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A Post-Keynesian Theory for Tobin’s q in a Stock-Flow Consistent Framework

Javier López Bernardo1
Engelbert Stockhammer
Félix López Martínez

Abstract: The paper proposes a post-Keynesian framework to explain Tobin’s q behaviour in the long run. The theoretical basis is informed by the Cambridge corporate model originally proposed by Kaldor (1966), which is reinterpreted here as a theory for q. The core of the ‘Kaldorian q theory’ is a negative long-run relation between q and growth rates, a negative relation between q and propensities to consume, and the fact that q can be different from 1 in the long-run equilibrium. We generalise this model through a medium-scale Stock-Flow Consistent (SFC) model, which introduces important post-Keynesian aspects missing in the Kaldorian model, such as endogenous money, a financial system and inflation. We extend the model to include a more realistic treatment of firms’ financial structure decisions and allow the interdependence between these decisions and dividend policy. Numerical simulations confirm that the original Kaldorian relations between q and growth rates and propensities to consume hold, but unlike the original model, in our model q is not independent of how firms finance their investment. We also confirm the possibility of q being different from 1 in the long-run. Finally, we contrast this ‘post-Keynesian q theory’ with the Miller-Modigliani dividend irrelevance proposition and the neoclassical investment and financial theory. It is shown that its validity depends crucially on the value taken by q: for q values different from 1 the proposition will not hold and dividend policy will be relevant for equity valuation. Therefore, post-Keynesian q theory stands against the main predictions of mainstream finance and constitutes an alternative for developing a macroeconomic theory for equity markets.

Keywords: Tobin’s q, post-Keynesian macroeconomic theory, stock-flow consistent models, Cambridge corporate models, Miller-Modigliani dividend irrelevance proposition.

JEL Classifications: E12, E22, E44, G10, O42.

1 Kingston University (London), Kingston University, and EOI (Madrid), respectively. Correspondence e-mail: javieribernardo@gmail.com. We gratefully acknowledge helpful comments and suggestions by Paul Auerbach, Malcolm Sawyer, Dimitris Sotiropoulos, Rafael Wildauer and participants of the 12th Conference, Developments in Economics Theory and Policy, held in Bilbao, June 2015. However, none of them are responsible for the final views expressed here. Javier López also acknowledges financial support by the Ramón Areces Foundation (Madrid).
1. Introduction

Tobin’s $q$ is defined as the ratio of the ‘going price in the market for exchanging existing assets’ to their ‘replacement or reproduction cost’ (Tobin & Brainard, 1977). Since the seminal works of Brainard and Tobin (Brainard & Tobin, 1968; Tobin, 1969; Tobin & Brainard, 1977), Tobin’s $q$ has become an important theoretical construct widely used both by financial practitioners to assess current stock market conditions (Montier, 2014a; Smithers, 2009) and by academics, who have used $q$ as the main explanatory variable in investment functions (Hayashi, 1982). However, none of the two groups have offered an explanation of the movements of $q$ through time: the first group has usually assumed mean-reversion for the $q$ series (with no strong theoretical justification) while the second has been more interested in the role of $q$ as an exogenous variable, not as an endogenous one.

The present paper offers an alternative macroeconomic vision of $q$ based on the ‘Cambridge corporate model’ developed by Kaldor and others in the 1960s and 1970s (Kaldor, 1966; Marris, 1972; Moore, 1975; Moss, 1978). The Cambridge corporate model was originally proposed as a solution for the Harrod-Domar knife-edge dilemma, where equity valuation (not technology, as in the neoclassical framework, nor income distribution, as in the original Cambridge model) was the adjusting variable that brought overall savings and investment in equilibrium. This model can be reinterpreted as a macroeconomic theory for the valuation of equity markets – i.e. as a theory explaining $q$. This new interpretation offers two important conclusions: first, it finds a negative long-run relationship at the macroeconomic level between growth rates and valuation ratios; this is in contrast to firm-level equity valuation models (e.g. dividend, residual income and free-cash-flow discounted models), which suggest the opposite. Second, the causality goes from investment and animal spirits to $q$, whereas the neoclassical model (Hayashi, 1982) stresses the importance of $q$ on investment decisions.

This simple Kaldorian framework has been able to explain remarkably well the experience of the last decades in developed countries, where lower growth rates have been associated with higher valuation ratios. However, the Kaldorian framework has at least two important shortcomings: first, it is based on a real economy framework without money where equities are the only financial asset (Davidson, 1968; Kregel, 1985) and, second, the modelling of firms’

2 In fact, first person to propose this ratio at the macroeconomic level was Kaldor (1966), who called it the ‘valuation ratio’. In this chapter, the words ‘Tobin’s $q$’ and ‘valuation ratio’ will be used interchangeably.

3 The insight that higher growth rates lead to lower valuation ratios has profound implications both for policy makers and market participants. The importance of market valuations for policy makers have been argued in length in Smithers (2009), who argues that central bankers should pay more attention to financial market valuations and not exclusively price inflation. For market participants, it is useful to have an idea whether markets are ‘expensive’ or not. However, at the macro level, traditional fundamental equity valuation methods applied to the valuation of whole indices will not work if the Kaldorian insight applies – because these discounted cash-flows methods will tell you that higher growth rates should lead to higher valuations.

4 Although Tobin and Brainard did not develop formally this reverse causation issue, they briefly hinted at this dependence of $q$ on investment decisions; ‘We agree that $q$’s are partly endogenous variables, that investments can influence $q$’s as well as vice versa, and that the lags between exogenous changes in $q$ and investment could be “long and variable”’ (Tobin & Brainard, 1990, p. 548).
financing decisions is simplistic in that it assumes fixed dividend payout and share issue ratio. In other words, dividend and financing decisions are made independent of financial market conditions.

This paper generalises and extends the Kaldorian model to address these shortcomings. This will be done through a medium-scale SFC model, which allows for a more sophisticated treatment of the financial aspects of the economy with a richer asset-liability structure. Our model is a generalisation because it contains Kaldor’s key behavioural function but also includes some realistic features missing in the original model, such as endogenous money, financial markets, cost-push inflation, corporate leverage and fiscal and monetary policy. We thus address the first shortcoming. In terms of behavioural assumptions, the model follows established post-Keynesian theory, but we deviate in one important aspect. In contrast to the Kaldorian model and to the standard SFC literature (Godley & Lavoie, 2007; Dos Santos & Zezza, 2008; Le Heron & Mouakil, 2008; Van Treeck, 2008), we consider financing and dividend policy decisions as interdependent following Gordon (1992, 1994). We have endogenous dividend payout and share issuance and thus address the second shortcoming.

The aim of the model is to demonstrate that post-Keynesian theory, using a reasonable set of assumptions, can offer a robust theoretical explanation for the behaviour of \( q \) over the long-run. While our SFC model features both short-term and long-term dynamics, our focus is, like Kaldor’s, on \textit{long-run} steady-state positions. Our modelling of short run dynamics remains minimalistic; in particular we bypass all the interesting asset price dynamics highlighted by the Minskyan theory of financial markets and behavioural finance (Thaler, 2005). We do so not because these issues would not be important – indeed they are – but because we argue that \textit{even in steady growth equilibrium without speculation or any other specific behavioural bias}, post-Keynesian theory offers a distinct explanation of \( q \).

