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Cash holdings and financing decisions under ambiguity

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Abstract: This paper addresses the following unresolved questions: Why do some firms issue equity instead of debt? Why did most firms retain their cash holdings instead of distributing them as dividends in recent times? How do firms change their financing policies during a period of severe financial constraints and ambiguity, or when facing the threat of an unpredictable financial crisis? We analyze how the values of the firm's equity and debt are affected by ambiguity. We also show that cash holdings are retained longer if the investors' ambiguity aversion bias is sufficiently large, while cash holdings become less attractive when the combined impact of ambiguity and ambiguity aversion is relatively low.

Keywords: corporate finance decisions; ambiguity; cash holdings; optimal dividends

JEL Classification Numbers: G30; G32; D01; D81;

1. INTRODUCTION

Over the last three decades, there have been many developments in decision theory that improved our understanding of uncertainty. In line with Knight (1921), uncertainty can be divided into two well-defined distinct parts, risk and ambiguity. “Risk” is used to refer to any sort of uncertainty that can be defined through the existence of a probabilistic model based on one single probability assessment, which is known to the decision maker (DM). “Ambiguity” is used to refer to situations in which the DM appears to be not fully confident that his/her beliefs apply. Practically, risk is mostly used when uncertainty is calculable, i.e. both outcomes and a subjective probability distribution over outcomes can be specified. Ambiguity applies to situations where uncertainty is incalculable, i.e. where there is no clear perception of the possible outcomes or of an estimate of a single plausible probability distribution. At least since Ellsberg (1961), experimental studies in ambiguous settings have repeatedly shown that DMs usually prefer to deal with known, rather than unknown probabilities, thereby revealing a form of ambiguity aversion. Estimating a quantified opportunity cost of acting now rather than later is particularly difficult when dealing with financing decisions under significant ambiguity.

Although the recent literature on ambiguity has provided a unified and elegant framework to address (and often solve) some financial puzzles (e.g. the equity premium puzzle and the interest rate puzzle, see Epstein and Schneider, 2010), there are still ill-understood phenomena in corporate finance. Recent studies document a secular increase in the cash holdings of some firms (Bates, Kahle and Stulz, 2009; Denis and Sibikov, 2010; Faulkender and Wang, 2006; Haushalter, Klasa and Maxwell, 2007; Holberg, Phillips and Prabhala, 2014). And yet, one would expect that the precautionary demand for cash should decrease when firms can hedge more effectively as more types of derivatives are available, e.g. as a consequence of improvements in information and financial technology since the early 1980s. The observed increase in cash holdings represents an anomaly that challenges existing theories. At the same time, it is not clear whether equity holders would rather increase or decrease equity and dividends in the presence of vague economic perspectives and different forms of uncertainty (see also Lins, Servaes and Tufano, 2010, about investor preferences and cross-country differences in cash holdings).

Various empirical studies are inconclusive about the hierarchy or “pecking order” among different sources of funds (see Leary and Roberts, 2010, and references there). Some have documented a significant heterogeneity in corporate decisions attributed to a divergence in beliefs about the firm’s value between managers and the market. Behavioural explanations of corporate decisions have recently come to consider “managers’ personality traits” (Hackbarth, 2008, 2009), which may include their attitude towards ambiguity. But whether the choice between equity or debt finance is affected by managers' personality traits and their perception biases is still controversial.

This paper sets out to answer the following unresolved questions: Why do certain firms issue equity instead of debt? Why did most firms retain their cash holdings instead of distributing them as dividends in recent times? How do firms change their financing policies during a period of severe financial constraints and ambiguity, or when facing the threat of a financial crisis in the foreseeable future? Our paper tries to provide answers within the framework of a dynamic model which incorporates ambiguity and the investor's attitude towards it. We model the corporate decisions as real options and apply the mathematics of mixed singular control/optimal stopping methods in stochastic settings under ambiguity. In particular, we analyse how the values of the firm's equity and debt are affected by ambiguity (Propositions 1 and 2) and relate our results to the pecking order puzzle (Proposition 3); moreover, we show how ambiguity affects cash holdings and optimal dividend policies (Propositions 4 and 5). We find that the presence of a standard pecking order or its reverse may depend on the relative ambiguity aversion biases of the managers and the investors: if managers have a stronger ambiguity aversion bias than the market, then a reversal of the standard pecking order preferences can be obtained. Finally, we find that cash holdings are retained longer if the impact of the ambiguity aversion bias is sufficiently large, which is consistent with the observed change in cash holdings in periods of turbulence and vague uncertainty. Cash holdings become less attractive with relatively small ambiguity aversion biases, in which case the DM prefers to receive dividends instead.

