 1* and *Phase 2* respectively while the latter corresponded with the short residual period post-*Phase 2*.

*Model 1* was therefore a direct test of Hypotheses 1 and 2, with feedback measured by portfolio returns over the latest period in addition to cumulative portfolio performance up to each point within the study period. The market volatility proxy represented the broad “environmental” variable in order to measure the extent to which behaviour was influenced by recent volatility in general asset prices. *Model 2* added a dummy variable capturing the broad division between *Technical (chartist)* and *Fundamental (fundamentalist)* styles as self-declared by participants. *Model 3* performed a similar analysis but broke the *Fundamental* category down into its various sub-styles (*Growth*, *Judgmental* and *Value*), again as self-

declared by participants. *Model 4* tested for additional explanatory power provided by market *phases*.

## 8.6. Description and Discussion of Results

The summary statistics (Table 22) and correlation matrix (Table 23) provide insights into the structure of the variables. Reflecting the nature of the test period, the summary statistics indicate relatively high levels of volatility and data ranges for the non-dummy variables. The characteristics of the return-related variables, *Pperf%* and *Cperf%*, in particular capture the volatile nature of the period, confirmed by the range for *Mvol*. The correlation matrix reveals a weak relationship between *Pperf%* and *Cperf%* and between *Pperf%* and *Mvol*. A moderate (inverse) relationship is found between *Cperf%* and *Mvol*. Positive correlations are found between the two portfolio return variables while both are negatively correlated with *Mvol*. The latter is to be expected as *Mvol* is strictly positive while the general return data reflected fundamental differences between the *Phases*; the average daily return and annualised standard deviation of daily returns during *Phase 1* (M = -0.03%, SD = 23.46%) differing substantially from those during *Phase 2* (M = 0.11%, SD = 12.10%).

Four regression models were run with *Model 1* testing the core hypotheses relating to the impact of feedback, in the form of portfolio returns (individual period and cumulative portfolio returns), and general volatility on the demand for information as measured by participant search activity. Each subsequent model tested additional variables as shown in Table 21. The results of the regression tests are shown in Table 24.

Table 22. Summary statistics: description of variables used in the four models

Variable	Observations	Obs. with missing data	Min.	Max.	Mean	Std. deviation
Number of Views $f(DS)$	2850	0	0.00	16.00	0.92	1.98
Absolute Performance ( $Pperf\%$ )	2850	0	-8.07	8.58	0.39	2.45
Cumulative Performance (Cperf%)	2850	0	-19.99	26.93	2.96	10.18
Return Volatility ( $Mvol$ )	2850	0	0.21	2.85	0.91	0.55
Styles <i>Technical, Fundamental (Istyle)</i>	2850	0	0.00	1.00	0.23	0.42
Self-Declared Style ( $Sstyle$ )	2850	0	1.00	4.00	2.44	1.04
Phases ( $P_{1,2}$ )	2850	0	0.00	2.00	1.32	0.71

Table 23. Correlation matrix for all variables used in the four models

Variables	Styles					
	Absolute Performance ( $Pperf\%$ )	Cumulative Performance (Cperf%)	Return Volatility ( $Mvol$ )	<i>Technical, Fundamental (Istyle)</i>	Self-Declared Style ( $Sstyle$ )	Phases ( $P_{1,2}$ )
Absolute Performance ( $Pperf\%$ )	<b>1.00</b>	0.22	-0.30	-0.02	-0.01	0.07
Cumulative Performance (Cperf%)	0.22	<b>1.00</b>	-0.51	-0.22	-0.08	0.04
Return Volatility ( $Mvol$ )	-0.30	-0.51	<b>1.00</b>	0.00	0.00	-0.13
Styles <i>Technical, Fundamental (Istyle)</i>	-0.02	-0.22	0.00	<b>1.00</b>	0.29	0.00
Self-Declared Style ( $Sstyle$ )	-0.01	-0.08	0.00	0.29	<b>1.00</b>	0.00
Phases ( $P_{1,2}$ )	0.07	0.04	-0.13	0.00	0.00	<b>1.00</b>

The goodness of fit statistics compare the performance of an independent (constant-only) model, in which the linear combination of explanatory variables reduces to zero, with a full model, including the combined effects of all of the IVs. These tests found that each of the models possessed considerable explanatory power; Cox and Snell  $R^2$  measures ranged from 0.49 (*Model 1*) to 0.51 (*Model 4*) while Nagelkerke  $R^2$  measures ranged from 0.50 (*Model 1*) to 0.53 (*Model 4*). The high level of significance for the models means that the null hypothesis (the coefficients of the IVs are all zero) is rejected. The Akaike Information Criterion (AIC) and Schwarz's Bayesian Information Criterion (BIC) provide tests of the quality of the models under consideration by measuring the amounts of information lost through parameter estimation. Both measures test goodness of fit adjusted for potential



overfitting as a result of adding additional variables. Lower values for AIC and BIC suggest better fits, providing a basis for comparing models.

Consistent with the  $R^2$  statistics, each model is found to be a significant improvement on the independent constant-only model. The confidence tests assess the extent to which the models under consideration maximize the value of the likelihood function which makes the observed data most likely given the model's parameters. Higher values for each of the measures suggest a greater degree of confidence. The Type II analysis evaluates the likelihood of generating a "false positive" with regard to the hypothesis associated with individual variables. Each of the variables across the four models is found to be highly significant with the likelihood of Type II error therefore low.

The coefficients for the IVs are found to be stable across the four models, consistent with the relatively low correlations between the dummy variables and other IVs. Crucially, they are all stable in terms of sign. Overdispersion tests derived the following  $p$ -values for each of the models: *Model 1*,  $p = .93$ ; *Model 2*,  $p = .98$ ; *Model 3*,  $p = .92$ ; *Model 4*,  $p = .89$ . As the computed  $p$ -value in each case exceeds the significance level,  $\alpha = .05$ , the null hypothesis that the data are Poisson distributed cannot be rejected.

Table 24. Results of log-linear (Poisson) regression tests of four overlapping models

Summary goodness of fit statistics:

	Model 1	Model 2	Model 3	Model 4	
	Independent (Constant only)	Full Model	Full Model	Full Model	Full Model
Statistic					
Observations	2850	2850	2850	2850	2850
DF	2849	2846	2845	2845	2844
-2 Log(Likelihood)	10274.94	8371.83	8252.40	8330.81	8214.98
R <sup>2</sup> (Cox and Snell)	0.00	0.49	0.51	0.49	0.51
R <sup>2</sup> (Nagelkerke)	0.00	0.50	0.52	0.51	0.53
Akaike Information Criterion	10276.94	8379.83	8262.40	8340.81	8226.98
Schwarz's Bayesian Criterion	10282.89	8403.65	8292.18	8370.59	8262.71

Confidence tests:

Statistic	Model 1		Model 2		Model 3		Model 4	
	Chi-square	Pr > X <sup>2</sup>	Chi-square	Pr > X <sup>2</sup>	Chi-square	Pr > X <sup>2</sup>	Chi-square	Pr > X <sup>2</sup>
-2 Log(Likelihood)	1903.10	< 0.0001	2022.53	< 0.0001	1944.13	< 0.0001	2059.95	< 0.0001
Score	2090.79	< 0.0001	2260.48	< 0.0001	2138.75	< 0.0001	2260.50	< 0.0001
Wald	1782.27	< 0.0001	1955.00	< 0.0001	1822.51	< 0.0001	1936.65	< 0.0001

Type II analysis

Source	Model 1		Model 2		Model 3		Model 4	
	Chi-square (LR)	Pr > LR	Chi-square (LR)	Pr > LR	Chi-square (LR)	Pr > LR	Chi-square (LR)	Pr > LR
Absolute Performance ( <i>Pperf%</i> )	131.53	< 0.0001	128.82	< 0.0001	130.35	< 0.0001	138.70	< 0.0001
Cumulative Performance ( <i>Cperf%</i> )	573.11	< 0.0001	434.37	< 0.0001	590.62	< 0.0001	464.23	< 0.0001
Return Volatility ( <i>Mvol</i> )	167.18	< 0.0001	203.43	< 0.0001	161.44	< 0.0001	226.34	< 0.0001
Style ( <i>Istyle</i> )			119.43	< 0.0001			108.33	< 0.0001
Self-Declared ( <i>Sstyle</i> )					41.02	< 0.0001		
Phases ( <i>P<sub>1,2</sub></i> )							37.42	< 0.0001

Model parameters for the components

Source	Model 1		Model 2		Model 3		Model 4	
	Value (SE)	Wald Chi-Square (Pr > X <sup>2</sup> )	Value (SE)	Wald Chi-Square (Pr > X <sup>2</sup> )	Value (SE)	Wald Chi-Square (Pr > X <sup>2</sup> )	Value (SE)	Wald Chi-Square (Pr > X <sup>2</sup> )
Intercept	-0.65 0.04	217.01 < 0.0001	-0.83 0.05	307.27 < 0.0001	-0.36 0.06	33.75 < 0.0001	-1.23 0.08	216.96 < 0.0001
Absolute Performance ( <i>Pperf%</i> )	-0.09 0.01	125.96 < 0.0001	-0.08 0.01	123.60 < 0.0001	-0.09 0.01	124.74 < 0.0001	-0.09 0.01	131.65 < 0.0001
Cumulative Performance ( <i>Cperf%</i> )	-0.06 0.00	519.42 < 0.0001	-0.05 0.00	398.53 < 0.0001	-0.06 0.00	533.57 < 0.0001	-0.06 0.00	401.83 < 0.0001
Return Volatility ( <i>Mvol</i> )	0.45 0.03	176.52 < 0.0001	0.50 0.03	216.19 < 0.0001	0.44 0.03	170.33 < 0.0001	0.53 0.03	239.42 < 0.0001
Style ( <i>Istyle</i> )			0.46 0.04	124.35 < 0.0001			0.44 0.04	112.57 < 0.0001
Self-Declared ( <i>Sstyle</i> )					-0.12 0.02	40.81 < 0.0001		
Phases ( <i>P<sub>1,2</sub></i> )							0.27 0.04	35.22 < 0.0001

The model results support the main hypotheses. Demand for information has been found to be influenced by the nature of short-term feedback in the form of portfolio returns; both recent and cumulative returns appear to impact behaviour. The negative regression coefficients for *Pperf%* and *Cperf%* indicate that search effort increases disproportionately in the domain of losses versus gains, consistent with risk aversion and response to uncertainty. Investors therefore appear to be highly sensitive to their own recent performance and the nature of short-term feedback received within the context of the overall market environment, varying information search accordingly. These results are consistent with the findings of Lurie, Jayashankar and Swaminathan (2009) who found the similar evidence relating to the impact of feedback on information demand. General short-term market volatility captured by *Mvol* was also found to impact search effort, the positive regression coefficients revealing that demand for information increases in periods of higher short-term volatility. These results are consistent with the empirical findings of Vlastakis and Markellos (2012) and Dimpfl and Jank (2016), but add important new information in terms of the effect of short-term

performance-related feedback in shaping behaviour as noted above. This behaviour was also found to vary according to investment style. Greater explanatory power was derived from the broader style classification (*Istyle*), which assigned participants to either *Technical* or *Fundamental* groupings. The latter category encompasses three sub-styles declared by participants (*Growth*, *Judgmental* and *Value*). Performing the analysis on the basis of each sub-style (*Model 3*) still found *style* to be highly significant. Adding a categorical variable to capture *Phases* had only a marginal effect on the overall explanatory power of the models, despite the significance of the variable. This would suggest that the return-based IVs along with the measure of short-term volatility substantially captured the variability of the DV. Overall, the models are broadly similar in terms of quality and explanatory power. However, adding both the style and phase dummies, *Istyle*, *Sstyle* and  $P_{1, 2}$ , improves the models. Based upon confidence tests and goodness of fit, *Model 4* is marginally the better.