The main findings of our post-Keynesian model are as follows. First, the original two long-run relationships of the Kaldorian model, between \( q \) and growth rates and \( q \) and propensities to consume, hold. Second, in contrast to the Kaldorian model, simulations show that the way investment is financed matters, not only for \( q \), but also for output, employment and prices. Finally, as in Kaldor’s, the level of \( q \) does not tend to 1 even in the long-run, contradicting thus the neoclassical \( q \) theory where the equilibrium level of \( q \) is 1. This last finding has far-reaching consequences for the Miller-Modigliani (M&M) dividend irrelevance proposition (Miller & Modigliani, 1961), which states that the value of a corporation is independent from its dividend policy. Although the theory was originally under attack by corporate finance theorists (Lintner, 1962; Gordon, 1963; Walter, 1963), now it is commonplace in finance and has been widely used as a micro-foundation for many neoclassical macro models. We show that the

5 For a discussion of a possible research agenda in common between post-Keynesian economies and behavioural finance, see Jefferson & King (2010).

6 This is the case in most real-business cycle and new-Keynesian models. See, for instance, Christiano et al. (2005) and Smets & Wouters (2007). In these models, the institutional setup is irrelevant (it does not matter who owns what), so that capital structure (and dividend policy) is irrelevant. In addition, no clear picture of the role of financial intermediaries in the system is provided.
M&M dividend proposition will only hold when \( q \) is equal to 1, a condition that in our post-Keynesian model will be only fulfilled by chance.

The structure of the paper is as follows. Section 2 reviews the literature on \( q \), with special emphasis on the theoretical literature and on the main features of the Cambridge corporate models. Section 3 presents a post-Keynesian SFC model and the simulations conducted to study its behaviour. Section 4 explains the implications of the post-Keynesian \( q \) theory for the validity of the M&M dividend proposition. Section 5 concludes.

2. Literature review

1. Tobin’s \( q \) in the neoclassical framework

Keynes (1936, chp. 12) admitted that ‘the daily revaluations of the stock exchange[...] inevitably exert a decisive influence on the rate of current investment. For there is no sense in building up a new enterprise at a cost greater than that at which a similar existing enterprise can be purchased [...] if it can be floated off on the Stock Exchange at an immediate profit’ (1936, chp. 12). Brainard and Tobin (1968) and Tobin (1969) took up Keynes’s idea in a formal model; the latter was an extension of the original Hicks IS-LM model with an LM curve depending on a vector of asset prices rather than on a single interest rate, while the former was one of the first contributions in dealing with macro models embedded in a rigorous accounting structure and can be regarded as an early forerunner of the SFC methodology. Another interpretation of Keynes’s idea was offered by Minsky (2008), whose framework is similar to Tobin’s in that investment depends on the difference between the demand price and the supply price of capital goods.7

In neoclassical theory, as in Brainard and Tobin’s seminal papers, \( q \) plays the main role in investment decisions, but uses a more restricted microeconomic rational behaviour setting.8 This implementation was developed by Lucas & Prescott (1971), Yoshikawa (1980) and Hayashi (1982), and since then \( q \) has become the ‘preferred theoretical description of investment’ (Fischer & Merton, 1984, p.29) in a neoclassical framework and is featured as such in advanced textbooks (Carlin & Soskice, 2006; Romer, 2012). One reason for its success is that the model can be derived from the maximising behaviour of a single representative firm operating in competitive markets and facing adjustment costs. Such adjustment costs can be either internal (installation and other costs) or external (new investment induced by a higher level of \( q \) bids up the price of capital goods), but the workings of the theory are the same in both cases (Romer, 2012, p. 408). The relevant \( q \) for the neoclassical theory of investment is marginal \( q \),

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7 For a discussion of the differences between Minsky and Brainard and Tobin’s framework, see Crotty (1990) and Palley (2001).

8 However, Brainard and Tobin’s framework is quite different from the neoclassical one. As they admit: ‘We are so far from being thorough-going neoclassicals that we are quite comfortable in believing that corporate managers [...] respond to market noise and are in any case sluggish in responding to the arbitrage opportunities of large deviations of “\( q \)” from par’ (Tobin & Brainard, 1990, p. 548). Furthermore, Tobin (1984, pp. 6–7) expressed serious reservations about the ‘efficiency of financial markets’, citing approvingly Keynes’s idea of markets driven by non-informed, herding behaviour.
that is, the ratio of the market value of a marginal unit of capital to its replacement cost. The equilibrium value for $q$ is 1; if, for whatever reason, the actual value is above that level, wealth-maximising firms will find profitable investment projects and then will push down the marginal efficiency of capital (i.e. the rate of profit), given the assumption of a production function with decreasing marginal factor returns.

There have been several theoretical criticisms to this framework. First, marginal $q$ is an unobservable variable, so ‘[t]he managerial investment decision-making process cannot possibly be guided by an unobservable variable’ (Crotty, 1990, p. 538, emphasis in the original). Second, perfect capital markets are assumed, and shareholders and managers are conflated into a single agent (Crotty, 1990). The assumption of perfect capital markets rules out the possibility of long periods of time where actual values deviate from fundamentals, so managers always receive relevant information from the stock market for their investment decisions. The conflation of shareholders with managers implies that firms do not exist in the neoclassical framework and that managers as a class do not have different goals from shareholders. Third, as Palley (2001, p. 665) notes, if firms and shareholders have different expectations about future cash-flows, $q$ equilibrium will be different from unity. Fourth, managers will maximise shareholders’ wealth choosing the most appropriate technique for a given technology – i.e. the rate of profit is given by a production function. However, it is well-known that the use of production functions for determining the rate of profit is problematic (Cohen & Harcourt, 2003; Felipe & Fisher, 2003; Felipe & McCombie, 2013).

The empirical evidence for the neoclassical investment function has been quite disappointing (Summers, 1981; Abel & Blanchard, 1986; Chirinko, 1993): ‘Their explanatory power is low and serial correlation or dynamic structures including the lagged dependent variable are common. In addition, other variables […] are often significant in the equations even though the standard formulation of $Q$ models does not provide a satisfactory rationale for their inclusion’ (Blundell et al., 1992). Even when the $q$ variable is found to be statistically significant (Blundell et al., 1992), its economic significance is very low. Furthermore, the adjustment costs estimates found in some studies are usually far too large to be reasonable (Summers, 1981). Some of these problems stem directly from the theoretical assumptions of the model. For instance, the assumption of perfect financial markets, where actual prices cannot deviate from fundamentals, does not reflect observed stock market behaviour: ‘Sentiment creates a problem for the $q$ model insofar as investment decisions are based on fundamentals’ (Chirinko, 1993, p. 1889). Another possible source of problems comes from the way capital stock at replacement cost is measured, because the perpetual inventory method used can be ‘highly inaccurate in the face of major structural shifts’, although it seems that the ‘extant evidence provides little support for the capital mismeasurement hypothesis’ (Chirinko, 1993, p. 1890).