2. MODEL SET-UP

When considering a decision maker (DM) facing ambiguity, the Choquet Expected Utility approach represents his/her beliefs by a non-additive unit measure, which is referred to as a capacity. Applying the Choquet integral of a capacity to a given vector of outcomes (an 'act') generates an implied probability distribution over the outcomes, on the basis of which the expected utility value is calculated. But in contrast to subjective expected utility theory, there may no longer exist a single implied probability distribution over the states of nature that applies for all acts. Rather, the implied probability distribution may change according to the ranking of the states of nature regarding the desirability of the outcomes obtained for them. For acts that generate the same ranking of the states of nature, i.e. co-monotonic acts, the same implied probability distribution applies. So if attention is restricted to a set of co-monotonic acts only, the results are indistinguishable from subject expected utility, with the subjective probability distribution equalling the implied probability distribution (see a.o. Schmeidler, 1989).

In the Choquet Expected Utility model a capacity simultaneously represents the ambiguity experienced by the decision maker and his/her attitude towards this ambiguity. The resulting complications tend to be circumvented by assuming (full) ambiguity aversion. Under this additional assumption – which applies throughout this paper, unless stated otherwise – the capacity only describes the ambiguity experienced by the DM (see e.g. Ghirardato, Maccheroni and Marinacci, 2004, and Chateauneuf, Eichberger and Grant, 2007). We refer to the combined effect of the perceived ambiguity and the DM's ambiguity aversion as his/her ambiguity aversion bias.

When capacities are updated in the light of new information, it is natural for dynamic inconsistency to arise between the initial beliefs and the updated beliefs (see Gilboa and Schmeidler, 1993). Dynamic consistency typically requires the decision problem with non-additive beliefs to be equivalent to one with additive beliefs, as stated in Kast and Lapied (2010) discussing Sarin and Wakker (1998). The problem of dynamic inconsistency carries over to ambiguous stochastic processes. Accordingly, dynamic consistency requires the ambiguous stochastic process to be equivalent to an additive one. For ambiguous random walks represented by a binomial tree, Kast and Lapied (2010) assume independence of the conditional capacities in order to derive the associated additive random walks. They show that additive random walks converge to Brownian motions for which an increase in ambiguity decreases both the drift and the variance, as outlined below.

Suppose that the firm's asset value, V_t, follows a Choquet–Brownian process[1]. It is defined on the basis of a binomial lattice, where for each s_t at time t, such that $0 \leq t \leq T$, ${s_{t+1}}^u$ and ${s_{t+1}}^d$ denote the possible successors at time $t + 1$ for an "up" and a "down" movement, respectively. If "up" and "down" movements have the same capacity, then $\upsilon({s_t}^u|s_t) = \upsilon({s_t}^d|s_t) = c$, where c, $0 < c < 1$, is a constant that represents the DM's ambiguity about the likelihood of the states to come. If the DM is ambiguity averse, the capacity is sub-linear, so that $c < 1/2$ (Gilboa, Postlewaite and Schmeidler, 2008). If the perceived ambiguity increases, the value of the parameter c moves further away from the anchor 1/2. Thus, the capacity becomes more convex (for an ambiguity averse DM) or more concave (for an ambiguity loving DM). The symmetric discrete process outlined above can be shown to converge to a continuous time generalized Wiener process with mean $m = 2c - 1$ and variance $s^2 = 4c(1\text{-}c)$. The absence of an ambiguity bias is obtained as a special case for $c = 1/2$. Thus, the firm's asset value is given by[2]:

$$dV_t/V_t = ((r - q) + m\sigma)dt + s\sigma dB_t \tag{1}$$

[1] A Choquet-Brownian process is a distorted Brownian process, where the distortion derives from the nature and intensity of preferences toward ambiguity (Kast, Lapied and Roubaud, 2014)

[2] Expression (1) is obtained from $dV_t / V_t = (r - q)dt + \sigma dW_t$, where $W_t = mt + sB_t$ and B_t is a Wiener process.

where r is the risk–free interest rate, q is the instantaneous rate of return on the firm's assets (determining the internal liquidity of the firm from its cash flows), σ is the volatility, and B_t is a Wiener process. For fully ambiguity averse DMs we have $-1 < m < 0$ and $0 < s < 1$, so $r - q + m\sigma < r - q$ and $0 < s\sigma < \sigma$. Both drift and volatility are reduced in comparison to the case where ambiguity is absent. We assume that the firm issues perpetual debt which pays a continuous coupon at the rate C. The firm uses its revenue to make the coupon payment or to pay equity holders' dividends. When revenues are not sufficient and in the absence of cash balances, the firm can decide either to issue new equity or to declare bankruptcy. Thus, if the revenue rate exceeds the coupon rate ($qV \geq C$), equity holders receive dividends; if the revenue rate falls below the coupon rate, the firm dilutes equity. Equity dilution is costly and we assume that the cost of equity dilution is proportional to the proceeds from issuance. Thus, following the argument about equity dilution in Asvanunt, Broadie and Sundaresan (2011), it is equivalent to a negative dividend of $\beta(qV - C)$. Below a critical value V_B the firm will declare bankruptcy: that is, V_B denotes the firm's endogenous default threshold and is obtained as a result of equity holders' optimization, as in Leland (1994). In the event of default, debt holders receive $(1-\alpha)V_B$, where α denotes the fraction of cash flows lost due to default costs. In the next section we compute the total values of the firm's equity and debt in the presence of ambiguity, for the current value of its assets.