### **8.6.1. Validating Self-Declared Styles**

In order to evaluate the validity of the use of self-declared styles within the models, additional analysis is needed. The categorical variables used in the models are based upon self-declared investment styles; while it might be expected that participants would at least adhere to the broad characteristics of the various styles at the outset of the experiment, there is no guarantee that they did not switch styles throughout its course. In addition, certain styles, particularly *Judgmental* and *Technical*, may adopt various biases over time as a consequence of the fluidity of their approaches (Shiller, 1999). For example, *Judgmental* investors may modify their perceptions of potential investments as market characteristics change, perhaps becoming more risk averse or risk seeking in different periods. As a result,

they may switch between growth and value over time, changing portfolio biases accordingly (Blackburn, Goetzmann & Ukhov, 2012). Examining frequency charts showing aggregate information search per period by self-declared style does suggest differences in search intensity across styles. (In order to take account of the differing number of participants in the various styles, the frequency data discussed below has been averaged.)

The frequency charts shown in Figure 12 provide insights into the information demand attributable to specific styles. The *Technical* style grouping exhibited the greatest level of demand for information across the study, averaging 76.6 total searches per style participant over the whole period. Demand was particularly strong during the initial period of market weakness and picked up again coincident with the reversal in trend of the broad market. The preponderance of all search activity occurred in *Phase 1* (69.3% of total searches) while 25.9% occurred during *Phase 2*. *Growth* and *Judgmental* style participants showed similar patterns of demand (averaging 44.3 and 44.7 overall searches per style participant respectively). Most search activity again occurred during the course of *Phase 1* (64.6% and 66.7% respectively), more than double the level of search activity in *Phase 2* (31.8% and 31.5%). The *Value* style showed the lowest level of search activity by a considerable margin, averaging just 18.6 searches per participant over the course of the study, with 54.7% occurring during *Phase 1* and 35.0% during *Phase 2*. Assuming consistency with regard to style biases, the above findings support the hypothesis that information demand is influenced by investment style. The clear distinction between *Technical* and *Value* style participants might be expected as the former tends to have a short-term focus while the latter is inherently longer term with regard to investment horizon. The findings for the *Judgmental* and *Growth* styles suggest a degree of similarity in terms of behaviour.

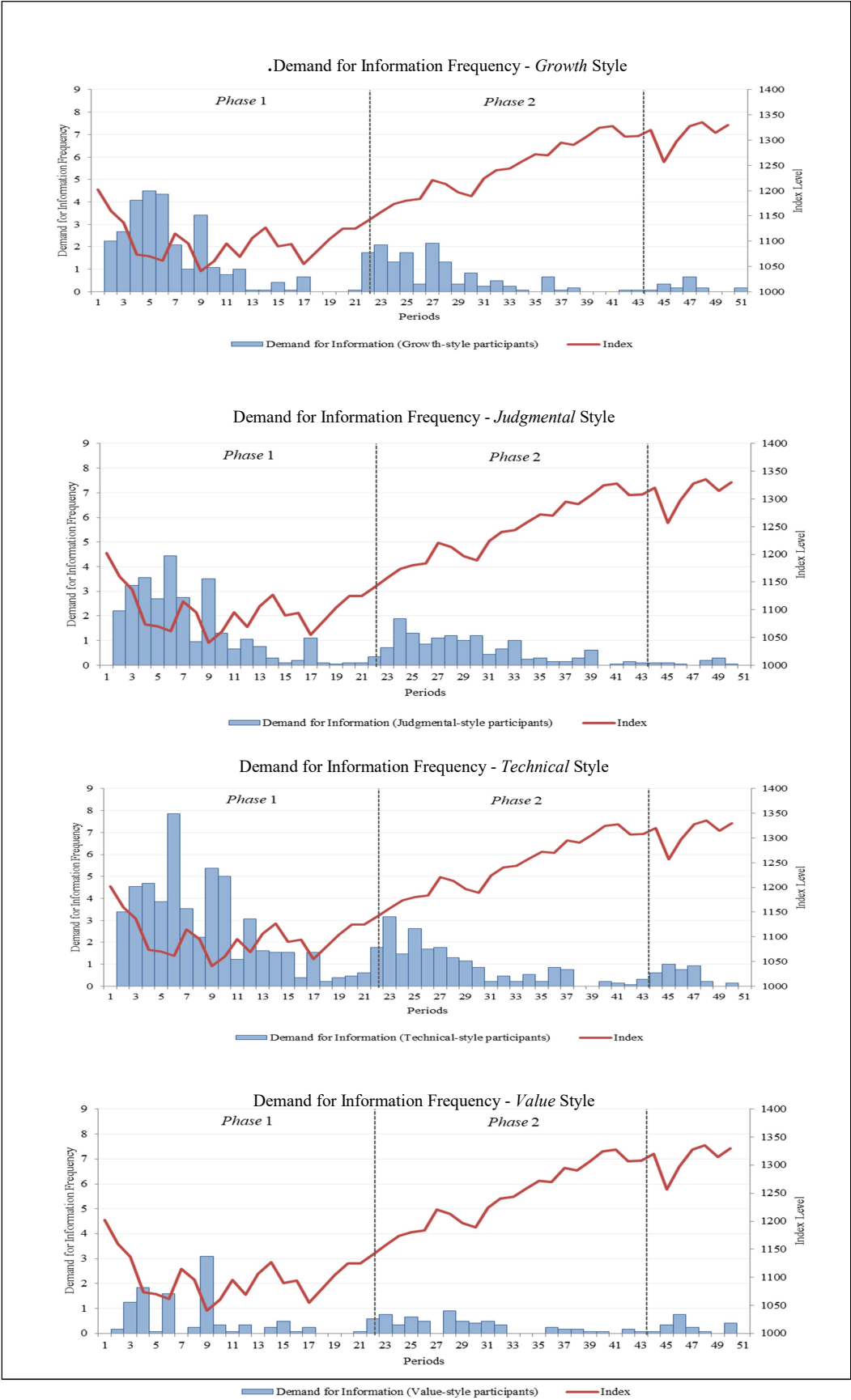


Figure 12. Frequency data of  $f(DS)$  by self-declared style overlaid with the market Index.

### 8.6.2. Using Factor Analysis to Deduce Style

A more formal investigation of characteristic portfolio biases which may be indicative of certain investment styles was undertaken using factor analysis. Factors are general characteristics which describe stocks, such as the fundamental variables described earlier in relation to the stock screens available through the study. Relative factor exposures would be expected to vary according to investment style (Taylor, 1999, Peltomaki, 2017). Therefore, value-biased portfolios would be expected to display above-average weighted exposures to some or all of the well-defined value metrics such as: *EP*; *DY*; *PBV*. Analysis of factor exposures therefore enables an assessment to be made about the key metrics which drive stock selection through time as well as the consistency with which those metrics are applied; the latter then providing an indication of the degree of adherence to a defined style.

The aggregate exposure of a portfolio at period  $t$  to a specific factor,  $a$ , can be derived from;

$$P'(a^t) = \sum_{i=1}^n w(i)^t N^t . a^t \quad \text{Equation 40}$$

where  $w(i)^t$  is the weighting of stock  $i$  in the portfolio at period  $t$  and  $N^t . a^t$  represents the normalised exposure of stock  $i$  to factor  $a$  at period  $t$ .

The normalised value for each stock is obtained from the usual standardisation procedure;

$$Z_{(F_i)} = (F_i - \hat{F}_i) / S_{F_i} \quad \text{Equation 41}$$

where  $\hat{F}_i$  is the mean of the distribution and  $S_{F_i}$  is the standard deviation.

Outliers which would distort the mean and standard deviation of the raw data, and hence the standardised scores, are excluded; means and standard deviations are therefore calculated without the impact of outliers. Winsorization is then applied so that outliers are bounded by standardised scores of 2.05, -2.05 (the 98<sup>th</sup> and 2<sup>nd</sup> percentiles of a standardised distribution respectively).

Factor analysis of *Fundamental* portfolios showed systematic biases towards a number of the core metrics which would be associated with growth and value styles. Summary data relating to factor exposures for *Fundamental* and *Technical* portfolios is shown in Table 25.

Table 25. Weighted average factor exposures according to style (Fundamental v Technical)

General Factor Characteristic	<i>Fundamental (Growth, Judgmental and Value) Style</i>					<i>Technical Style</i>					
	Max Exposure	Min Exposure	Average Exposure	SD	Coeff of Var	Max Exposure	Min Exposure	Average Exposure	SD	Coeff of Var	
<i>Value</i>	<i>EP</i>	0.71	0.23	0.48	0.16	0.33	0.06	-0.07	-0.02	0.04	<i>NM</i>
	<i>DY</i>	0.64	0.50	0.57	0.04	0.07	0.09	-0.11	0.02	0.06	<i>NM</i>
	<i>PBV</i>	0.13	-0.01	0.06	0.04	0.59	-0.13	-0.22	-0.18	0.03	<i>NM</i>
	<i>PCF</i>	0.30	0.01	0.16	0.10	0.61	0.08	-0.13	-0.02	0.07	<i>NM</i>
<i>Relative Value</i>	<i>PEG</i>	1.08	0.81	0.94	0.09	0.10	0.52	0.43	0.48	0.02	0.05
<i>Growth</i>	<i>EPSG</i>	0.66	0.35	0.64	0.04	0.07	0.66	0.41	0.44	0.01	0.03
<i>Technical</i>	<i>RSPERC</i>	0.53	0.20	0.35	0.09	0.25	0.93	0.18	0.44	0.17	0.38

The maximum and minimum exposures in the above Table are calculated from the individual period weighted portfolio exposures across the entire study period (51 periods). As has been noted above, the *Fundamental* category covers three distinct styles (*Growth, Judgmental and Value*). This aggregated grouping might therefore be expected to be something of a hybrid of the characteristics of those individual styles. This proposition is supported by the high average exposure to the relative value measure, *PEG* (earnings growth relative to P/E ratio) and well above average exposure to *EP, DY* and *EPSG*. The coefficient of variation for *DY*



and *EPSG* is low, suggesting that these characteristics are possibly significant determinants of stock screening, biases which remained stable across the course of the study.