2. **Cambridge corporate models**

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9 Under constant returns on the adjustment costs, it can be shown that marginal and average $q$ coincide (Hayashi, 1982). Moreover, other influences such as monopoly power, downward-sloping product demand curves and a large share of dated capital can produce discrepancies between the marginal and the average $q$. See Romer (2012, p. 415).
The simplest post-Keynesian long-run macroeconomic model that deals with the determination of the business profit rate is the basic dual-class Cambridge model (Kaldor, 1955; Robinson, 1956; Pasinetti, 1962). In this model, the rate of profit is given by the growth rate of investment divided by capitalists’ propensity to consume. In such a framework the main results are framed in a distributive context of workers and capitalists. The model has been extended to include a government sector (Dalziel, 1991; Pasinetti, 1989; Steedman, 1972), and a financial sector (Palley, 1996; Park, 2006).10

In a strand of this literature launched by Kaldor (1966), this ‘dual-class structure’ was changed by a ‘corporate structure’, in which the relevant distinction is no longer between workers and capitalists but rather between households and firms.11 This change in the scope of the institutional setup was motivated by the criticisms of Samuelson & Modigliani (1966) directed towards Pasinetti’s result of workers’ savings irrelevance for the profit rate, and more precisely to what they regarded as the assumption of ‘the existence of identifiable classes of capitalists and workers with “permanent membership” – even as rough first approximation’ (Samuelson & Modigliani, 1966, p. 271). In his rejoinder, Kaldor (1966, p.310) considered the high propensity to save out of profits ‘something which attaches to the nature of business income, and not to the wealth (or other peculiarities) of the individuals who own property.’

The change from a dual-class structure to a corporate one was not mere window dressing, but had important theoretical implications. In the dual-class model, the adjustment to full-employment output occurs through the change in the average propensity to save of the economy – weighted by workers’ and capitalists’ participation in total saving. For instance, an increase in the growth rate raises investment needs, which will be fulfilled through an increase in the rate of profit and thus an increase of the profit share in total income. On the other hand, in the Cambridge corporate model the adjustment occurs in the stock market: consumption has to reach a certain level (through the capital gains component embedded in the consumption function) in order to close the gap between full-employment output and investment. The valuation ratio plays a crucial role in this process, reconciling corporations’ desire for growth and households’ desire to consume. Households’ savings play a buffer role here, but now through the volume of capital gains, so the relevant measure making the adjustment is households’ comprehensive savings. However, as Davidson (1968, p.259, emphasis in the original) pointed out, Kaldor ‘has unwittingly reinstated the *deux ex machina* of the neoclassical system – the rate of interest – as the balancing mechanism, not only for maintaining equilibrium in the securities market, but also for ensuring a level of effective demand always ample to secure full employment.’12

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10 For a thorough review of the Cambridge model literature, see Baranzini & Mirante (2013).

11 See Moss (1978) for a model with dual-class income distribution analysis in the framework of a corporate economy.

12 The terminology used in the Cambridge literature is misleading. What is labelled ‘the rate of interest’ really is the equity yield. In fact, there is neither money nor debt in these models, thus no ‘rate of interest’ in the Keynesian sense. The origin of this confusion could be due the neoclassical institutional structure where there are no households or firms, but a representative agent, so the difference between the rate of profit and equity yield vanishes, because the agent as household will equalise the equity yield to the rate of profit obtained as
Therefore, the introduction of the corporate sector adds a high dose of realism to the Cambridge model but it also adds a new set of theoretical problems, especially those related to corporate behaviour and stock market valuation. It is no wonder that the literature has been concerned with the valuation ratio and its relationship with the macroeconomic profit rate (Marris, 1972; Moore, 1973, 1975; Lavoie, 1998; Commendatore, 2003). Moreover, this corporate framework is in stark contrast to the neoclassical framework. In the later, firms are veils, and the production process is a black box – a production function. In contrast, the Cambridge corporate model allows for corporations to have their own existence and to make decisions independent from households.

Several propositions can be derived from the basic model: first, there is a negative relation between \( q \) and growth rates; second, there is a negative relation between \( q \) and capital-output ratios; and third, there is a positive relationship between \( q \) and households’ savings rates.13 Regarding the profit rate, higher growth rates have a positive effect on profit rates, whereas higher retention ratios and higher new share issues have a detrimental effect on the profit rate. Finally, higher growth rates have an unequivocal positive effect on the equity yield – both through higher profit rates and a lower Tobin’s \( q \).

The pair of relationships between \( q \) and growth rates, and \( q \) and savings rates constitutes the core of what we call the ‘Kaldorian theory of \( q \)’. Despite its simplicity, the model is able to explain remarkably well the long-run trend of \( q \) in, for instance, the US economy and other developed economies during the last 40 years.14 The evidence shows that the recent higher level of \( q \) (Montier, 2014a; Piketty, 2014, p. 189) has been coupled with lower accumulation rates and higher propensities to save, the latter due to income redistribution to the top percentile; these effects, inter alia, are usually associated in the post-Keynesian literature with the financialisation process (Stockhammer, 2004; Orhangazi, 2008; Van Treeck, 2008). Moreover, there is nothing in the Kaldorian theory to preclude \( q \) from being persistently different from unity. Admittedly, there have been other factors that have undoubtedly played a role in the evolution of \( q \), notably the increase in leverage and the change in institutions, which are not included in the model. But the evidence taken at face value between growth rates, \( q \) and savings rates is in principle favourable to the post-Keynesian theory.

Another remarkable feature of the Kaldorian theory that has gone unnoticed in the post-Keynesian literature15 is that it reverses the causation compared to neoclassical theory: while

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13 Kaldor (1966) assumed that households’ propensity to save was homogenous across all income classes (i.e. wages, dividends and capital gains). However, in a model with different propensities to save the link between every propensity and the valuation ratio is still positive (Moore 1973, 1975).

14 Full disclaimer: all these relationships should be properly understood in a long-run context.

15 See the discussion in the previous section. Post-Keynesians disagree on the neoclassical investment functions (those solely with \( q \) as an independent variable), on the grounds that the rationality imposed is absurd (e.g. radical
mainstream theory predicts a causal link running from \( q \) to investment, the Kaldorian theory posits a link running from investment to \( q \). While the mainstream theory supports stock market booms as drivers of corporate investment (\( q \) values higher than one), in the Kaldorian theory such a mechanism is assumed to be irrelevant. The Kaldorian theory features the Keynesian principle that investment is given by animal spirits, while in the mainstream theory investment is given by the production function through the law of the one price – the entrepreneur will carefully equalise the marginal efficiency of capital to the marginal productivity of capital (rate of profit), the latter given by the production function.

However, there are other, less favourable features in the basic Kaldorian model. There are problems of omission as well as commission. The problem of omission is that Kaldor offers an explanation of \( q \) based on a model without a proper financial sector. There are no banks, there is no money and there is only one financial asset. The problem of commission is in the modelling of firms’ financing decisions and dividend policy. Kaldor assumes that a fixed part of profits is paid out as dividends and a fixed share of investment is financed by equity issue, independent of financial market conditions. However, in the real world, the financing decision and the dividend decision are neither independent nor completely fixed regardless of the state of capital markets. Moreover, a higher proportion of investment financed through new shares should lead to lower valuation ratios – given the higher supply of shares. Finally, one would expect that other factors not included in the model (most notably, corporate leverage, inflation, and fiscal and monetary policy) to affect the evolution of \( q \). These issues will be addressed in the model presented in the next section.