3. EQUITY AND DEBT UNDER AMBIGUITY AVERSION

For a given value of the firm's assets V, denote the total value of its equity by $E(V)$ and the total value of its debt by $D(V)$. Following the standard contingent claims literature, we derive the value of equity by solving the system:

$$\frac{1}{2}s^2\sigma^2V^2E''(V) + ((r-q)+m\sigma)VE'(V) - rE(V) + \beta(qV - C) = 0 \quad \text{if } V_B \leq V < \frac{C}{q} \tag{2}$$

$$\frac{1}{2}s^2\sigma^2V^2E''(V) + ((r-q)+m\sigma)VE'(V) - rE(V) + (qV - C) = 0 \quad \text{if } V \geq \frac{C}{q} \quad . \tag{3}$$

In addition, the following boundary conditions must be satisfied:

BC: $E(V_B) = 0$

$VM: E(\frac{C}{q})^- = E(\frac{C}{q})^+$

$$SP: \; E'(\frac{C}{q})^{-} = E'(\frac{C}{q})^{+}$$

that is, equity holders receive nothing at bankruptcy (*BC*), and the value matching (*VM*) and smooth pasting (*SP*) conditions hold at *C/q* (see also Asvanunt, Broadie and Sundaresan, 2011). Moreover, we impose the condition that *E*(*V*) behaves like *V* when the firm's value approaches infinity. In the Appendix we derive the following expression for the value of the firm's equity:

$$E(V) = \begin{cases} \beta(\rho V - \frac{C}{r}) + A_2 V^{-\omega_2} + A_3 V^{-\omega_1} & if \quad V_B \le V < \frac{C}{q} \\ (\rho V - \frac{C}{r}) + \hat{A}_3 V^{-\omega_1} & if \quad V \ge \frac{C}{q} \end{cases} \tag{4}$$

where:

$$A_2 = (\beta - 1)\left(\frac{C}{q}\right)^{\omega_2}\left(\frac{\omega_1}{\omega_1 - \omega_2}\right)\left[\frac{C}{r} - \left(\frac{1+\omega_1}{\omega_1}\right)\frac{C}{q - m\sigma}\right]$$

$$A_3 = -\beta\left(\rho V_B^{\omega_1 + 1} - \frac{C}{r} V_B^{\omega_1}\right) - A_2 V_B^{\omega_1 - \omega_2}$$

$$\hat{A}_3 = A_2 \frac{\omega_2}{\omega_1}\left(\frac{C}{q}\right)^{\omega_1 - \omega_2} + A_3 - (\beta - 1)\left(\frac{C}{q}\right)^{\omega_1}\frac{C}{(q - m\sigma)\omega_1},$$

with $\rho = \frac{q}{q - m\sigma}$ and $\omega_{1,2} = \frac{(r - q + m\sigma) - \frac{s^2\sigma^2}{2} \pm \sqrt{\left[\frac{s^2\sigma^2}{2} - (r - q + m\sigma)\right]^2 + 2s^2\sigma^2 r}}{s^2\sigma^2}$.

Here we use ω_1 to denote the positive solution and ω_2 to denote the negative solution. The bankruptcy threshold V_B is derived by maximizing the value of equity with respect to V_B. Since *E*(*V*) depends on V_B only through A_3, the optimal level V_B^* is derived solving $\partial A_3/\partial V_B = 0$. This value is given by the following implicit expression:

$$\omega_1 \beta(\frac{C}{r})V_B^{-1} - (1+\omega_1)\rho\beta - (\omega_1 - \omega_2)A_2 V_B^{-\omega_2 - 1} = 0 \tag{5}$$

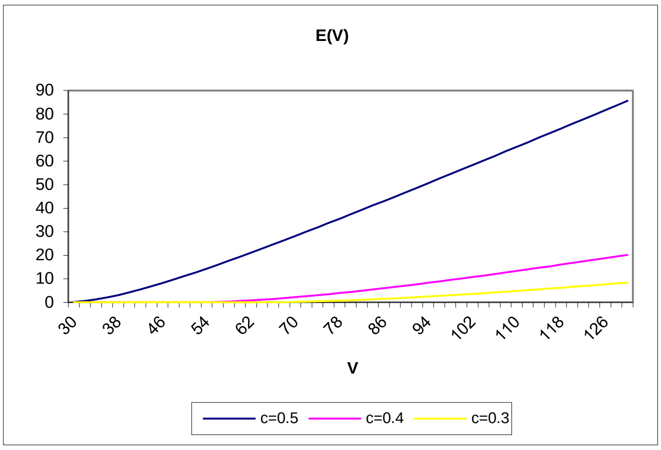
Straightforward computation on (4) leads to the following:

Proposition 1. *The value of the firm's equity decreases as the ambiguity perceived by the ambiguity averse DM increases.*

An example[3] is depicted in Figure 1, where the equity curve shifts downwards monotonically as c decreases.

Figure 1. Value of the firm's equity under ambiguity

$C = 2.3,\ \sigma = 0.2,\ r = 0.05,\ q = 0.03,\ \alpha = 0.3,\ \beta = 1$



This result contrasts with what is usually obtained for an increase in uncertainty as measured by the volatility σ: indeed, the value of the firm's equity increases with volatility, while it decreases as ambiguity increases. Moreover, the results in Remarks 1 and 2 follow:

Remark 1. *The default threshold* V_B *increases as the ambiguity perceived by the ambiguity averse DM increases.*

As perceived ambiguity increases, equity holders choose a higher default level and hence enter financial distress earlier. This occurs because the value of the option to keep the firm open decreases with a higher ambiguity aversion bias, which reduces the variance in the Choquet-Brownian motion.

Remark 2. *The value of the firm's equity decreases as the cost of equity dilution (* β *) increases.*

Let us now determine the value of the firm's debt. If the firm liquidates its assets upon bankruptcy, a fraction α is lost due to liquidation costs. Thus, due to limited liability, debt holders will only receive $D(V_B) = (1-\alpha)V_B$. The value of the firm's debt $D(V)$ is determined solving the following equation:

[3] The parameter values are similar to Asvanunt, Broadie and Sundaresan (2011) and consistent with previous works (see Leland, 1994).

$$\frac{1}{2}s^2\sigma^2V^2D''(V)+((r-q)+m\sigma)VD'(V)-rD(V)+C=0 \quad \text{for } V\geq V_B \ . \tag{6}$$

From this, we obtain (see the Appendix):

$$D(V)=\frac{C}{r}+\left((1-\alpha)V_B-\frac{C}{r}\right)\left(\frac{V}{V_B}\right)^{-\omega_1} \quad \text{if } V\geq V_B \ . \tag{7}$$

The following Proposition holds:

Proposition 2. *The value of the firm's debt increases as the ambiguity perceived by the ambiguity averse DM increases.*

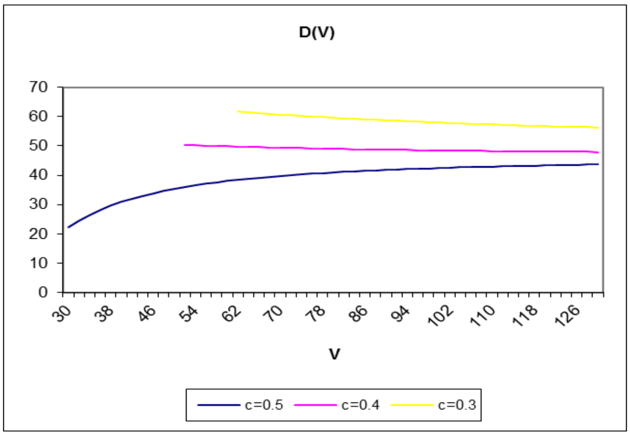
An example is depicted in Figure 2, where the debt curve shifts upwards as c decreases.

Also this result contrasts with what is usually obtained for an increase in uncertainty as measured by the volatility σ: the value of debt decreases with volatility, while it increases with ambiguity. So the effects of risk and ambiguity go in opposite directions.

The intuition of our results resembles that of a firm facing the corresponding ambiguous static decision problem. Following the requirement for dynamic consistency, assume the decision maker's beliefs can equivalently be represented by a probability distribution. An increase in the level of ambiguity changes the initial equivalent probability distribution into one whose mean and variance are decreased.

Figure 2 – Value of the firm's debt under ambiguity

C = 2.3, σ =0.2, r =0.05, q = 0.03, α =0.3, $\beta = 1$



The firm is financed by equity and debt and is vulnerable to bankruptcy. The value of its equity is determined by limited liability considerations. An increase in ambiguity now tends to decrease the probability weight on high returns. As a consequence, the value of equity decreases. As in the results for the ambiguous stochastic process (Propositions 1 and 2), the effect of an increase in ambiguity is opposite to the standard effect of an increase in volatility.