Based upon the data, there are significant differences between the characteristic factor exposures of *Fundamental* and *Technical* style portfolios. Whereas the *Fundamental* classification shows significant and consistent bias towards particular metrics (*DY*, *EPSG* and *PEG*), the same profile is not apparent for the *Technical* style. This might well be expected: in the case of price-driven investors, consideration of fundamental data is likely to be of secondary importance, suggesting that fundamental factor exposures are more likely to be the residual outcome of some other primary processes which determine stock selection. In essence, technical investors are typically considered to base decisions upon relatively short-term price movements or defined historical patterns. Only one variable in the above analysis, *RSperc*, has any direct bearing as a technical input as it is a measure of relative price momentum. Nevertheless, it will be noted that this self-declared style still exhibited some consistency in terms of certain fundamental factor exposures; *PEG* and *EPSG* both show positive factor exposure bias with consistency over the course of the study. It is possible that technical analysis based upon emerging price momentum, as opposed to a contrarian bias, may naturally carry some systematic factor biases in common with growth investing. It is also possible that technical investors use certain fundamental metrics as part of their filtering process. Therefore, while price patterns may represent necessary conditions for potential stock selection, a secondary screening process may differentiate on the basis of specific fundamental factors as a means of reducing stock selection risk. Similarly, an inherently contrarian technical style may at times create a fundamental value factor bias. These possibilities again highlight the potential breadth and diversity of the potential investment strategies which become grouped together under broad style classifications.

Based upon the observed data, there are significant overall differences in terms of factor exposures between participants classified either as *Fundamental* or *Technical*. Evidence that there is a degree of consistency with regard to core factor exposures suggests that self-declared styles can be used with some validity in the regression models. For most of the fundamental factors, there is a clear difference in terms of exposures between the two classifications. Even in the case where biases are in the same direction, as with *PEG* and *EPSG*, the degree of factor bias is significantly higher within the *Fundamental* style. Formal t-tests show a significant effect for style;  $t(51) = 34.02, p < .00001$  in the case of *PEG* and;  $t(51) = 86.72, p < .00001$  in the case of *EPSG*.

A breakdown of the *Fundamental* category into its style-constituent parts using the same methodology as above is shown in Table 26. The two most clearly defined styles, *Growth* and *Value*, reflect certain factor biases consistent with those styles. *Growth* shows a clear aggregate bias towards *PEG* and *EPSG*. There is some bias towards *EP* alone from the value factors while the technical indicator, *RSperc*, appears to be coincidental to the primary drivers of stock selection. *Value* shows a clear bias towards *DY* while exposure to *EP* varies considerably over the period. There is a consistently positive exposure to *EPSG* suggesting that above average value could be paired with positive earnings dynamics for the period of the study. This is further confirmed by the positive exposure to *PEG*, indicating that stocks with attractive rates of earnings growth could be acquired at reasonable valuations at times during this period. Such a factor profile is therefore consistent with value investors able to meet thresholds for core value characteristics while gaining exposure to other positive fundamental factors related to earnings trends, again diversifying risk.

Table 26. Weighted average factor exposures based upon Fundamental sub-styles

		<i>Growth Style</i>					<i>Judgmental Style</i>				
Geeral Factor Characteristic		Max Exposure	Min Exposure	Average Exposure	SD	Coeff of Var	Max Exposure	Min Exposure	Average Exposure	SD	Coeff of Var
<i>Value</i>	<i>EP</i>	0.46	0.10	0.32	0.12	<i>0.37</i>	0.88	0.21	0.50	0.22	<i>0.44</i>
	<i>DY</i>	0.09	-0.03	0.03	0.03	<i>NM</i>	1.00	0.79	0.89	0.06	<i>0.07</i>
	<i>PBV</i>	0.07	-0.03	0.03	0.03	<i>NM</i>	0.11	-0.08	0.01	0.05	<i>NM</i>
	<i>PCF</i>	0.16	-0.23	-0.01	0.15	<i>NM</i>	0.30	0.09	0.19	0.06	<i>0.33</i>
<i>Relative Value</i>	<i>PEG</i>	1.37	1.18	1.26	0.06	<i>0.05</i>	0.83	0.60	0.70	0.06	<i>0.09</i>
<i>Growth</i>	<i>EPSPG</i>	0.94	0.00	0.89	0.13	<i>0.14</i>	0.64	0.56	0.61	0.02	<i>0.04</i>
<i>Technical</i>	<i>RSPERC</i>	0.61	0.04	0.40	-0.13	<i>NM</i>	0.73	-0.11	0.36	0.17	<i>NM</i>

		<i>Value Style</i>				
Geeral Factor Characteristic		Max Exposure	Min Exposure	Average Exposure	SD	Coeff of Var
<i>Value</i>	<i>EP</i>	0.86	0.37	0.61	0.15	<i>0.24</i>
	<i>DY</i>	0.94	0.72	0.81	0.07	<i>0.08</i>
	<i>PBV</i>	0.25	0.02	0.15	0.07	<i>0.49</i>
	<i>PCF</i>	0.44	0.14	0.30	0.10	<i>0.33</i>
<i>Relative Value</i>	<i>PEG</i>	1.09	0.64	0.87	0.17	<i>0.19</i>
<i>Growth</i>	<i>EPSPG</i>	0.49	0.36	0.42	0.04	<i>0.09</i>
<i>Technical</i>	<i>RSPERC</i>	0.59	0.03	0.31	0.16	<i>0.50</i>

As suggested by the earlier analysis, the *Judgmental* style category appears to be something of a hybrid of *Growth* and *Value* styles, with some participants perhaps deliberately diversifying stock selection according to sub-styles (mixing both growth and value characteristics in the portfolios). *DY*, *PEG* and *EPSPG* display positive and consistent factor exposures with *EP* showing positive but varying bias over the period.

Overall, the three styles show some distinctive elements in terms of emphasis upon specific factors. However, there is undoubtedly some commonality in terms of direction of bias and focus. For example, *DY* appears to be the preferred value metric within the timeframe of the study. In addition, the volatile nature of the early period appeared to present opportunities to diversify factor exposures, particularly in the case of investors with a value

bias; therefore, individual investments appeared to be available offering attractive combined characteristics in terms of both value and growth factors. While *Value* and *Growth* styles could be inferred on the basis of observed common biases in portfolio factor exposures, certain participants assigned to the *Judgmental* category could possibly be reallocated to either the *Growth* or *Value* categories based upon their portfolio factor profiles. Overall, however, there is sufficient distinction between the three styles, and a degree of consistency within each individual style, to enable self-defined style groupings to stand.

### **8.7. Trading Propensity**

While investor style has been found to influence relative levels of demand for information, numerous studies have also found a strong link between investment style and trading propensity. In part, it might be expected that levels of trading would be associated with the degree of adherence to particular styles since underlying changes in stock characteristics determine the need to rebalance portfolios (Froot & Teo, 2008). Thus, Blackburn, Goetzmann & Ukhov (2012) found that value investors rebalance less frequently than growth investors, reflecting the fact that a number of standard valuation metrics tend to change relatively slowly. For example, accounting-related measures, such as book value and total asset value, are typically revised in accordance with the reporting of newly announced balance sheet information by companies, often semi-annually or annually. Similarly, dividend pay-outs are typically less volatile than earnings per share over time, although investors may clearly form expectations about the likelihood of dividend cuts or increases and respond accordingly (Surendranath, Jory & Hamid, 2017). While many valuation metrics include price as an element, a large change in price is often needed to radically alter

underlying valuation metrics, meaning that, under relatively normal market conditions, fundamental values tend to evolve over time rather than experience sudden changes. Where sudden changes in valuation do occur, it is therefore more likely to be due to specific events, such as a significant dividend cut or major write-offs affecting book and asset values. As a consequence, the need for portfolio rebalancing based upon changing underlying valuation metrics is relatively limited in comparison with certain other styles.

In the case of technical investors, potential trading signals are likely to occur with much greater frequency. While particular patterns of stock behaviour may evolve over time, other triggers, for example, prices breaking above or below key moving averages, might occur much more regularly (Huang & Huang, 2018). To illustrate, during the course of *Phase 1* of the study, closing stock prices were found to have broken above their 55-day moving averages on 136 occasions, while breaking below the moving averages on 142 separate occasions; similarly, the 21-day moving broke above the 55-day moving average on 41 occasions, breaking below 57 times<sup>9</sup>. Technical investors using moving averages in order to derive signals would therefore have been exposed to a large number of potential triggers. While trading propensity may be a function of the need to rebalance portfolios in order to maintain adherence to a particular fundamental style, there can be many other motivations to trade, including controlling risk within a portfolio or changing expectations. Over time, investors may become more or less risk-averse. For example, declines in asset prices have been associated with changes in subjective expected returns as well as perceptions of risk (Beaud & Willinger, 2014). Cohn, Engelmann, Fehr and Maréchal (2015) found strong evidence of countercyclical risk aversion following the priming of investment professionals in

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<sup>9</sup> Many of these events would have related to the same stocks over the course of the study. For example, many of the stocks which experienced prices breaking below moving averages during the initial period of price weakness would subsequently have seen them rise above the same moving averages as the market recovered.

relation to booms and crashes; risk aversion rising significantly in the case of the latter. This, in turn, was found to influence the level of trading. The data gathered during the course of the main study made it possible to examine trading propensity as a function of information demand, feedback, fundamental and technical data. Casual examination of the underlying data showed significant differences between the number of trades entered into in aggregate across the various styles. Sales and purchase transactions were also distributed somewhat differently throughout the study. Figure 13 summarises the aggregate data for each investment style. While it can immediately be seen that the number of purchase and sale transactions differed substantially across styles, it is also noticeable that sale transactions outweighed purchase transactions during the period of market weakness, denoted as *Phase 1*. This pattern was reversed as the broad market, and therefore asset prices generally, started to recover with more evidence of same-period switching. Net-selling versus net-buying may therefore be a function of intermediate market trends with each contributing in some degree to the extension of those trends.