3. **A post-Keynesian SFC model for Tobin’s \( q \) behaviour**

The Cambridge model allows gleaning very general relationships between \( q \) and other macroeconomic variables, but at the cost of simplification. The model presented here depicts a more sophisticated economic system with post-Keynesian sectoral behaviour. The main post-Keynesian features are: endogenous money, mark-up pricing, sectors with independent motivations (especially households, firms and banks) and a theoretical framework that is demand-led. Because the model presented is too large to be solved analytically, simulations will be performed to analyse \( q \)’s behaviour over time. We are specifically interested in three sets of shocks that will allow us to evaluate whether the conclusions of the original Kaldorian model still hold: a change in the growth rate of the economy, a change in households’ propensity to consume and a change in the willingness of corporations to issue new shares.

1. **The model**

The model consists of 56 equations. The three matrices that lay out the full accounting structure of the model (stocks, flows and price revaluations) can be found in the Appendix. Except for firms’ financing decisions, no pretension of originality in the behavioural assumptions is made; rather, the aim is to set up a model based on established aspects of uncertainty and animal spirits) or on the grounds that no quantities (e.g. cash-flows, utilisation) are included. Thus \( q \) is not a key determinant of investment. Our point is that investment is a determinant of \( q \).
post-Keynesian theory as far as possible. Given the size of the model, only the main behavioural equations will be discussed, the full list of equations can be found in the Appendix. For the sake of convenience, each sector is discussed separately.

Firms’ behaviour

Firms’ behaviour is characterised as follows. Investment grows in real terms at a constant rate, $g r_k$, given by Keynesian animal spirits. More complicated investment functions have been extensively used in the literature (Dos Santos & Zezza, 2008; Lavoie & Godley, 2001; Van Treeck, 2008; Zezza, 2008), but here a simpler form has been preferred. Our specification of firms’ financing decisions and dividend policy differs substantially from the standard treatment of the SFC literature (Godley & Lavoie, 2007; Dos Santos & Zezza, 2008; Le Heron & Mouakil, 2008; Van Treeck, 2008), which typically assumes that a fixed percentage of investment is financed through new share issues, regardless of financial market conditions, and that dividend policy is a fixed percentage of total profits. In other words, dividend policy is considered to be independent of investment financing decisions. We regard both assumptions as problematic and follow Gordon’s (1994) investment financing and dividend theory instead. In Gordon’s framework, the sale of shares is a supplement, and not a substitute for retained earnings in investment financing decisions; dividend policy is regarded as subordinate to investment policy and one cannot be varied independently of the other. At the mathematical level, firms’ financing decisions are modelled here very much in the manner of Tobinesque households’ portfolio decisions: the share of every financing method (retained profits, debt and equity issues) will depend on the interest rate on loans, on the share price in the stock market and on the degree of leverage. The relative equity price is modelled using the price-earnings ratio (using the trailing twelve months earnings), $p e r_{ttm}$, so firms will opt for equity issues and retained earnings when this ratio is high and shares are expensive. Once the share of retained profits is given by equation (2), firms will distribute the excess over retained profits as dividends (equation 5), so dividends will vary depending on financing decisions. Finally, equation (6) depicts firms’ price policy decision as a mark-up over the unit costs of the previous period.

\[
i = k_{-1} \cdot gr_k \tag{1}
\]
\[
\frac{\Delta L_d}{I} = f_{2o} + f_{21} \cdot r_{t-1} + f_{22} \cdot \frac{1}{p e r_{ttm-1}} + f_{23} \cdot \left(\frac{L}{R}\right)_{-1} \tag{2}
\]
\[
\frac{p e r \cdot \Delta e f}{L} = f_{3o} + f_{31} \cdot r_{t-1} + f_{32} \cdot \frac{1}{p e r_{ttm-1}} + f_{33} \cdot \left(\frac{L}{R}\right)_{-1} \tag{3}
\]
\[
\Pi_d^f = \Pi^f - \Pi_r^f \tag{5}
\]
\[
p = (1 + \varphi) \cdot U C_{-1} \tag{6}
\]

Inflation

16 For the post-Keynesian debate on investment functions, see Hein et al. (2011), Hein et al. (2012) and Lavoie (2014, chap. 6).
The labour market follows mainly Godley & Lavoie (2007, chp. 9, 10, 11): equation (7) says that unions have a desired, targeted real wage that is a function of the previous target level, labour productivity and the rate of employment. Labour population, \(N_{fe}\), is assumed to be fixed and does not grow. On the other hand, equation (8) depicts the part of the negotiation process that is included into the nominal wage rate of the current period. Overall, our labour market is different from Kaldor’s, because it allows for the possibility of unemployment in the long-run and because growth in labour population here is zero. However, our model is similar in spirit to the Kaldorian theory of income distribution because wage bargaining is nominal and real wages will adjust in the long run. Finally, equation (9) deals with labour productivity: following Zezza (2008), it is assumed that productivity grows at an exogenous rate, \(gr_{pr0}\), minus a parameter that reflects that higher levels of capacity utilization will lead to lower levels of productivity growth.

\[
\omega^T = \left(\frac{W}{P}\right)^T = \omega^T_1,\left[1 + \Omega_0 + \Omega_1,\frac{pr_{-1}}{\omega_{-1}} + \Omega_2,\left(\frac{N}{N_{fe}}\right)_{-1}\right]
\] (7)

\[
W = W_{-1} \cdot \left[1 + \Omega_3,\left(\frac{\omega^T_{-1}}{w_{-1}}\right)\right]
\] (8)

\[
gr_{pr} = gr_{pr0} - gr_{pr1} \cdot u
\] (9)

**Households’ behaviour**

The most important household decisions are regarding consumption and their portfolio allocation. Equation (10) says that households’ consumption decisions are assumed to be in real terms, depending on expected real disposable income and one-period-lagged real wealth. Equation (11) shows how to calculate the deflated value for disposable income.17 Households’ portfolio decision is a two-step process: in the first round (equation 12), households will decide how much wealth to allocate as deposits, and in the second round households will decide how to allocate the rest between equities and bills following *Tobinesque* principles: households will have some previous preferences for such allocation (parameters \(\lambda_{10}\) and \(\lambda_{20}\)), which will be modulated by the equity yield and the rate of bills of the previous period.

\[
c = \alpha_1, yd^e + \alpha_2, vh_{-1}
\] (10)

\[
yd = \frac{YD}{p} - \frac{\pi.Vh_{-1}}{p}
\] (11)

\[
D_h = \sigma.Vh
\] (12)

\[
\frac{pe \cdot eh}{Vh - D_h} = \lambda_{10} + \lambda_{11} \cdot y_{-1} + \lambda_{12} \cdot rb_{-1}
\] (13)

\[
B_h = Vh - D_h - pe \cdot eh
\] (14)

**Banks, government’s behaviour and financial markets**

17 Real disposable income is not simply the deflated value of nominal disposable income, but has to be adjusted for the erosion in wealth produced by inflation. For a formal proof, see Godley & Lavoie (2007, pp.293-294).
Turning to banks, government, central bank and financial markets, equation (15) states the very well-known post-Keynesian principle that in credit-based economies money is endogenous and largely the result of commercial banks’ decisions. Equations (16) and (17) determine banks’ profits as the amount of interest payments of the current period and banks’ dividend decisions, which distribute all their profits to households. This decision, together with that of setting the interest rate on loans (equation 18), are the only decisions that banks in this model can autonomously take. Equations (18) to (22) depict government and central bank decisions. Government decides on the growth of government expenditures based on the level of its debt in real terms as a share of real income and on the level of the unemployment in the economy. The former depicts the extent to which the government has public debt target while the latter indicates the strength of anti-cyclical fiscal measures. Equation (21) says that the central bank is a residual buyer of government’s debt and (22) is central bank’s monetary policy decision – deciding the level of the interest rate on bills.