In a nutshell: the limited liability effect of enhanced risk makes equity holders more aggressive, which increases default risk and reduces the value of debt. An increase in ambiguity, on the other hand, changes the stochastic process perceived by the DM by reducing both its mean and its variance, making equity holders less aggressive. This mitigates against the limited liability effect, by reducing the perceived likelihood of default and increasing the value of the firm's debt.

4. PECKING ORDER FINANCING DECISIONS

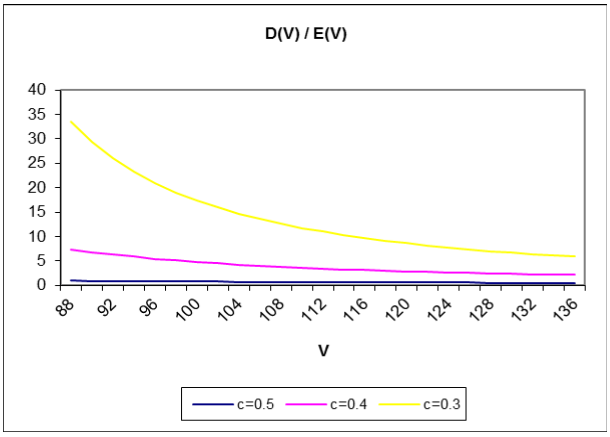
It is well-know that the general rule of the pecking order hypothesis for the issuance of securities suggests the order of preference to be: firstly internal funds, if available; then debt, if external funds are needed; and finally equity. This rule is discussed in the seminal contribution by Myers and Majluf (1984), and suggests that firms issue the securities that carry the smallest adverse selection cost, i.e. are least likely to be mispriced by imperfectly informed outside investors. Debt dominates new equity, because it is considered to be robust against mispricing.

However, debt can create information problems of its own if there is a significant probability of default. In order to obtain the standard pecking order one needs to assume that either *(i)* debt is risk free (as in Myers and Majluf, 1984, where there is no investment risk), or *(ii)* debt is not mispriced (because uninformed outside investors do not care about risk when making decisions, as in Nachman and Noe, 1994). The pecking order hypothesis has been challenged within the theory of optimal design of securities under asymmetric information (see Giammarino and Neave (1982), Nachman and Noe (1994), and Fulghieri and Lukin, 2001). In addition, much empirical work challenges the pecking order hypothesis (see, e.g. Hennessy, Livdan and Miranda, 2010, Leary and Roberts, 2010, and Halov and Heider, 2011).

In this section we reconsider the pecking order puzzle within the framework of our model with ambiguity. Let us consider a situation where the DM's valuation reflects his/her ambiguity aversion bias, while the market's valuation is not biased by ambiguity aversion. So the DM believes that cash flows are described by expression (1) with $c < 1/2$, whereas the market's valuation is as if the cash flows are described by expression (1) with c = 1/2. By plotting $D(V)/E(V)$, we find for any value V of the firm's assets, that when the ambiguity perceived by the ambiguity averse DM increases, $D(V)/E(V)$ increases as well (see Figure 3). This implies that the DM believes that equity is more overvalued by the market than debt. Hence, the DM will prefer issuing equity rather than debt, which results in a reversal of the standard pecking order financing behaviour.

Figure 3 – Debt/Equity Value under ambiguity

$C = 2.3,\ \sigma = 0.2,\ r = 0.05,\ q = 0.03, \alpha = 0.3,\ \beta = 1$



The argument can be summarized as follows:

Proposition 3. *A reversal of the standard pecking order may occur if the ambiguity aversion bias of the DM exceeds that of market.*

The result in Proposition 3 poses a challenge for the standard pecking order: managers may not (may) follow a pecking order if they have a larger (smaller) ambiguity aversion bias than the market's valuations. The way this managerial bias affects the pecking order preferences may help explain the inconclusive cross-sectional findings on the observed heterogeneity in capital structures and standard pecking order predictions.

Our result is also in keeping with the theoretical literature on the pecking order, showing that asymmetries relating to the information available to managers and to investors may lead to a reversal in the preferences (Giammarino and Neave, 1982, and Nachman and Noe, 1994, Hackbarth, 2008). These contributions, however, consider forms of asymmetric information, rather than differences in the ambiguity aversion bias.