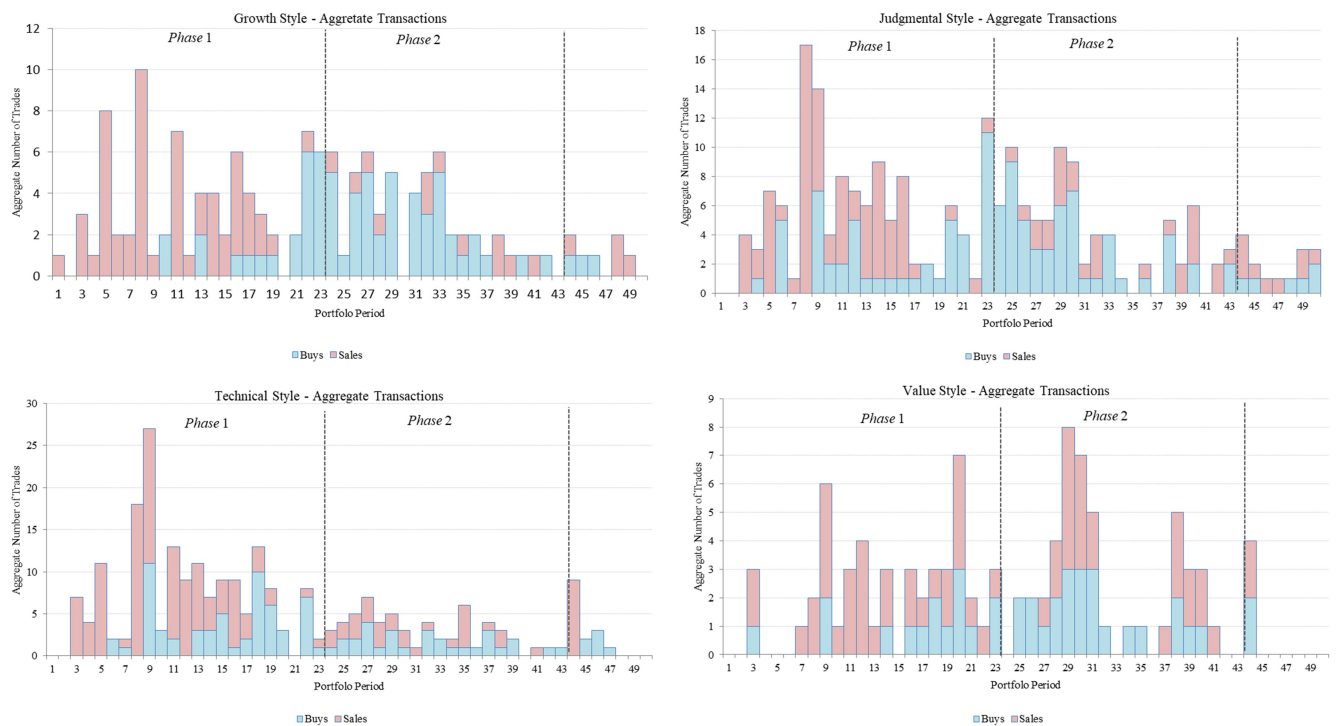


Figure 13. Aggregate number of trades (buys and sales) by investment style.

### 8.7.1. Empirical Tests of Trading Propensity as a Function of Investment Style

Trading propensity of the defined investment styles was investigated using logistic regressions, adopting an approach broadly similar to that employed by Grinblatt and Keloharju (2001), described in the previous chapter. While that study used very extensive data relating to virtually all Finnish investors and sought to distinguish behaviour based upon general investor type, we are more interested in examining the effects of investment style and feedback on trading propensity. Nevertheless, certain comparisons are possible with regard to the results.

Logistic regression computes estimates of parameters which maximise the likelihood of observing the sample values of a binary outcome variable. Regression coefficients therefore represent logit-transformed probabilities of a linear equation defining the relationship between the binary outcome variable and predictor variables. If  $\lambda$  is the binary outcome variable taking the values of 0/1 (failure/success),  $p$  is the probability that  $\lambda = 1$ . For a set of predictor variables,  $x_1, \dots, x_n$ , the parameters  $\beta_0, \dots, \beta_n$  are maximum likelihood estimates which form the linear equation;

$$\text{logit}(p) = \log(p/(1-p)) = \beta_0 + \beta_1 x_1 + \dots + \beta_n x_n \quad \text{Equation 42}$$

The probability associated with the above equation is;

$$p = \exp(\beta_0 + \beta_1 x_1 + \dots + \beta_n x_n) / (1 + \exp(\beta_0 + \beta_1 x_1 + \dots + \beta_n x_n)) \quad \text{Equation 43}$$

Individual coefficients measure the expected change in the log odds of  $\lambda = 1$  given a unit increase in one predictor variable holding all other variables constant. Standardisation of variable coefficients allows for easy comparison of the relative importance of variables expressed in different units, the coefficients then measuring the expected change in the log odds of  $\lambda = 1$  for a 1 standard deviation change in the predictor variable, holding all other variables constant.

As implied by Equation 42, probabilities of individual variables can be calculated from the non-standardised coefficients as follows;  $p_i = \exp(x_i) / (1 + \exp(x_i))$ . The odds ratio, showing the relative likelihood of success ( $\lambda = 1$ ) versus failure ( $\lambda = 0$ ), is related directly to  $p$  and can be derived from;  $p/(1-p)$ , the ratio increasing directly with  $p$ . While probability is bounded by the range 0 to 1, the odds ratio can assume any value between  $-\infty$  and  $+\infty$ . Both the probability and the odds ratio therefore provide indications of the likelihood of a predictor variable coinciding with observed events where  $\lambda = 1$ . Positive predictor variable coefficients are characterised by  $p > .5$  with odds ratios  $> 1$  while negative coefficients are characterised by  $p < .5$  and odds ratios  $< 1$ .

### 8.7.2. Sell-versus-Hold Model

The first logistic regression model tested selling propensity within each investment style using a *sell-versus-hold* output variable. This variable was derived from all holding data



across all portfolios of every participant assigned to one of the four investment styles. The binary outcome variable took the value of 0 when a stock was held in a particular period and 1 in the event of a sale transaction. A number of predictor variables (PVs) were applied relating to stock-specific returns for relatively short-term, non-overlapping periods, fundamental factors and other relevant metrics (specific factors were added for *Technical*-style participants to capture some unique elements of that style). Stock return measures, including volatility, were used to test the impact of short-term feedback on selling propensity along with potential evidence of contrarian trading; relatively large short-term returns of either sign may create attention bias leading to a behavioural response affecting selling propensity while levels of volatility may have their own independent effects on the outcome variable due to changing levels of risk aversion and uncertainty. Fundamental variables were used to test certain valuation anchors or hurdles which may lead to rebalancing decisions. Demand for information was also tested; it has already been found that the level of demand for information is affected significantly by the nature of short-term feedback. Direct testing of information demand therefore examines the extent to which the amount of data gathered and evaluated translates into actionable decisions.

The PVs were defined as follows;

$P.t_{0-5}$ ; measures the return to each security over the previous 5-trading days (the time interval between portfolio rebalancings in the main study above);

$P.t_{5-10}$ ; measures the return over the previous 5-trading days ( $t - 5$  and  $t - 10$ );

$P.t_{10-30}$ ; measures the stock return over the period from  $t - 30$  to  $t - 10$ .

*S.Vol*; is a proxy for stock price volatility, defined as the 21-day standard deviation of previous daily returns for each stock held in a portfolio.

A number of factor exposures are included, together with the change in each factor over rolling 5-day periods; the factors chosen were the ones found to be actively weighted within the style portfolios, consistent with the analysis described above, specifically: *RSPERC*; *PEG*; *EP*; *DY*; *PBV* and their 5-day changes:  $\Delta RSPERC$ ;  $\Delta PEG$ ;  $\Delta EP$ ;  $\Delta DY$ ;  $\Delta PBV$ .

Other PVs include;

*Hold period*; the length of time the security has been held (investors may be reluctant to sell a position only recently acquired);

*BV+/-*; is the return to each security measured against its cost of acquisition (book cost). It is therefore a measure of cumulative gain or loss over the entire holding period and may provide an indication of possible disposition effects;

*Views*; reflects the number of fundamental or technical views of specific stocks held in the portfolios in each period.

Specific technical factors were included in the analysis for *Technical*-style participants as follows;

*MA-21 Break*; a dummy variable assigned to each stock capturing price breaks of the 21-day moving average (downward and upward breaks considered separately as indicated by the up and down arrows next to the variable label. Positive (negative) breaks are deemed to occur when;  $P^t > (<) MA_{-21}^t$ , given that  $P^{t-1} < (>) MA_{-21}^{t-1}$ .

*MA-55 Break*; a dummy variable assigned to each stock capturing price breaks of the 55-day moving average;

*MA-89 Break*; a dummy variable assigned to each stock capturing price breaks of the 89-day moving average;

*GC-21-55*; a dummy variable denoting a 21-day moving average breaking *above* the 55-day moving average;

*GC-55-89*; a dummy variable denoting a 55-day moving average breaking *above* the 89-day moving average;

*DC-21-55*; a dummy variable denoting a 21-day moving average breaking *below* the 55-day moving average;

*DC-55-89*; a dummy variable denoting a 55-day moving average breaking *below* the 89-day moving average;

Results of the analysis are shown in Table 27.

The binary output variable has *sell* events as the reference variable, each such event taking the value of 1; all hold events take the value of zero. Predictor variable coefficients are shown in standardised form to facilitate comparisons between variables measured in different units; none of the significance or confidence data change as a result of standardisation. Positive coefficients attached to a predictive variable indicate that the probability of the reference event (*sell*) increases (decreases) as the value of the predictive variable rises (falls). Negative coefficients indicate that the probability of the reference event

declines (increases) as the PV coefficient increases (decreases). Coefficients significant at the 5% level are highlighted in bold.

Table 27. Standardised regression coefficients and t-statistics for sell-versus-hold binary DV.