Finally, equations (23) to (25) are simply definitions of well-known financial ratios: Tobin’s $q$, price-earnings ratio and equity yield, respectively. The price-earnings ratio in equation (24) anchors on the trailing-twelve-months corporate earnings. Finally, equation (25) is the common definition of the equity yield.

\[
\Delta L_s = \Delta L_d \tag{15}
\]
\[
\Pi^b = r_{t-1}.L_{s-1} \tag{16}
\]
\[
\Pi^b = \Pi^b \tag{17}
\]
\[
\eta = \bar{r}_t \tag{18}
\]
\[
g = g_{-1}.(1 + gr_{gov}) \tag{19}
\]
\[
gr_{gov} = gr_0 - gr_1.\left(\frac{B}{p}\right) + gr_2.\left(1 - \frac{N}{N_fe}\right)_{-1} \tag{20}
\]
\[
\Delta B_{cb} = \Delta B - \Delta B_h \tag{21}
\]
\[
r_b = \bar{r}_b \tag{22}
\]
\[
q = \frac{p_e.e_h + L_d}{K} \tag{23}
\]
\[
per_{ttm} = \frac{p_e.e_h}{\Pi_f} \tag{24}
\]
\[
\gamma = \frac{\Pi_d + CG}{p_{e-1}.e_{h-1}} \tag{25}
\]

As usual in a SFC model, a ‘redundant equation’ is left: an accounting identity is implied in the set of logical relations with variables already explained in some other equation. This equation says that Central Bank’s high-powered money is equal to banks’ reserves:

\[
H_b = H_{cb}
\]
The only price-equilibrating mechanism in the model takes place in the equity market, where the equity price fluctuates to accommodate the equity supply (given by firms and households who wish to sell their shares) with the equity demand (given by households who wish to buy shares).

2. Simulations

An increase in the growth rate of the economy

The first simulation will deal with an increase in the growth rate of the capital stock, $gr_k$, which from a Keynesian point of view can be regarded as an increase in animal spirits. Figures 1 to 4 show the results. The first chart confirms the Kaldorian conclusion that higher growth rates yield lower valuation ratios. However, not much attention should be placed in this case to short-term results, given the way financial markets have been introduced in the picture, because one should expect that financial markets should include higher growth rate expectations into equity prices in the short-run – the empirical evidence suggests that markets almost always overreact. In any case, the secular decline in the long-run can be explained by the increase in the inflation rate, which affects not only financial market indicators but corporations’ return on equity as well, through higher values of capital at replacement cost. This result is in contrast to the Cambridge model, where higher growth rates lead to higher profit rates. Here, although economic activity improves (both in the short-run and in the long-run), the fact that the return on equity is measured with capital in nominal values (as it should) leads to a decline of the return on equity over time.

Figure 1a. First simulation: increase in animal spirits, $gr_k$.

Figure 1b. First simulation: increase in animal spirits, $gr_k$. 
An increase in the propensity to consume out of wealth

Our second simulation will deal with another parameter of the Cambridge corporate model, a change in the propensity to consume out of wealth. Figures 5 to 8 shows the implications of an increase in this parameter. This increase has the expected Keynesian results affecting positively consumption and disposable income – both in the short and in the long-run. The unemployment rate improves due to higher levels of income and firms enjoy a higher level of profitability as well. This is in contrast to the Kaldorian model, where households’ behaviour does not have any impact on the long-run profit rate. However, higher levels of consumption are balanced with lower levels of real wealth, which provoke the sale of shares and a fall in stock market valuations, confirming the negative relationship between propensities to
consume and valuation ratios. Higher levels of inflation push up the replacement cost of fixed capital and contribute additionally to the fall in $q$. Although the equity yield improves both in the short and in the long-run (higher dividend yields are the result of both higher profits and lower equity prices), which creates a rebalancing effect in households’ portfolio towards equities (increasing then their price), this effect is not enough to compensate for households’ desire to reduce their wealth level relative to their income.

**Figure 2a. Second simulation: increase in the propensity to consume out of wealth, $\alpha_2$.**

**Figure 2b. Second simulation: increase in the propensity to consume out of wealth, $\alpha_2$.**

**Figure 2c. Second simulation: increase in the propensity to consume out of wealth, $\alpha_2$.**
A decrease in the proportion of new share issuance

The third simulation will deal with a change in the policy of new share issues. Figures 9 to 12 summarise the main effects of a permanent reduction in the proportion of investment financed out of new issues. Given the reduction in the supply of shares, the valuation metrics increase notably in the short run and they keep increasing in the long-run. The Kaldorian model suggested that firms’ financing policy should not have any long-run effect on $q$, but here that is not the case. The increase in valuation metrics has negative effects on the equity yield, given that shareholders have to buy the same assets at higher prices. In turn, a lower equity yield leads to a lower share of equities in households’ portfolio. Finally, the way investment is financed matters for aggregate output: the last chart shows that unemployment is higher both in the short and in the long-run, which impact on corporate profitability through lower levels of return on equity. This effect is the opposite expected by the Cambridge model, where lower levels of share issuance should lead to an increase in the profit rate. However, the unemployment rate here is not fixed and matters for the level of profitability. This simulation can be conceptually thought as an increase in the degree of ‘financialisation’ (Stockhammer, 2004; Orhangazi, 2008), and the results track the main predictions of the literature: higher valuation ratios in the new steady-state and lower levels of output and employment. An
additional feature could be added: lower levels of shareholder profitability, because of higher stock market valuations.18

Summing up, in our SFC post-Keynesian $q$ model we confirm two of the insights of Kaldor’s original model: higher growth rates and higher propensities to consume lead to lower levels of $q$. However, our richer model with an explicit financial sector and treatment of firms’ financing decisions (that regards investment and financing decisions of firms as interdependent) does find that share issuance affects $q$. Unsurprisingly, in our model firms’ investment decisions affect output, employment and income distribution. Thus, one of the key features of our model is that firms influence $q$ through investment decisions as well as through their financing policy. Finally, as in Kaldor’s, $q$ does not tend to 1 even in the long-run, so accumulation can proceed persistently above or below that level.

Figure 3a. Third simulation: decrease in new share issuance, $f_{30}$

---

18 Montier (2014b) presents additional evidence against shareholder value maximisation, showing how equity returns were higher in the period 1940-1990 than since then.
4. Post-Keynesian $q$ and the M&M dividend irrelevance proposition

Our post-Keynesian model and its findings for $q$ are at variance with those of neoclassical theory. There, households and firms are mixed (so firms’ decisions as such do not exist) and a single representative rational agent takes their place. While in the post-Keynesian tradition investment is driven by animal spirits and quantity-variables (e.g. capacity utilization or output), in the neoclassical framework all that is needed is the (unobservable) marginal $q$. The fulfilment of this maximising rule will assure that in equilibrium $q$ will be 1 and that any discrepancy from this level will be corrected by individual agents adjusting their capital stocks. On the other hand, the previous section suggests that in a post-Keynesian model the condition of $q$ to be equal to 1 in the long-run will be only fulfilled by chance, given that no equilibrium mechanism exists in the model to bring $q$ back to unity; firms take their investment, dividend and financing decisions not solely having in mind equity prices (as in the neoclassical model), and the behaviour of the rest of the sectors taken together does not guarantee that $q$ should

Moreover, there is nothing in our model (or in the post-Keynesian tradition) that suggests that equity prices only incorporate the relevant information for managers so as they can make ‘rational’ investment decisions. In other words, no efficient market hypothesis is assumed here.
converge to 1. We are going to show that the implications of this non-convergence for the Miller & Modigliani (M&M) dividend irrelevan

t proposition are profound.