5. CORPORATE LIQUIDITY POLICIES

Now suppose that the firm can hold cash reserves and assume that the accumulated net revenues up to time *t* can be described by a Bachelier additive model. Its use simplifies the mathematical structure of our model and allows us to consider net cash flows that may become negative when the firm's revenues are insufficient to cover its costs. We assume the firm acts in the best interest of its equity holders and maximizes the expected present value of dividends up to default. We further assume that the firm has no access to capital markets and that equity dilution is not possible. As a consequence, default occurs as soon as the cash process net of the coupon payment, hits the threshold 0. Denote the total dividends distributed up to time *t* by Z_t, where $dZ_t \geq 0$. Now the firm's cash reserve X_t evolves according[4] to:

$$dX_t = (r - q - m\sigma)dt + s\sigma dB_t - dZ_t, 0 \leq t \leq t_0 \qquad (8)$$

$$dX_t = dZ_t = 0, \ \ t \geq t_0$$

with $X_0 = x \geq 0$ being given. The firm chooses its liquidity policy to maximize its total profits, i.e the expected total discounted dividends

$$\mathrm{E}_x \int_0^\infty e^{-rt} dZ_t \, .$$

Accordingly, we define $V(x) = sup \ \mathrm{E}_x \int_0^\infty e^{-rt} dZ_t$ where the *sup* is taken over all admissible control policies. The problem reduces to the classical dividend policy problem, formulated as a mixed singular control/optimal stopping problem, as in Jeanblanc-Picquè and Shiraev (1995), Radner and Shepp (1996) and Decamps and Villeneuve (2007, 2013), but here the effects of ambiguity are incorporated into the model. Calculations following the same arguments as in this literature lead to the following solution:

Proposition 4. *The value of the firm's assets is given by:*

$$V(x) = \max(0, \frac{f(x)}{f'(x^*)}), \quad for \qquad 0 \leq x \leq x^*$$

$$V(x) = x - x^* + V(x^*), \ \ for \qquad x \geq x^*$$

where

$$f(x) = e^{Ax} - e^{Bx},$$

[4] Expression (8) is obtained from $dX_t = (r-q)dt + \sigma dW_t - dZ_t$, where $W_t = mt + sB_t$ and B_t is a Wiener process.

$$x^* = \frac{1}{A-B}\ln(\frac{B}{A})^2$$

and

$$\frac{1}{A-B} = \frac{s^2\sigma^2}{2\sqrt{(r-q+m\sigma)^2 + 2rs^2\sigma^2}}$$

$$(\frac{B}{A})^2 = \frac{(-(r-q+m\sigma) - \sqrt{(r-q+m\sigma)^2 + 2rs^2\sigma^2})^2}{(-(r-q+m\sigma) + \sqrt{(r-q+m\sigma)^2 + 2rs^2\sigma^2})^2} .$$

The interpretation of this result is as follows. If the cash process falls below the coupon payment, this leads to immediate bankruptcy. Whenever the cash process exceeds the threshold value for paying out dividends, $x \geq x^*$, the optimal policy pays out all cash in excess of the threshold value x^*. If the cash process falls below the critical level x^*, but exceeds the coupon payment, then no dividends are paid, but bankruptcy is avoided.

This "all-or-nothing" policy is common to the classical dividend policy of the literature mentioned above. It finds that the optimal choice of dZ_t is singular: in the "dividend" region where $x \geq x^*$, that is, where the liquidity reserve becomes too high, it is optimal to pay dividends as quickly as possible, reducing the cash holdings until either the liquidity reserve returns to the "save" region, where the firm will not pay dividends, or until the firm is bankrupt.

This formulation can be extended to allow the firm to issue new equity. In this case, another "issue" region will be added, lying below the "save" region, such that if the liquidity reserve becomes too low, then new equity is issued to return to the "save" region (see also Anderson and Carverhill, 2011). Another extension could consider lumpy investments and uncertain capital supply, which not only affects the pecking order of sources of finance, but also leads to several different regions relating to the firm's dividend policy (see Hugonnier, Malamud and Morellec, 2013). For the sake of simplicity, we do not pursue these possibilities here and instead focus on the effect of ambiguity within the classical framework.

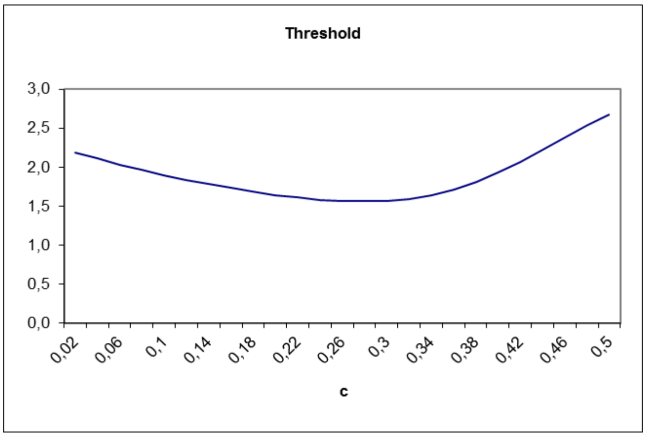
To see how ambiguity affects the optimal dividend policy, we need to know how the critical threshold x^* changes as c changes. Calculation of $\partial x^*/\partial c$ leads to the following:

Proposition 5. *The critical threshold value of the cash process increases (decreases) for high (low) levels of ambiguity aversion.*

An example is provided in Figure 4, where the threshold x* + C is plotted for 0 < c < ½.