Independent variables		Dependent variable: Sell v Hold Dummy							
		Standardised Coefficients				t-statistics			
		Growth	Judgmental	Technical	Value	Growth	Judgmental	Technical	Value
<i>Stock return variables</i>	P.t 0 - 5	<b>-0.45</b>	<b>-0.40</b>	<b>-0.50</b>	-0.06	<b>-4.33</b>	<b>-5.05</b>	<b>-5.85</b>	-0.56
	P.t 5-10	0.08	0.02	<b>-0.21</b>	-0.10	1.05	0.27	<b>-3.33</b>	-1.10
	P.t 10-30	-0.12	-0.04	<b>-0.20</b>	0.06	-1.27	-0.56	<b>-3.25</b>	0.71
<i>Volatility</i>	S. Vol	-0.17	-0.04	<b>-0.26</b>	-0.34	-1.64	-0.70	<b>-3.45</b>	-1.25
<i>Fundamental factors</i>	PEG	<b>-0.26</b>	-0.06	-0.07	-0.10	<b>-2.68</b>	-0.75	-1.05	-0.83
	EP	0.01	-0.14	0.02	0.08	0.07	-1.73	0.31	0.98
	DY	-0.10	0.03	<b>-0.17</b>	-0.09	-1.06	0.43	<b>-2.50</b>	-0.90
	PBV	0.09	0.13	0.05	0.03	1.06	1.82	0.79	0.35
<i>Momentum factors</i>	RSPEC	0.05	-0.10	0.03	-0.14	0.50	-1.42	0.41	-1.31
$\Delta$ Factors	$\Delta$ PEG-5	-0.02	-0.03	0.02	<b>-0.26</b>	-0.23	-0.44	0.24	<b>-4.49</b>
	$\Delta$ EP-5	-0.05	-0.06	-0.12	-0.02	-0.05	-0.68	-1.56	-0.26
	$\Delta$ DY-5	-0.04	-0.08	0.02	0.06	-0.37	-1.06	0.22	0.76
	$\Delta$ PBV-5	-0.04	-0.01	0.10	-0.10	-0.37	-0.15	1.38	-1.12
	$\Delta$ RSPEC-5	<b>-0.24</b>	0.05	<b>-0.16</b>	-0.15	<b>-2.36</b>	0.69	<b>-2.36</b>	-1.49
<i>Other variables</i>	Hold Period	0.20	0.01	-0.06	0.03	1.81	0.16	-0.76	0.29
	BV +/-	0.02	0.00	-0.18	-0.20	0.23	0.02	-1.65	-1.14
	Views	<b>0.26</b>	<b>0.23</b>	<b>0.29</b>	<b>0.24</b>	<b>5.00</b>	<b>5.46</b>	<b>7.38</b>	<b>5.40</b>
<i>Explicit technical variables</i>	MA-21 Break ▼			<b>0.12</b>				<b>2.56</b>	
	MA-55 Break ▼			0.00				0.02	
	MA-89 Break ▼			0.01				0.22	
	MA-21 Break ▲			-0.04				-0.48	
	MA-55 Break ▲			-0.04				-0.62	
	MA-89 Break ▲			<b>-0.12</b>				<b>-2.13</b>	
	GC-21/55			<b>-0.12</b>				<b>-3.02</b>	
	GC-55/89			-0.28				-0.01	
	DC-21/55			0.01				0.29	
	DC-55/89			-				-	
<i>Pseudo R<sup>2</sup></i>	McFadden	0.26	0.17	0.22	0.13				
	Nagelkerke	0.29	0.20	0.27	0.15				
<i>Confidence</i>	-2 Log(Likelihood)	145.49	129.97	238.11	65.88				
	Score	209.13	179.40	346.83	99.08				
	Wald	111.31	115.70	177.81	60.67				

All confidence measures significant,  $p > \chi^2 < .0001$

Short-term return variables showed some significance, particularly  $P.t_{0-5}$  (the latest period return); this variable was found to be significant for all styles other than *Value*.  $P.t_{5-10}$  and  $P.t_{10-30}$ , were found to be significant for the Technical style alone. In each case, the negative coefficients indicate that positive short-term stock returns reduce the likelihood of sell events while price weakness tends to increase selling propensity. The aggregate impact

of short-term returns is strongest for the *Technical* style, as might be expected given the assumed importance of short-term price changes to adherents of that style. Conversely, *Value*-style investors appear to have been relatively indifferent to short-term price returns, again consistent with perceptions of that approach as being inherently longer-term and driven primarily by fundamental factors. General stock price volatility was found to be significant for *Technical*-style investors. Note, however, that the sign of the coefficient is negative for each style, indicating that higher volatility by itself tended to make selling activity less likely. This is consistent with the effects of uncertainty. Few of the fundamental variables were found to be significant, although *PEG* and *DY* were significant for *Growth* and *Value* style investors respectively. Rates of change in the variables, while generally lacking significance, were found to have negative coefficients; improving fundamentals therefore tended to reduce the probability of a sale resulting.

No significance was found for either the length of the holding period (*Hold Period*) or gains and losses in relation to book costs (*BV+/-*). These findings are consistent with a proposition that investors tended to re-evaluated investments with a degree of objectivity to the extent that a short holding period was no barrier to potentially selling a position. The absence of bias in relation to cumulative gains or losses means that there is no evidence of a disposition effect as captured by *BV+/-*. Significance was found for *Views*, the predictive variable which measures the level of information demand. The coefficients were positive for all styles meaning that the likelihood of *sells* tended to increase in line with rising demand for information. This is again consistent with the proposition that investors re-evaluated portfolio holdings objectively on the basis of the latest available information before final sell decisions were executed. Comparing standardised coefficients reveals that short-term returns and *Views* carried the greatest significance across all styles other than *Value*.

Specifically with regard to technically-driven investors, a number of price-based metrics were tested. The behaviour of prices in relation to moving averages of various durations were used to identify positive and negative turning or “break” points in price trend. Thus, the *MA-21 Break ▼* variable identified events where a share price closed below the 21-day moving average of closing prices having been above the moving average on the basis of the prior closing price. Both downward breaks, denoted by the ▼ suffix to the variable and positive breaks (denoted by ▲) were examined. In addition, similar breaks associated with the moving average themselves were considered. Thus, *GC-21/55* recorded events where the 21-day moving average closed above the 55-day moving average, having closed below it on the previous day. Such positive breaks are often referred to as “Golden Crosses” within the framework of technical analysis and are considered to be bullish signals; equivalent downward breaks are often known as “Death Crosses” and are taken as negative signals with regard to future price evolution (Huang & Huang, 2018). Each of these technical predictive variables is binary, taking the value 1 when the event occurs and 0 otherwise. Downward breaks were found to be significant for the shortest duration measure (*MA-21*), the positive coefficient indicating that the likelihood of a sale increased with such breaks. No such significance was found for negative breaks in relation to either the 55- or 89-day moving averages. Upward price breaks did not mirror the evidence for downward breaks; neither *MA-21* nor *MA-55* was found to be significant. *MA-89* and *GC-21/55* were found to be significant with negative coefficients, as would be expected. The finding with regard to *MA-89* appears somewhat anomalous, however, given the lack of significance for the other shorter time-period variables. (No significance was found for the other variables (*DC-55/89* was removed from the model as there were too few such events to be meaningful).

The individual regressions models generated significant improvements over the null hypothesis (constant-only), as evidenced by the *Pseudo R<sup>2</sup>* measures (McFadden and Nagelkerke). The *Judgmental* and *Value* models represent moderate fits of the data while the *Growth* and *Technical* models showed good explanatory power in relation to the output variable. The confidence measures were all highly significant, again indicating that each of the models was a significant improvement over the constant-only model. Despite some apparent anomalies in relation to certain regression coefficients, most of the findings appear intuitive. In general, positive short-term returns and improving fundamentals tend to reduce the likelihood of a stock being sold. The most important finding relates to demand for information with heightened search activity associated with a greater likelihood of sell decisions being made for all styles of investors. This is consistent with the proposition that investors employ new information when re-evaluating holdings and appear to behave objectively in terms of the decisions taken. As a result, holding period and cumulative gains or losses were not found to be biasing factors and there is no evidence of any irrational bias such as the sunk cost effect

### **8.7.3. Sell-versus-Buy Model**

It will be recalled that no shorting (selling of stocks not already held) was allowed in the main study. For that reason, it was possible to consider the sell-versus-hold decision described above based solely upon the individual holdings of portfolios created by the participants. Purchases, on the other hand, can be of two types; either additions to existing holdings or purchases of stocks not already held. On that basis, the predictor variables must be modified to include relevant data relating to new stocks entering the various portfolios.

The output variable in the sell-versus-buy model mirrors that used in the sell-versus-hold model above. In this case, however, stocks which are held in a portfolio in any period for which there is no transaction (purchase or sale) are excluded from the analysis. The binary output variable is then constructed such that *sell* events again take the value of 1 (as above) while *buy* events take the value of 0. The description of the predictor variables remains the same as before. The results of the model are shown in Table 28.

Table 28. Standardised regression coefficients and t-statistics for sell-versus-buy binary DV.

Independent variables		Dependent variable: Sell v Buy Dummy							
		Coefficients				t-statistics			
		Growth	Judgmental	Technical	Value	Growth	Judgmental	Technical	Value
<i>Stock return variables</i>	P.t 0-5	<b>-1.06</b>	<b>-0.95</b>	<b>-0.77</b>	0.34	<b>-3.00</b>	<b>-5.96</b>	<b>-3.33</b>	1.24
	P.t 5-10	-0.38	0.00	<b>-0.23</b>	0.20	-1.89	-0.04	<b>-2.13</b>	1.14
	P.t 10-30	-0.44	-0.38	<b>-0.34</b>	-0.07	-1.76	<b>-3.23</b>	<b>-2.65</b>	-0.37
<i>Volatility</i>	S. Vol	<b>0.80</b>	0.18	0.26	0.47	<b>2.22</b>	0.85	1.67	1.84
<i>Fundamental factors</i>	PEG	-0.36	0.20	-0.09	-0.52	-1.37	1.31	-0.74	-1.88
	EP	-0.42	-0.26	0.24	0.16	-1.65	-1.84	1.92	0.92
	DY	0.20	-0.06	-0.25	-0.22	1.06	-0.47	-2.02	-0.98
	PBV	<b>0.75</b>	<b>0.33</b>	0.07	0.00	<b>2.72</b>	<b>2.79</b>	0.56	0.00
<i>Momentum factors</i>	RSPERC	<b>0.86</b>	-0.13	0.01	-0.11	<b>3.14</b>	-1.02	0.11	-0.52
$\Delta$ Factors	$\Delta$ PEG-5	0.13	0.09	-0.02	-0.32	0.68	0.83	-0.16	-1.69
	$\Delta$ EP-5	-0.07	-0.28	-0.23	-0.29	-0.27	-1.73	-1.05	-1.46
	$\Delta$ DY-5	<b>-0.72</b>	0.25	0.08	<b>-0.66</b>	<b>-2.20</b>	1.73	0.46	<b>-2.35</b>
	$\Delta$ PBV-5	-0.65	-0.14	0.28	0.28	-2.03	-1.05	1.67	1.42
	$\Delta$ RSPERC-5	-0.51	<b>0.29</b>	<b>0.34</b>	0.20	-2.07	<b>2.40</b>	<b>2.22</b>	1.08
<i>Other variables</i>	Hold Period	-0.35	0.02	0.06	-0.16	-1.59	0.21	0.55	-0.82
	BV +/-	-0.53	0.01	0.14	0.03	-2.08	0.11	1.39	0.16
	Views	0.11	0.06	-0.12	-0.22	0.42	0.46	-0.98	-1.30
<i>Explicit technical variables</i>	MA-21 Break ▼			<b>0.41</b>				<b>2.20</b>	
	MA-55 Break ▼			-0.07				-0.48	
	MA-89 Break ▼				2.36			0.01	
	MA-21 Break ▲			-0.05				-0.42	
	MA-55 Break ▲			0.00				-0.04	
	MA-89 Break ▲			0.03				0.24	
	GC-21/55			0.12				1.41	
	GC-55/89								
	DC-21/55								
	DC-55/89								
<i>Pseudo R<sup>2</sup></i>	McFadden	0.48	0.31	0.34	0.24				
	Nagelkerke	0.56	0.46	0.50	0.37				
<i>Confidence</i>	-2 Log(Likelihood)	111.09	92.78	108.10	0.32**				
	Score	74.53	72.26	83.04	26.13				
	Wald	29.27	49.28	46.93**	18.65				

Confidence measures significant,  $p > \chi^2 < .0001$ , for *Growth* and *Judgmental* styles and -2Log and Score for the *Technical* style; significance at the 5% is shown by \*\*. The Score and Wald values for *Value* are not significant.