The M&M dividend irrelevance proposition was first put forward by Miller & Modigliani (1961) as a companion to the capital structure irrelevance proposition presented three years before (Modigliani & Miller, 1958). The M&M dividend proposition states that the value of a company is independent of its dividend policy. Intuitively, the reason is as follows: an individual investor, given its portfolio constraints and risk-return objectives, will be indifferent between receiving cash-flows as dividends or as capital gains and, moreover, he will be able to undo corporate decisions by creating ‘home-made’ dividends. This arbitrage argument is no different in essence to the one proposed by M&M for corporate financial structure irrelevance: there, personal leverage was supposed to be a perfect substitute for corporate leverage, so if there were an ‘undesired’ change in the corporate financial structure policy, the investor could still borrow or lend to attain his portfolio risk-return objectives again and ‘undo’ corporate decisions.

The conventional critiques of the M&M are well known. Instead of rehearsing them, we want to focus on what happens to the M&M proposition when \( q \) is different from unity. The M&M dividend proposition and \( q \) can be linked through a valuation formula which says that, in equilibrium, \( q \) can be expressed as a function of the rate of profit (return on equity here), \( r \), the equity yield, \( y \), and the growth rate of the economy, \( g \).

\[
q = \frac{r - g}{y - g}
\]

Only in the case \( r = y \), then \( q \) will be one. It turns out that the effect of dividend policy on company valuation depends on the values taken by \( r \) and \( y \). Table 4 shows the valuation of a hypothetical common share under four different scenarios.

---

20 Most of them are based on market imperfections, among others: different tax rates for dividends and capital gains, asymmetric information (managers may want to signal corporate prospects through dividend policy) and other corporate imperfections such as inefficient managers who may squander cash – making it preferable to pay out dividends. Recently, experiments in the behavioural finance literature have shown that individuals pay attention to the source from which they receive income, engaging in mental accounting (Thaler, 1990, 1999): the way an investor receives his income matters. Finally, the M&M proposition, which is an argument derived from micro-conditions, may not necessarily be applicable at a macro level, due to well-known fallacy of composition problems (Taylor, 2004; King, 2012). For additional critiques of the M&M framework, see Gordon (1992, 1994), Glickman (1997), Pasinetti (2012) and Wood (2013).

21 A precision has to be made. For convenience, the \( q \) used in this section and computed with this formula is the ‘equity \( q' \) – i.e. market value of equity to its replacement cost (assets net of debt). The equity \( q \) (or ‘leveraged \( q' \)) is related to the traditional \( q \) in the following way: \( q_e = \frac{q - l}{1 - l} \), where \( l \) is the leverage ratio (debt to total assets). As one would expect, when \( q \) is equal to 1, then equity \( q \) will be equal to 1 as well. Therefore, for the M&M discussion and its validity when \( q \) is different from 1, it does not matter to use the traditional \( q \) or the equity \( q \).

22 The example is taken from Penman (2011, ch. 2), but modified and adapted for our purposes. However, Penman does not explicitly discuss the case when \( r \neq y \). The technical details of the four scenarios can be found in Appendix II.
Table 4. Summary of valuations under different scenarios

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Nil pay-out, ( r = \gamma )</th>
<th>Full pay-out, ( r = \gamma )</th>
<th>Half pay-out, ( r &gt; \gamma )</th>
<th>Full pay-out, ( r &gt; \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Book value [1]</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Total value [1+2=3]</td>
<td>100.0</td>
<td>100.0</td>
<td>140.0</td>
<td>116.7</td>
</tr>
<tr>
<td>Price to book (Equity q) [3/1]</td>
<td>1.00</td>
<td>1.00</td>
<td>1.40</td>
<td>1.17</td>
</tr>
</tbody>
</table>

Table 4 shows that dividend policy is irrelevant only when the rate of profit (return on equity) is equal to the equity yield or, in other words, when \( q \) is equal to 1: in this case, the pay-out ratio chosen by the firm does not matter, because the value of the enterprise will remain constant. However, this is not the case when the previous equality does not hold and \( q \) is different from 1: changes in pay-out ratios will affect the value of the company,23 because the difference between \( r \) and \( \gamma \) makes that dividends and capital gains are not any longer in the same footing. In the first two scenarios, the value remains the same because it is financially equivalent to receive dividends and reinvest them at the market rate than to accumulate unrealised capital gains (through higher equity prices) because of higher retained profits. However, in the other two cases, the rate of return is higher than the equity yield, so the investor is better off if the company decides to reinvest the earnings rather than to distribute them as dividends – i.e. the investor would obtain a lower reinvestment rate in the market in the latter case.

Therefore, from an empirical standpoint, as long as \( q \) is not equal to unity the M&M dividend irrelevance proposition will not hold, because dividends and unrealised capital gains cannot be treated as financially alike. An empirical analysis of \( q \) is beyond the scope of this paper, but suffice it to say that the historical evidence in the developed countries since 1950 shows that \( q \) has been persistently different from 1 – and trending up or down for whole decades. This is crucial empirical evidence for the relevance of corporations’ dividend decisions on equity valuations.

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23 For brevity’s sake, only the case when \( r > \gamma \) is considered here.
5. Conclusions

The present essay has proposed a post-Keynesian $q$ theory at the macroeconomic level based on Kaldor’s (1966) seminal paper. The Kaldorian model provides two important macroeconomic long-run relationships, between $q$ and the growth rate of the economy and $q$ and propensities to consume. We claim that these relationships alone can provide new valuable insights on long-run relationships between financial (equity) markets and macroeconomics. Our medium-scale SFC post-Keynesian model has improved the simplistic monetary and financial framework of the Kaldorian model and has shown that in this enriched setup these two long-run relationships still hold. Our model has also addressed, following Gordon (1992), the interdependence between firms’ financing decisions and dividend policy, and aspect often overlooked but crucial for the understanding of financial markets. On the other hand, our model does find that share issuance (and more generally, firms’ financing decisions) affects $q$, whereas in the Kaldorian model $q$ was independent of firms’ financing policy. Furthermore, the Kaldorian insight that, in general, $q$ will be different from 1 in the long run, is confirmed by the numerical simulations. Independent sectors with different motivations make possible that accumulation can proceed with $q$ levels different from unity.

Finally, this non-convergence impinges on the validity of the Miller-Modigliani dividend irrelevance proposition. As long as $q$ is different from 1, the proposition does not hold, because in this case capital gains and dividends cannot be considered financially equivalent, so firms’ dividend policy will affect equity valuation. The empirical evidence is against the M&M proposition. Economic theory should consider a $q$ different from 1 as part of the financial markets stylized facts. Post-Keynesian macroeconomic theory can explain this, even in the absence of speculation or other persistent behavioural biases.