Figure 4 – The threshold for different values of ambiguity

C = 2.3, σ =0.2, r =0.05, q = 0.03

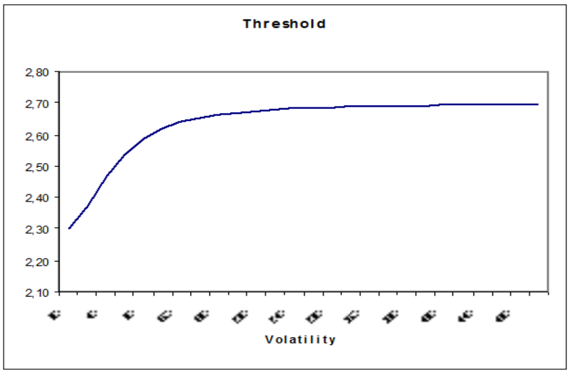


If the DM is ambiguity averse, we find that cash holdings are retained longer if his/her ambiguity aversion bias is sufficiently large. This result is consistent both with the observed change in cash holdings in periods of turbulence and vague uncertainty, and with the literature justifying large cash holdings because of the precautionary motive. However, for relatively small ambiguity aversion biases, cash holdings become less attractive. In this case, the DM prefers to receive dividends instead.

Notice that the effects of an increase in ambiguity aversion bias and an increase in volatility on the threshold do not align. Figure 5 depicts the critical threshold for c=1/2, which is monotonically increasing in volatility.

Figure 5 – The threshold for different values of volatility

C = 2.3, r =0.05, q = 0.03, c=1/2.



6. CONCLUSION

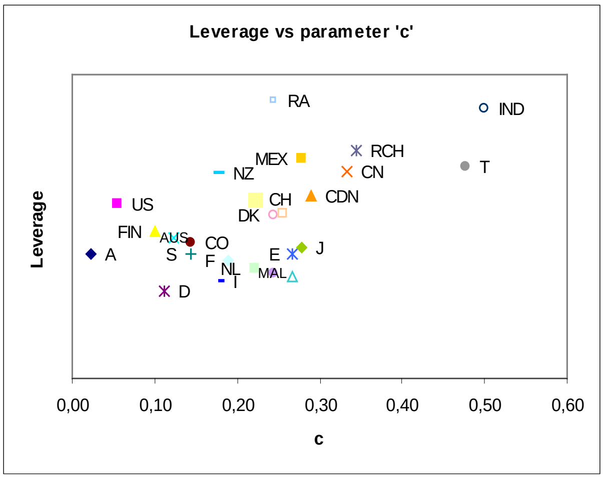
Providing empirical support for our results is not straightforward, due to the difficulty in finding a convincing proxy for the size of the ambiguity aversion bias. Recent work by Rieger, Wang and Hens (2014) employs a methodology to measure the average ambiguity aversion across different countries, and this methodology might be adapted to our framework in order to estimate the level of the ambiguity aversion.

As a preliminary step we analyzed cross-country average leverage values - where leverage is defined as book value of long term debt (item 106 in Compustat Global database) over market value of total assets, calculated as book value of total assets (item 89) minus book value of equity (item 146) plus market value of equity (item MKVAL) - over a period of five years for 24 countries[5]. The average ambiguity aversion across these countries is provided by Rieger, Wang and Hens (2014) and is mapped into the parameter c. We found a positive correlation (0.566) between leverage and ambiguity bias (see Figure 6), which is consistent with our results.

Initial empirical evidence showing that ambiguity aversion is positively associated with cash holdings is provided in Neamtiu, Shroff, White and Williams (2014). They use the dispersion in forecasts of corporate profits from the Survey of Professional Forecasters as a proxy for the level of ambiguity. In contrast, Breuer, Rieger and Soypak (2014) find that for financially constrained firms cash holdings decrease with increasing ambiguity aversion, while they get inconclusive results for unconstrained firms. Although such results seem conflicting, they might benefit from being interpreted within the context of our model, which offers a general framework for understanding corporate decisions under ambiguity.

[5] Australia, Austria, US, UK, Finland, Germany, Colombia, Sweden, Italy, New Zealand, France, The Netherlands, Switzerland, Argentina, Denmark, Malaysia, Portugal, Spain, Japan, Mexico, China, Chile, Canada, Thailand.