The coefficients in the Table indicate the extent to which predictor variables make a sell decision more likely in comparison with buy decisions. Once again, coefficients significant at the 5% level are shown in bold.

Significance is found for the short-term return variable, P.t 0 –5, for every style other than Value. The other period return variables (P.t 5 –10 and P.t 10 –30) are also significant for the *Technical* style alone. Once again, the more significant values attach to negative coefficients indicating that positive short-term performance reduces the likelihood of sales, all other things being equal. The measure of short-term volatility (*S.Vol*) has a positive coefficient, indicating that volatility makes sales somewhat more likely than buys; the coefficient is significant in the case of the *Growth* style. The fundamental factors are generally not significant, the exceptions being *PBV* for the *Growth* and *Judgmental* styles. However, this factor was not found to have a consistent positive skew within the aggregate portfolios corresponding to those styles; it is therefore unclear how to interpret this finding. The rate-of-change variables relating to the fundamental factors again suggest a general pattern whereby fundamental improvement reduces the likelihood of a sale decision relative to a buy. Significant, negative coefficients are found in relation to *DY* for the *Growth* and *Value* styles, as might be expected, particularly in the case of the latter. The price momentum factor (*RSperc*) is significant only in the case of the *Growth* style. The coefficient is positive, suggesting that relatively positive price momentum increases the likelihood of a stock being sold. This might be consistent with a contrarian approach but could well be anomalous in the context of the particular style for which this output has arisen. The rate-of-change of this variable is also significant for the *Judgmental* and *Technical* styles. The positive coefficients again indicate that short-term improvements in relative price momentum increase the likelihood of sales rather than buys, consistent with a contrarian bias. Holding periods and

gains or losses with respect to book costs are again insignificant across all style, suggesting an absence of behavioural bias with respect to these factors. There is no evidence for the disposition effect, therefore. Demand for information, as captured by *Views*, was found not be significant for any style, indicating that search effort had no obvious bias in terms of impacting the likelihood of sales versus purchase decisions. The variables unique to the Technical style were not found to be significant with the exception of *MA-21 Break ▼*; stock prices breaking down through the short-term moving average therefore increased the likelihood of sales relative to buys, as would be expected .

The *Pseudo R<sup>2</sup>* values appear to indicate that the models offer reasonable explanatory power for the observed data. However, the confidence statistics are somewhat mixed. While the majority are highly significant for each of the measures ( $p < .0001$ ), the Score and Wald statistics are not significant for the *Value* model while the -2 Log Likelihood and Wald statistics are only significant at the 5% level for the *Value* and *Technical* styles respectively. Overall, the *Value* model is found to be the weakest in terms of goodness of fit, confidence and therefore specification.

#### **8.7.4. Discussion**

Interpreting the results of the above analysis should take account of the relatively limited data sets from which they are drawn. For example, the study by Grinblatt and Keloharju (2001), referenced earlier, used over 290,000 binary data points relating to portfolio holdings and transactions and applied over 200 regressors; data for virtually all investors in the Finnish

market, both institutional and retail, were included. The analysis presented here was based upon 57 investors each generating 51 portfolios over the course of the study (50 portfolios from which sell, buy and hold decisions could be extracted as the initial portfolios created in the initial period are ignored for these purposes). The sell-versus-hold model, extracting data relating to the individual holdings of every portfolio, analysed 6843 data points for the output variable (sells and holds) comprising 389 reference points (sells), the latter representing 5.68% of total data points. The sell-versus-buy model analysed 698 data points (sells and buys), the reference points therefore making up 55.73% of total data points. Notwithstanding, a number of interesting observations are possible.

Both the sell-versus-hold and the sell-versus-buy models found strong evidence that short-term stock returns affected the likelihood of transactions occurring; the likelihood of sales increasing following short-term price weakness, the opposite for buys. The most recent period return data generally carried the most significance. Grinblatt and Keloharju found evidence that very recent gains in stock prices increased the likelihood of stock sales, a behaviour they found common to all categories of investors studied; the opposite of the results shown above. Thus, whereas Grinblatt and Keloharju found systematic evidence of contrarian behaviour, the study presented here found evidence more associated with momentum investing, consistent with the findings of Nagel (2005). Similarly, Grinblatt and Keloharju were able to find significant evidence of a disposition effect (the preference for selling stocks with cumulative gains versus losses); no such evidence is found here. Indeed, from the above analysis, investors appear to be highly objective in their decision-making, as evidenced by the heightened search for new information prior to action. There are some similarities between the studies in terms of general stock price volatility. Similar to the Finnish study, the results above show a relatively weak effect for volatility on selling

propensity within the sell-versus-hold model, the exception being for *Technical*-style investors perhaps affected more by the fact that higher volatility is likely to generate more potential trading signals. A slightly stronger effect is found in relation to the sell-versus-buy model, although the signs of coefficients are reversed. Therefore, while the presence of greater short-term volatility would tend to reduce the likelihood of sales within the sell-versus-hold comparison, the likelihood was increased for all styles when comparing sell-versus-buy decisions.

When drawing comparisons between studies, it is important to recognise key differences in how the models are framed. The Finnish study considered classes of investors, such as Finance and Insurance companies, Governmental institutions and non-profit organisations. In contrast, the studies conducted here differentiate according to investment style as opposed to investor type. We are therefore interested in observing characteristics which different investor types have in common as well as those which differentiate them. Further, the studies presented here are based upon a somewhat untypical market environment. That market environment has been shown to affect the behaviour of participants due to the significance of the effects of feedback on behaviour, particularly the level of demand for information. These factors alone may all help to explain the different findings.

Notwithstanding caveats, relating primarily to the limited size of the available data set, we can nevertheless draw some potentially important conclusions. The most significant of these is evidence that investors across all styles appear to demand current information before making trading decisions within their portfolios. The search for new, current data as part of the evaluation process suggests that investors seek to

behave with a high degree of objectivity notwithstanding any uncertainty which short-term market characteristics may present. The evidence suggests therefore that, while demand for information is in part a function of risk aversion and uncertainty (as evidenced by the volatility measures), the new information also informs final decisions. Furthermore, evidence relating to the fundamental variables suggests that information sought is applied systematically. Thus, while the levels of fundamental factors, in terms of normalised exposures of individual stocks to key fundamental metrics, are generally not significant, the rate of change measures of those same variables provide some evidence that improvements in short-term fundamentals reduce the likelihood of a stock being sold while increasing the likelihood of a purchase occurring. This is again consistent with the results found by Nagel (2005) based upon an analysis of mutual fund data. The results do not show evidence of a disposition effect; there was therefore no apparent preference to sell stocks with larger short- or cumulative gains. This suggests that, at the very least, effect may be subject to context. Similarly, there was no evidence of other biases such as the sunk cost effect.

## **8.8. Conclusions**

The studies are found to support the proposition that response behaviour and levels of demand for information are affected by the nature of short-term feedback and the context in which it is received. Losses appear to amplify information demand more than gains, potentially consistent with risk aversion. The studies also suggest that participants were substantially objective in their decision-making, generally seeking the latest available information before making decisions. The overall level of demand for information, in

combination with short-term return metrics, was therefore found to influence selling propensity.

The pattern of sale versus buy transactions suggests that the overall market environment tended to influence the degree of switching in a portfolio (selling one position in order to buy another). In the period of general weakness, a degree of net selling was apparent, although this was reversed once the market, and general price levels, recovered. This would suggest that heightened market volatility, particularly when it is associated with losses, does lead to a certain modification of expectations. It would also appear that price volatility leads to certain triggers, more so for some investment styles than others. Thus, the Technical style, most associated with short-term trading based upon price signals, would ordinarily be most affected by significant short-term volatility. In contrast, Value-style investors are likely to be much less affected. The results of the study are therefore consistent with other empirical findings with regard to patterns of volatility and trend-reinforcing reactions (Lobato & Velasco, 2000; Mandelbrot, 1971). Thus, the emergence of volatility in the marketplace can trigger a behavioural response which itself further contributes to that volatility; the same effect can be seen in relation to trends to the extent that short-term weakness affects current net selling while the emergence of positive trends are more likely to encourage the commitment of any net cash.

Since short-term performance and cumulative returns have been shown to play a significant role in terms of affecting current demand for information, the results described above should also be interpreted within the context of the overall portfolio return distributions of the participants. Examination of the entire data from the study revealed that the average return across all 57 participant portfolios was 18.03%, higher than the Monte Carlo simulated

mean of 16.02% and significantly higher than the 13.16% generated by the market Index over the period. Based upon the return distribution parameters derived from the randomly generated Monte Carlo portfolios, such average outperformance of the Index has only a 6.26% probability of occurring by chance, strongly implying a degree of skill on the collective part of the participants. (The highest individual portfolio return was 26.93%, an outcome with only a 1.29% probability of occurring by chance.)

Based upon self-declared investment styles, significant differences were found between average portfolio returns: Judgmental (M = 22.18%, SD = 2.83%) and Value (M = 20.19%, SD = 2.67%) styles generated portfolio returns significantly in excess of the returns achieved by Growth (M = 15.57%, SD = 2.25%) and Technical (M = 11.92%, SD = 3.45%) styles ( $p < .002$ ), with Growth styles significantly outperforming the Technical style ( $p < .02$ ). The high level of skill implied by these general results may therefore have consequences for the type of behavioural responses observed in the study; it is not clear, therefore, how far the general findings might be extended to a broader array of investors where skill sets differ.