6. References


Appendix I. List of equations of the model, values used in the simulations and matrices of the model

List of equations

\[ i = k_{-1} \cdot gr_k \]  
Real investment \hspace{1cm} (1)

\[ \Pi^f_r = I_d - p_e \cdot \Delta e_r - \Delta L_d \]  
Firms' profits \hspace{1cm} (2)

\[ \frac{\Pi^f_r}{I} = f_{10} + f_{11} \cdot r_{t-1} + f_{12} \cdot \frac{1}{\text{per}_{ttm-1}} + f_{13} \cdot \left(\frac{L}{K}\right)_{-1} \]  
Share of investment financed by retained profits \hspace{1cm} (2a)

\[ \frac{\Delta L_d}{I} = f_{20} + f_{21} \cdot r_{t-1} + f_{22} \cdot \frac{1}{\text{per}_{ttm-1}} + f_{23} \cdot \left(\frac{L}{K}\right)_{-1} \]  
Share of investment financed by new loans \hspace{1cm} (3)

\[ \frac{p_e \cdot \Delta e_f}{I} = f_{30} + f_{31} \cdot r_{t-1} \]  
Share of investment financed by new equity \hspace{1cm} (4)

\[ \Pi^d_f = \Pi^f_r - \Pi^f_r \]  
Firms' dividends \hspace{1cm} (5)

\[ p = (1 + \varphi) \cdot UC_{-1} \]  
Level of prices \hspace{1cm} (6)

\[ \omega^T = \left(\frac{W}{P}\right)^T = \omega^T_{-1} \cdot \left[ 1 + \Omega_0 + \Omega_1 \cdot \frac{\text{pr}_{t-1}}{\omega^T_{t-1}} \right] \]  
Targeted real wage \hspace{1cm} (7)

\[ W = W_{-1} \cdot \left[ 1 + \Omega_3 \cdot \frac{\omega^T_{-1}}{w_{-1}} \right] \]  
Nominal wage \hspace{1cm} (8)

\[ gr_{pr} = gr_{p,0} - gr_{p,1} \cdot u \]  
Growth in labour productivity \hspace{1cm} (9)

\[ c = \alpha_1 \cdot yd^e + \alpha_2 \cdot v_{h-1} \]  
Real consumption \hspace{1cm} (10)

\[ yd = \frac{YD}{p} - \frac{\pi \cdot V_{h-1}}{p} \]  
Real disposable income \hspace{1cm} (11)

\[ D_h = \sigma \cdot V_h \]  
Share of deposits in wealth \hspace{1cm} (12)

\[ \frac{p_e \cdot e_h}{V_h - D_h} = \lambda_{10} + \lambda_{11} \cdot y_{-1} + \lambda_{12} \cdot r_{b-1} \]  
Share of equities in wealth \hspace{1cm} (13)
\[ B_h = V_h - D_h - p_e e_h \]  
\[ \frac{B_h}{V_h - D_h} = \lambda_{20} + \lambda_{21} \cdot \gamma_{-1} + \lambda_{22} \cdot r_{b_{-1}} \]  
\[ \Delta L_s = \Delta L_d \]  
\[ \Pi^b = r_{i_{-1}} \cdot L_{s_{-1}} \]  
\[ \Pi^b_d = \Pi^b \]  
\[ r_i = \bar{r}_{i} \]  
\[ g = g_{-1} \cdot (1 + g_{gov}) \]  
\[ gr_{gov} = gr_0 - gr_1 \cdot \left( \frac{B}{\bar{P}} \right)^{-1} + gr_2 \cdot \left( 1 - \frac{N}{N_{fe}} \right)^{-1} \]  
\[ \Delta B_{cb} = \Delta B - \Delta B_h \]  
\[ r_b = \bar{r}_{b} \]  
\[ q = \frac{p_e \cdot e_h + L_d}{K} \]  
\[ per_{ttm} = \frac{p_e \cdot e_h}{\bar{P}} \]  
\[ \gamma = \frac{\Pi_d^f + CG}{p_{e_{-1}} \cdot e_{h_{-1}}} \]  
\[ C_c = C_d \]  
\[ I_s = I_d \]  
\[ G_s = G_d \]  
\[ WB_s = WB_d \]  
\[ T_d = T_h^d + T_s^f \]  
\[ Y = C_s + I_s + G_s \]  
\[ YD = WB_d + \Pi_d^f + \Pi_d^b \]  
\[ + (r_{b_{-1}} \cdot B_{h_{-1}}) - T_s^h \]  
\[ y = c + i + g \]  
\[ K = k \cdot p \]  
\[ k = k_{-1} + i \]  
\[ I_a = i \cdot p \]  
\[ \Pi_{d}^f = I_d - p_e \cdot \Delta e_f - \Delta L_d \]
\[ \Pi_d^f = \Pi^f - \Pi_r^f \quad \text{Firms' dividends} \]  
\[ T_s^f = \theta_f \cdot [Y - WB_d - (r_{i-1} \cdot L_{d-1})] \quad \text{Taxes paid by firms} \]  
\[ UC = \frac{WB_d}{y} \quad \text{Unit costs} \]  
\[ WB_d = N \cdot W \quad \text{Wage bill} \]  
\[ \pi = \frac{(p - p_{-1})}{p_{-1}} \quad \text{Inflation} \]  
\[ pr = pr_{-1} \cdot (1 + gr_{pr}) \quad \text{Labour productivity} \]  
\[ u = \frac{y}{k_{-1}} \quad \text{Capacity utilisation} \]  
\[ N = \frac{y}{pr} \quad \text{Number of workers} \]  
\[ \Delta V_h = YD - C_d + CG \quad \text{Households' wealth} \]  
\[ T_s^h = \theta_h \cdot [WB_d + \Pi_d^f + \Pi_d^b + (r_{h-1} \cdot B_{h-1})] \quad \text{Taxes paid by households} \]  
\[ C_d = c \cdot p \quad \text{Nominal consumption} \]  
\[ yd^e = yd_{-1} \cdot (1 + gr_k) \quad \text{Expected disposable income} \]  
\[ v_h = \frac{V_h}{p} \quad \text{Real wealth} \]  
\[ \Delta D_d = \Delta D_h \quad \text{Banks' deposits} \]  
\[ \Delta H_b = \Delta D_d - \Delta L_s \quad \text{Banks' reserves} \]  
\[ G_d = g \cdot p \quad \text{Nominal gov. expenditure} \]  
\[ \Delta B = (G_d + r_{b-1} \cdot B_{-1}) - (T_d + r_{b-1} \cdot B_{cb-1}) \quad \text{Government balance} \]  
\[ \Delta H_{cb} = \Delta B_{cb} \quad \text{Central Bank's reserves} \]  
\[ CG = e_{-1} \cdot (p_e - p_{e-1}) \quad \text{Capital gains accrued to equities} \]  
\[ H_h = H_{cb} \quad \text{(redundant)} \]

List of parameter values:

- \( gr_k = 0.0294 \)
- \( \theta_h = 0.236 \)
- \( \alpha_2 = 0.02 \)
- \( f_{20} = 0.125 \)
- \( \varphi = 0.32 \)
- \( \sigma = 0.4 \)
- \( f_{21} = -13.1 \)
- \( \Omega_0 = 0.003 \)
- \( \lambda_{10} = 0.5 \)
- \( f_{22} = 15.1 \)
- \( \Omega_1 = 0.0110 \)
- \( \lambda_{11} = 4 \)
\[
\begin{align*}
f_{23} &= -2 & \Omega_2 &= 0.0150 & \lambda_{12} &= -4 \\
f_{30} &= 0.1 & \Omega_3 &= 0.046 & gr_0 &= 0.035 \\
f_{31} &= 3.5 & N_{fe} &= 1.1 & gr_1 &= 0.01 \\
f_{32} &= -5 & gr_{pr0} &= 0.031 & gr_2 &= 0.012 \\
f_{33} &= 1.5 & gr_{pr1} &= 0.005 & r_l &= 0.035 \\
\theta_f &= 0.27 & \alpha_1 &= 0.85 & r_b &= 0.03
\end{align*}
\]