Figure 6. Cross-country leverage vs ambiguity aversion



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Appendix

Equity Value under Ambiguity

We start by determining, the first and the second derivatives of the general solution to (2),

$$E(V) = A_0 + A_1 V + A_2 V^{-\omega_2} + A_3 V^{-\omega_1}:$$

$$E'(V) = A_1 - \omega_2 A_2 V^{-\omega_2 - 1} - \omega_1 A_3 V^{-\omega_1 - 1}$$

$$E''(V) = \omega_2(\omega_2 + 1) A_2 V^{-\omega_2 - 2} + \omega_1(\omega_1 + 1) A_3 V^{-\omega_1 - 2}$$

Substituting them into (2), we obtain:

$$\frac{1}{2}s^2\sigma^2 V^2\left[\omega_2(\omega_2+1)A_2 V^{-\omega_2-2} + \omega_1(\omega_1+1)A_3 V^{-\omega_1-2}\right]$$
$$+\left((r-q)+m\sigma\right)V\left[A_1 - \omega_2 A_2 V^{-\omega_2-1} - \omega_1 A_3 V^{-\omega_1-1}\right]$$
$$-r\left[A_0 + A_1 V^{-\omega_2} + A_3 V^{-\omega_1}\right] + \beta(qV - C) = 0$$

Rearranging this expression by gathering the terms relating to V, $V^{-\omega_1}$ and $V^{-\omega_2}$ we find:

$$V^{-\omega_2}\left[\frac{1}{2}s^2\sigma^2\omega_2(\omega_2+1)A_2 - (r-q)\omega_2 A_2 - m\sigma\omega_2 A_2 - rA_2\right]$$
$$+V^{-\omega_1}\left[\frac{1}{2}s^2\sigma^2\omega_1(\omega_1+1)A_3 - (r-q)\omega_1 A_3 - m\sigma\omega_1 A_3 - rA_3\right]$$
$$+ \quad V(\beta q - qA_1 + m\sigma A_1) - rA_0 - \beta C = 0$$

Setting the coefficients equal to zero we obtain $A_0 = -\beta\frac{C}{r}$, $A_1 = \beta\frac{q}{q - m\sigma}$, and can determine ω_1 and ω_2 as specified in (4), where ω_1 is positive and ω_2 negative.

Applying the same procedure to the general solution of (3), $E(V) = \hat{A}_0 + \hat{A}_1 V + \hat{A}_2 V^{-\omega_2} + \hat{A}_3 V^{-\omega_1}$, we find $\hat{A}_0 = -\frac{C}{r}$ and $\hat{A}_1 = \frac{q}{q - m\sigma}$. Since ω_2 is negative, it follows that $\hat{A}_2 = 0$.

The remaining coefficients, A_2, A_3, and $\hat{A}_3$, as in (4), are now determined by solving the conditions:

$$BC: \quad \beta\left(\frac{q}{q-m\sigma}V_B - \frac{C}{r}\right) + A_2 V_B^{-\omega_2} + A_3 V_B^{-\omega_1} = 0$$

$$VM: \quad \beta\left(\frac{C}{q-m\sigma} - \frac{C}{r}\right) + A_2\left(\frac{C}{q}\right)^{-\omega_2} + A_3\left(\frac{C}{q}\right)^{-\omega_1} = \frac{C}{q-m\sigma} - \frac{C}{r} + \hat{A}_3\left(\frac{C}{q}\right)^{-\omega_1}$$

$$SP: \quad \beta \frac{q}{q-m\sigma} - A_2\omega_2(\frac{C}{q})^{-\omega_2-1} - A_3\omega_1(\frac{C}{q})^{-\omega_1-1} = \frac{q}{q-m\sigma} - \hat{A}_3\omega_1(\frac{C}{q})^{-\omega_1-1}$$

Debt Value under Ambiguity

Considering $D(V) = B_0 + B_1V + B_2V^{-\omega_1}$ as the general solution to (5), we obtain:

$$\frac{1}{2}\sigma^2\omega_1^2 B_2V^{-\omega_1} + \frac{1}{2}\sigma^2\omega_1 B_2V^{-\omega_1} + (r-q)VB_1 - (r-q)\omega_1 B_2V^{-\omega_1} + m\sigma VB_1 - m\sigma\omega_1 B_2V^{-\omega_1}$$

$$-rB_0 - rB_1V - rB_2V^{-\omega_1} + C = 0$$

The coefficients B_0 and B_1 are determined by gathering the constant and the coefficient of the V term and setting them equal to zero:

$$-rB_0 + C = 0 \Rightarrow B_0 = \frac{C}{r}$$

$$(r-q)B_1 + m\sigma B_1 - rB_1 = 0 \Rightarrow B_1 = 0.$$

Finally, the coefficient B_2 is obtained from the boundary condition $D(V_B) = (1-\alpha)V_B$.

Indeed, $\frac{C}{r} + B_2V_B^{-\omega_1} = (1-\alpha)V_B \Rightarrow B_2 = \left((1-\alpha)V_B - \frac{C}{r}\right)V_B^{\omega_1}$.

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