With regard to Technical style investors, it is also worth noting that the structure of the study may have inadvertently affected their performance more than other style investors. Portfolio trading was possible only on portfolio valuation periods; it will be recalled that these periods covered underlying intervals of five trading days. This means that the one style most associated with short-term trading might have been denied access to better trading opportunities, reducing their collective performance.

### **8.8.1. Limitations of the Study**

There are a number of limitations which should be considered relating to the study. While a considerable amount of data was made available to participants, it remained well short of an actual market environment in terms of both the breadth and depth of data. In particular, the investable universe was limited to just 98 stocks, a fraction of what would ordinarily be available to even the largest institutional investor. In addition, while substantial company-specific data was presented, the absence of relevant news, company announcements and analyst opinions meant that these potentially important sources were absent.

The decision to anonymise the data was necessary in order to prevent participants from potentially identifying the period and changing their behaviour accordingly. However, it also removed any specific expertise or knowledge which may have provided some participants with a potential competitive advantage. Similarly, while it was possible to run many filters on the data in order to narrow down potential investments, there was a limit to how far participants could define their own screens. Therefore, any participants used to running their own stock selection models using different formulations and types of data may have been somewhat limited by the design. Nevertheless, it should also be recognised that, overall, the participants appeared to show a considerable amount of skill, generating returns at levels which would be unlikely to occur by chance, suggesting that, whatever limitations were faced, most participants were able to overcome them.

The study was designed to include distinguishable phases of markets. Since real data was used, this meant covering a large timespan of actual data and shrinking it into a much more condensed timeframe for the purposes of the study. This is inevitably unrealistic and



prevents participants from absorbing any information that intraday movements in prices may confer. Similarly, all trading was assumed to take place at daily closing prices and was only possible with certain frequencies (the 5-day timescale relating to portfolio valuation periods). Therefore, participants had no ability to trade on intraday prices, which could have significantly improved performance in some cases. It has already been noted that this might have adversely affected the performance of Technical style investors in particular.

The study used mouse tracking in order to capture data on certain behaviour of participants. This generated a considerable amount of useful data. Nevertheless, more extensive process tracing techniques could have proved particularly useful. Foremost among these would be eye-tracking as that would have provided even greater insight into the exact data referenced by participants and its sequence. Similarly, there is a possibility that analysing data in the manner conducted here placed undue weight upon “current” over potential longer-term signals, although certain attempts were made to consider information and feedback data over periods other than the latest, for example using return data relating to previous periods. More generally, as an in-case study, caution should be used when extrapolating results. This might be particularly relevant given the apparent level of skill of the participants in the current study.

## CHAPTER 9

### CONCLUSION & DISCUSSION

#### 9.1 Overview

This thesis has explored the role which feedback plays in shaping decision-making behaviour and has also examined factors which affect the level of demand for information within complex environments. In many complex decision-making environments, uncertainty and ambiguity with regard to available choices can be profound. This is perhaps readily apparent within financial markets, particularly in periods of heightened volatility. In such cases, decision-makers, potentially able to access vast amounts of information often in real time, must attempt to distil which elements provide valuable information of relevance to the decision-making process and which are merely noise. To the extent that decision-makers are rational, the effectiveness of decisions may then depend in part upon how new information informs current expectations, the challenge being to seek processes which minimise certain biases which could lead to erroneous decisions.

The major empirical study on financial markets presented in the thesis provides support for the proposition that short-term feedback relating to portfolio performance and recent price volatility are significant factors affecting demand for information and response behaviour. The effect is magnified in the domain of losses relative to gains. The extent to which investors seek new information is, however, conditioned by the investment style adopted (Blackburn, Goetzmann & Ukhov (2014)). Therefore, technical (chartist) investors are found to be significantly more reactive to short-term feedback than judgmental investors, with value-style investors found to be the least reactive among the sub-categories of the latter general

classification. The relative preponderance of investment styles at any one time would therefore appear important in relation to observed phenomena such as volatility clustering (Mandelbrot, 1971) as short-term traders in particular are likely to further contribute to price volatility due to the nature of their reactive behaviour.

Similarly, the study found evidence that reaction to short-term feedback has a tendency to reinforce short-term trends. Thus, negative feedback with heightened volatility triggered a degree of net selling, a process which reversed once the overall market environment stabilised. This would appear to imply that, under conditions of market weakness and relatively high levels of stock price volatility, contrarian investors do not offset more reactive traders who are more likely to reduce overall market exposure to some degree by raising an amount of cash. The analysis of trading propensity was consistent with this proposition and provided some support for the proposition that general market volatility increases uncertainty. Investors may therefore re-evaluate their risk propensity within that context, becoming somewhat more risk averse at least in the short term. The evidence from the studies therefore provides further possible explanations for some of the apparent anomalies observed in financial markets.

## **9.2. Contribution of the Thesis and Suggested Future Research**

This thesis sought to add to existing knowledge in the area of judgment and decision making by focussing upon the role which feedback plays in shaping response behaviour. It has considered numerous core theories, traced their evolution and examined empirical research relating to a range of behavioural traits and biases. The empirical studies used mouse tracking techniques to generate data on behaviour as a prelude to decision-making rather than

the decisions themselves. That data suggested that decision-makers were systematic in their search for, and use of data. In both studies, participants appeared to behave objectively.

In the case of the financial markets study, participants appeared to adopt consistent approaches which could largely be differentiated on the basis of investment styles. This then appeared to shape response behaviour to a sufficient extent that investment style was found to be an explanatory or predictor variable in relation to both the demand for information and trading propensity. Beyond that, the study found no evidence of other systematic biases. This contrasts with other studies which continue to find fairly pervasive evidence of the sunk cost effect, for example (Agarwal, Green, Rosenblatt & Yao, 2015; Bao, Chunming Meng & Wu, 2017). As a result, further research could profitably be undertaken in order to distinguish between the contexts within which sunk cost biases are likely to persist from those where the systematic nature of decision-making minimises or eliminates them. The apparent skill of the participants was reported in the main study, evidenced by the generally strong investment returns which were generated. The question of whether skill and expertise minimises or eliminates irrational bias remains an important area for research.

Much could be done to extend the financial markets study. The use of eye tracking would have added yet more important information relating to the information gathering behaviour of participants over and above that which was captured by mouse tracking alone. More generally, additional development of the simulated environment itself could prove useful, perhaps allowing investors to interact or at least see the trades of other participants. The simulation could also be made more realistic by substantially broadening the data sets with many more potential investments made available and perhaps other sources of information

relating to news and company announcements being included, although the latter would be problematic should a high level of anonymity be required.

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## APPENDIX A :

### SAMPLE PAGE ILLUSTRATING THE SIMULATED ONLINE AUCTION

This sample page format, although heavily simplified in terms of content, would likely be familiar to participants in mainstream online auctions. Navigation buttons are added to ease switching between auctions (top right) while the buttons for bidding and viewing bid histories are contained towards the bottom of the page.

#### Super Deal!

#### Auction Sale Item - No Reserve!

*There is another, identical auction ending at the same time!*

Switch to other auction



Brand New, Suzuki Alto 1.0L. OTR Price £5,995

The Alto's fuel efficient 1.0L engine keeps running costs low; complete with 3 year or 60,000 mile warranty.

Auction Start Time : 08 February 2014 12:10:07  
Auction End Time : 08 February 2014 12:12:07  
Current Time : 10 September 2014 16:28:14  
Auction Time Remaining : **31:12**

Current Highest Bid : £2,400.00

Number of bids: **17**

Current Highest Bidder : **throgmorton**

Click to see bid history

Please click to enter bid of: **£2450**

BID

Figure A1: Sample online auction screen

## APPENDIX B:

### SCHEDULE OF PRE-PROGRAMMED COMPUTER BIDS (STANDARD AUCTIONS)

Table B.2. Schedule of pre-programmed timings and bid levels by auction type.

Slow 1		Slow 2		Fast 1		Fast 2		Slow (Bid Increment : £100)		Fast 3 (With Active switching)		Fast 4 (With Active Switching)	
Seconds		Seconds		Seconds		Seconds		Seconds		Seconds		Seconds	
From	Bid	From	Bid	From	Bid	From	Bid	From	Bid	From	Bid	From	Bid
Start		Start		Start		Start		Start		Start		Start	
2.14	2000	4.87	2000	1.50	850	0.84	850	6.56	900	2.43	850	1.50	850
17.35	2050	16.88	2050	7.55	900	5.45	900	14.34	1000	7.23	900	6.55	900
38.96	2100	39.87	2100	15.43	950	14.77	950	29.55	1100	10.54	950	8.77	950
58.32	2150	64.46	2150	23.43	1000	19.55	1000	58.00	1200	15.65	1000	11.23	1000
75.46	2200	77.55	2200	32.23	1050	27.55	1050	74.54	1300	17.50	1050	18.66	1050
94.54	2250	88.50	2250	41.23	1100	40.23	1100	90.50	1400	20.50	1100	24.53	1100
112.15	2300	101.12	2300	48.76	1150	47.66	1150	106.00	1500	24.50	1150	37.66	1150
127.56	2350	116.55	2350	57.00	1200	52.35	1200	112.34	1600	34.00	1200	51.23	1200
146.00	2400	134.23	2400	60.35	1250	56.55	1250	128.00	1700	39.70	1250	63.45	1250
171.65	2450	167.75	2450	66.45	1300	60.12	1300	144.70	1800	46.50	1300	69.88	1300
196.55	2500	192.23	2500	75.46	1350	72.23	1350	172.32	1900	60.43	1350	77.00	1350
215.32	2550	217.60	2550	84.50	1400	87.35	1400	188.60	2000	66.00	1400	84.30	1400
230.40	2600	231.23	2600	93.68	1450	92.35	1450	200.61	2100	82.50	1450	101.34	1450
244.55	2650	246.00	2650	102.50	1500	101.23	1500	213.40	2200	84.30	1500	106.55	1500
256.45	2700	259.80	2700	108.66	1550	104.80	1550	219.30	2300	86.90	1550	111.23	1550
265.45	2750	264.00	2750	112.35	1600	114.35	1600	223.00	2400	104.20	1600	112.35	1600
274.54	2800	277.69	2800	122.34	1650	122.45	1650	245.40	2500	118.23	1650	122.34	1650
281.23	2850	286.57	2850	128.76	1700	131.88	1700	251.40	2600	142.32	1700	127.54	1700
286.45	2900	288.32	2900	130.23	1750	141.12	1750	255.00	2700	155.45	1750	133.20	1750
292.34	2950	290.23	2950	134.34	1800	149.88	1800	263.40	2800	159.80	1800	137.65	1800
296.55	3000	296.00	3000	139.00	1850	163.23	1850	275.40	2900	164.00	1850	142.34	1850
298.32	3050	297.40	3050	143.00	1900	172.35	1900	290.00	3000	169.70	1900	158.54	1900
299.20	3100	298.90	3100	149.57	1950	182.23	1950	299.80	3100	188.40	1950	175.40	1950
				158.66	2000	190.35	2000			190.30	2000	189.34	2000
				172.40	2050	193.92	2050			194.50	2050	198.35	2050
				181.55	2100	210.23	2100			200.50	2100	209.54	2100
				189.35	2150	214.35	2150			206.50	2150	216.44	2150
				198.35	2200	224.35	2200			212.40	2200	228.66	2200
				209.54	2250	236.46	2250			224.50	2250	236.56	2250
				216.44	2300	238.70	2300			247.50	2300	238.65	2300
				228.66	2350	242.00	2350			255.40	2350	244.34	2350
				236.56	2400	245.35	2400			267.00	2400	249.76	2400
				241.34	2450	248.92	2450			268.40	2450	262.60	2450
				249.66	2500	252.49	2500			270.00	2500	264.43	2500
				254.54	2550	267.57	2550			271.50	2550	266.76	2550
				262.60	2600	271.14	2600			275.40	2600	271.23	2600
				263.10	2650	274.71	2650			277.40	2650	276.45	2650
				264.50	2700	278.28	2700			279.50	2700	277.40	2700
				266.00	2750	282.12	2750			281.40	2750	278.40	2750
				268.30	2800	286.55	2800			282.30	2800	279.50	2800
				274.30	2850	290.23	2850			285.00	2850	283.34	2850
				281.50	2900	291.45	2900			286.50	2900	288.43	2900
				288.40	2950	295.34	2950			288.00	2950	291.00	2950
				292.32	3000	297.54	3000			290.40	3000	295.53	3000
				297.50	3050	298.30	3050			296.50	3050	298.00	3050
				299.60	3100	299.00	3100			299.70	3100	299.60	3100