**Table 1. Balance-sheet matrix**

<table>
<thead>
<tr>
<th>Balance-sheet</th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Government</th>
<th>Central Bank</th>
<th>(\Sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real capital</td>
<td>+K</td>
<td></td>
<td></td>
<td></td>
<td>+K</td>
<td></td>
</tr>
<tr>
<td>Equities</td>
<td>+(p_e\cdot e_h)</td>
<td></td>
<td></td>
<td></td>
<td>+(p_e\cdot e_h)</td>
<td></td>
</tr>
<tr>
<td>Reserves (HPM)</td>
<td>+(H_b)</td>
<td></td>
<td>-(B)</td>
<td></td>
<td>-(H_{cb})</td>
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<tr>
<td>Bills</td>
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<td></td>
<td></td>
<td>+(B_{cb})</td>
<td>0</td>
</tr>
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<td>-(D_d)</td>
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<td>+(L_s)</td>
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<td>Net worth</td>
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<td>-(V_f)</td>
<td></td>
<td>+(V_g)</td>
<td></td>
<td>-((V_h + V_f - V_g))</td>
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<td>0</td>
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**Table 2. Revaluation matrix**

<table>
<thead>
<tr>
<th>Revaluation matrix</th>
<th>Households</th>
<th>Firms</th>
<th>Banks</th>
<th>Government</th>
<th>Central Bank</th>
<th>(\Sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real capital</td>
<td>+(\Delta p \cdot k_{-1})</td>
<td></td>
<td></td>
<td></td>
<td>+(\Delta p \cdot k_{-1})</td>
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<td>Equities</td>
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<td>+(\Delta p_e \cdot e_{h-1})</td>
<td></td>
</tr>
<tr>
<td>(\Sigma)</td>
<td>+(\Delta p_e \cdot e_{h-1})</td>
<td>+(\Delta p \cdot k_{-1})</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(+ \left(\Delta p_e \cdot e_{h-1} + \Delta p \cdot k_{-1}\right))</td>
</tr>
<tr>
<td>Transactions-flow matrix</td>
<td>Households</td>
<td>Firms</td>
<td>Banks</td>
<td>Government</td>
<td>Central Bank</td>
<td>Σ</td>
</tr>
<tr>
<td>--------------------------</td>
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<td>Current</td>
<td>Capital</td>
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<td>Capital</td>
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<td></td>
<td>−G_d</td>
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<td>[Y]</td>
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<td></td>
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<td>−WB_d</td>
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<td></td>
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<td>Firms' profits after tax</td>
<td>+Π^f_d</td>
<td>−Π^f</td>
<td>+Π^f_r</td>
<td></td>
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<td>−T^f_s</td>
<td></td>
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<tr>
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<td>+r_{t−1}.L_{s−1}</td>
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<tr>
<td>Interest on bills</td>
<td>+r_{b−1}.B_{h−1}</td>
<td></td>
<td></td>
<td>−Π^b</td>
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<tr>
<td>Central Bank profits</td>
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<td></td>
<td></td>
<td></td>
<td>+r_{b−1}.B_{cb−1}</td>
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<tr>
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<td>+p_e.Δe_f</td>
<td></td>
<td>−ΔH_b</td>
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<tr>
<td>Change in equities</td>
<td>−ΔB_h</td>
<td>+Δe_f</td>
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<tr>
<td>Change in bills</td>
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<td>+ΔL_s</td>
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<tr>
<td>Change in deposits</td>
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<td>+ΔD_d</td>
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<td>Change in loans</td>
<td></td>
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<td></td>
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Appendix II. Equity valuation scenarios for the M&M theorem

The main assumptions for our equity valuation exercise are as follows.

In order to value the stock, a method of residual earnings is used, given that in one scenario the pay-out is zero, so a Gordon-dividend model (Gordon & Shapiro, 1956) would not perform the task, given that there are no dividends to discount. In the residual earnings method, the value of a stock is the sum of current book value plus future discounted residual earnings, which are defined as the difference between the return on equity, \( r \), and the equity yield, \( y \), times previous period book value (Penman, 2011). Because the firm in question is an ongoing concern, a terminal value has to be added in order to take into account the part of value accruing in the distant future; for such terminal value, a formula for continuous compounding growth is applied. In any case, it is important to note that the residual earnings valuation model is financially equivalent to the sum of the discounted book value, plus the discount value of dividends in the projected horizon plus the value of discounted residual earnings beyond the projected horizon. The tables below show that both methods yield the same results.

For simplicity, it is assumed that future earnings are known with certainty. It is not implied whatsoever that in the real world equity analysts face such an easy task, but rather it is a useful device for assessing a stock’s *intrinsic value* – the common assumption in the M&M literature. Book value is equity at historic cost in the balance sheet. The difference between earnings and dividends in every period cumulates into the book value figure. The return on equity, the level of current earnings divided by the book value of the previous period, is 7%, and is equal to the equity yield in the first two scenarios – in the other two scenarios the equity yield is lower than the return on equity.

Table 5 reports the first two valuation scenarios. The first scenario assumes a valuation with a nil pay-out ratio. Starting with a book value of 100, this book value will be increased through retained earnings. Although the book value increases every year, residual earnings are every year zero (because of the assumption \( r = y \)), so that the value of this stock is simply the book value. On the other hand, the second scenario is assumed that dividends are paid in full. In such a case, the book value remains flat *sine die* because dividends flow out of the company. It can be seen that dividend policy in both scenarios does not matter, because the value of the company remains unchanged.

Table 6 shows a similar story but now for \( r \neq y \). It can be seen that in this case dividend policy matters, because the value of the company is affected by the change in dividend policy – from a value of 116.7 with a full pay-out ratio to a value of 140 with 50% as a pay-out ratio).
Table 5. Value of the stock under the assumption r = γ

<table>
<thead>
<tr>
<th>Pay-out ratio</th>
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<tbody>
<tr>
<td>Equity yield [γ]</td>
<td>7%</td>
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<tr>
<td>Return on equity [r]</td>
<td>7%</td>
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<td>-</td>
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<tr>
<td>% in residual earnings</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Dividends</td>
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<td>-</td>
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<td>Book value</td>
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<td>Discounted RE 2016</td>
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</tr>
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<td>Discounted RE 2018</td>
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<tr>
<td>Dividends discounted</td>
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<table>
<thead>
<tr>
<th>Pay-out ratio</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Required return [γ]</td>
<td>7%</td>
</tr>
<tr>
<td>Return on equity [r]</td>
<td>7%</td>
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<td>7.0</td>
<td>7.0</td>
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<tr>
<td>Residual earnings (RE)</td>
<td>-</td>
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<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>% in residual earnings</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Dividends</td>
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### Table 6. Value of the stock under the assumption $r \neq \gamma$

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<td>Return on equity ([r])</td>
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<td>3.5%</td>
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<td>Dividends</td>
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### Value

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