In order to auctions of the same intensity (fast or slow) to end at the same level, they also had to use the same opening bid level given that the number of pre-programmed computer bids were equal. It will be noted that bid patterns in auctions with the same pre-programmed computed bidding intensity varied in terms of the planned timing of those computer bids (all times were measured from the start of the auction using the computer's internal clock).

Intervals between computer bids were irregular although the gap, in terms of number of seconds, tended to decline as the auctions approached their climax, in order to simulate greater competitive bidding action.

## APPENDIX C

### SCHEDULE OF PRE-PROGRAMMED COMPUTER BIDS (BIN AUCTIONS)

Table C.1 Pre-programmed “Buy It Now” sequences used for concurrent auctions

Auction: Fast 1			Auction: Fast 3		
Pre-Programmed Bid Timings (Seconds from Start)	Bid Price	"Buy It Now" Price	Pre-Programmed Bid Timings (Seconds From Start)	Bid Price	"Buy It Now" Price
1.50	850	2500	2.43	850	2500
7.55	900	2500	7.23	900	2500
15.43	950	2500	10.54	950	2500
23.43	1000	2500	15.65	1000	2500
32.23	1050	2500	17.50	1050	2500
41.23	1100	2500	20.50	1100	2500
48.76	1150	2500	24.50	1150	2500
57.00	1200	2500	34.00	1200	2500
60.35	1250	2650	39.70	1250	2650
66.45	1300	2650	46.50	1300	2650
75.46	1350	2650	60.43	1350	2650
84.50	1400	2650	66.00	1400	2650
93.68	1450	2650	82.50	1450	2650
102.50	1500	2650	84.30	1500	2650
108.66	1550	2650	86.90	1550	2650
112.35	1600	2650	104.20	1600	2650
122.34	1650	2800	118.23	1650	2800
128.76	1700	2800	142.32	1700	2800
130.23	1750	2800	155.45	1750	2800
134.34	1800	2800	159.80	1800	2800
139.00	1850	2800	164.00	1850	2800
143.00	1900	2800	169.70	1900	2800
149.57	1950	2800	188.40	1950	2800
158.66	2000	2800	190.30	2000	2800
172.40	2050	2950	194.50	2050	2950
181.55	2100	2950	200.50	2100	2950
189.35	2150	2950	206.50	2150	2950
198.35	2200	2950	212.40	2200	2950
209.54	2250	2950	224.50	2250	2950
216.44	2300	2950	247.50	2300	2950
228.66	2350	2950	255.40	2350	2950
236.56	2400	2950	267.00	2400	2950
241.34	2450		268.40	2450	
249.66	2500		270.00	2500	
254.54	2550		271.50	2550	
262.60	2600		275.40	2600	
263.10	2650		277.40	2650	
264.50	2700		279.50	2700	
266.00	2750		281.40	2750	
268.30	2800		282.30	2800	
274.30	2850		285.00	2850	
281.50	2900		286.50	2900	
288.40	2950		288.00	2950	
292.32	3000		290.40	3000	
297.50	3050		296.50	3050	
299.60	3100		299.70	3100	

## APPENDIX D

### EXTRACT FROM TECHNICAL AND FUNDAMENTAL INDICATOR TABLES

Table D.1 Extract of data with summary technical (price-based) indicators.

Name	Price	Return Data				Summary Technical Indicators						
		1 Mo	3 Mo	6 Mo	12 Mo	Rel.Str.	RS Perc.	Stoch.	P/MA20	P/MA50	P/MA100	P/MA200
Alsom	17.55	2.7%	9.5%	20.5%	35.1%	1.016	0.560	0.972	1.014	1.037	1.068	1.142
Aplian Healthcare	22.02	-3.2%	-3.8%	1.6%	12.6%	1.003	0.160	0.205	0.984	0.963	0.955	1.008
Bard Healthcare	55.13	-5.3%	-1.9%	6.1%	9.1%	1.000	0.100	0.274	0.967	0.981	0.996	0.989
Blake Oil & Gas	39.05	14.8%	22.9%	17.5%	46.4%	1.025	0.800	1.000	1.051	1.122	1.155	1.209
Bosphur Financial	44.82	13.2%	24.1%	30.6%	108.1%	1.032	0.940	0.917	1.033	1.117	1.150	1.255
Boulton Biotech	43.37	-18.4%	-17.5%	-11.6%	-8.6%	0.986	0.020	0.022	0.870	0.836	0.827	0.844
Carlisle Banking	29.24	0.5%	6.2%	12.1%	10.5%	1.010	0.340	0.796	0.986	1.024	1.068	1.095
Catsom Industrials	68.54	1.9%	21.4%	54.8%	80.7%	1.035	0.960	0.925	1.023	1.058	1.157	1.318
Circle Q	42.80	-5.8%	8.4%	3.2%	16.0%	1.006	0.220	0.223	0.966	0.972	1.002	1.029
Conan Bio	19.88	-12.9%	-16.9%	-7.4%	-15.1%	0.987	0.040	0.006	0.917	0.872	0.872	0.867
Console Freight	55.57	7.3%	28.1%	31.6%	63.6%	1.027	0.860	1.000	1.026	1.087	1.141	1.220

The Table shows participants the current price for each security along with a summary of performance over previous periods (1 month, 3 months, 6 months and 12 months). The technical indicators shown are:

*Rel.Str.* a measure of relative strength, which measures the capital return of each individual security relative to that of the market over the prior 12-month period;

*RS Perc.* expresses the relative strength score as a percentile of the relative strength scores for all of the available securities. *RS.Perc.* ranges from 0 (the security is in the bottom percentile on the basis of performance) to 1 (it is in the top percentile as one of the strongest performing securities over the period);

*Stoch* measures a price stochastic calculated using data for the previous 60 days. This is computed from the general formula;

$$(P_t - P_{min}^{t,t-n}) / (P_{max}^{t,t-n} - P_{min}^{t,t-n})$$

where,  $P_{max}^{t,t-n}$  is the maximum closing price over the period for which the stochastic is being calculated (from  $t-n$  to  $t$ ),  $P_{min}^{t,t-n}$  is the minimum closing price over the same period;  $P_t$  is the latest closing price. The stochastic can therefore range between 0 (the

current price is equal to the lowest price over the period) and 1 (the current price is equal to the highest price over the period);

*P/MA20*, *P/MA50*, *P/MA100*/*P/MA200* refers to the current price divided by a moving average of prices. The period over which the moving average price is calculated is denoted by the numerical suffix applied (e.g. 20, 20, 100 and 200 trading days).

Table D.2. Extract showing fundamental data table.

Fundamental Data										
Name	ESPG	PE	Forecast PE	PEG	DY	Forecast DY	P/BV	P/CF	ROE	ROA
Alsom	21.4%	11.40	9.39	2.28	7.52%	8.32%	8.95	11.47	92.97	10.04
Aplian Healthcare	-19.8%	5.97	7.44	-2.66	7.08%	7.81%	1.49	5.51	28.49	12.12
Bard Healthcare	6.2%	12.22	11.51	0.54	0.00%	0.00%	2.42	9.69	21.39	12.11
Blake Oil & Gas	135.2%	12.05	5.12	26.38	4.89%	5.51%	0.96	36.16	8.26	3.29
Bosphur Financial	117.5%	29.10	13.38	8.78	1.61%	1.61%	3.71	-121.14	16.23	1.7
Boulton Biotech	-33.4%	12.08	18.15	-1.84	2.47%	2.72%	3.62	14.04	32.86	13.46
Carlisle Banking	-276.7%	-25.21	14.26	-19.40	1.74%	1.23%	1.22	-17.10	-5.04	-0.61
Catsom Industrials	141.8%	37.25	15.40	9.21	2.45%	2.45%	23.39	11.07	314.63	2.27
Circle Q	21.0%	13.21	10.92	1.92	5.23%	5.70%	14.13	11.63	95.97	18.78
Conan Bio	17.7%	14.10	11.98	1.48	0.00%	0.00%	5.62	12.99	50.11	31.53
Console Freight	44.9%	20.13	13.89	3.23	2.45%	2.52%	1.98	36.32	10.36	3.85

The data items shown are:

*ESPG*, measuring the year-over-year percentage change in earnings per share;

*PE* is the current share price divided by last reported earnings per share;

*Forecast PE* is the current share price divided by next year's forecast earnings per share;

*PEG* measures the PE relative to earnings per share growth ( $ESPG / PE$ );



*DY* is the dividend yield (last year's dividend per share divided by the share price, expressed as a percentage);

*Forecast DY* uses forecast dividends per share in the calculation;

*P/BV* measure the current share price divided by book value (shareholders' funds) per share;

*P/CF* represents the current share price divided by cash flow per share;

*ROE* measures return on equity (pre-tax earnings divided by book value);

*ROA* measure the return on assets (pre-tax earnings divided by total asset value).