Wage inequality and skill asymmetries*

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Abstract

Using a simple model with two levels of skill, we assume that high-skill workers who fail to get high-skill jobs may accept low-skill positions; low-skill workers do not have the analogous option of filling high-skill positions. This asymmetry implies that an adverse, skill-neutral shock to aggregate employment may cause an increase in wage inequality, both between and within skill categories, as well as an increase in unemployment, especially among low-skill workers. Movements in productivity, unemployment and inequality may thus be linked to induced overeducation and credentialism.

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1 Introduction

This paper focuses on interactions between unemployment and wage inequality. The main facts are well known. In the 1970s and 1980s unemployment rose in all OECD countries; in most of Europe it has remained high throughout the 1990s and into the 2000s. Inequality has been increasing as well, although here the picture is somewhat more complicated. Changes in the wage structure and in personal income distribution have been particularly striking in the US and UK, while so far some continental European countries have avoided significant increases in wage inequality. More recently, US unemployment rates have returned to the levels of the 1960s and there are signs that these developments have been accompanied by stable or declining wage inequality from the mid 1990s. Some European countries, including the UK, Netherlands, Denmark and Ireland, have also experienced substantial reductions in unemployment since the early 1990s.

It is our purpose to present and analyse a mechanism which may have contributed to these trends. At the centre of this mechanism is an asymmetry between high and low skill. A high-skill worker who fails to get a high-skill job may accept a low-skill position; a low-skill worker, on the other hand, does not have the analogous option of filling a high-skill position. Thus, in line with models of "job competition" (Thurow (1975)), a distinction is made between the skill requirement of a job and the skill of the worker. The asymmetry between the options facing the two kinds of workers implies that the rate of unemployment among low-skill workers will be more sensitive to changes in aggregate activity than will unemployment among high-skill workers, some of whom will take low-skill jobs rather than become unemployed. Since increasing numbers of high-skill workers will move into low-skill jobs when times are bad, the dispersion in their incomes will increase, that is, within-group inequality will increase. More importantly, a high average rate of unemployment will cause unemployment among low-skill workers to be particularly high and put pressure on wage rates for low-skill jobs, thus tending to increase the skill premium in high-skill jobs.

The distinction between the skill requirement of a job and the skills of the worker filling the job has been discussed in the empirical literature in terms of "overeducation" and "credentialism". A worker is overeducated if his education exceeds the requirements set by the employer. Credentialism, on the other hand, arises when a change in the pool of applicants leads employers to raise the skills required for recruitment to an otherwise unchanged job. Using this terminology, our argument focuses on endogenous changes in overeducation and credentialism. An adverse, skill-neutral shock to aggregate activity may reduce employment of both high- and low-skill workers, but induced changes in overeducation and credentialism imply that low-skill workers will be hit harder than high-skill workers.

The effects of induced overeducation and credentialism may be complementary to other, existing explanations of increased inequality. In the interest of analytical simplicity - and to highlight the potential contribution of induced overeducation - we shall assume, however, that the economy is closed, that technical change is Hicks-neutral in its effects on different types of labour, and that the institutional structure of the labour market as well as the (trend increase in the) supply of skill are unchanging. Thus, we exclude by assumption the factors which are typically held responsible for the rise in inequality.

The remainder of this paper is in 5 sections. Section 2 briefly describes the empirical evidence on overeducation. Section 3 sets out a standard model in which biased technical progress is needed in order to generate increases in both the relative wage and the relative employment of high-skill workers. Section 4 examines the implications of introducing the asymmetry between the job options of high- and low-skill workers. We derive the implications of changes in the employment pattern for wage inequality, both between and within skill categories. Section 5 endogenizes the changes in employment. It is shown that a negative, Hicks-neutral shock to aggregate demand may produce both increased wage disparities and an increase in the relative employment of high-skill workers. Section 6 contains a few concluding remarks. All proofs have been collected in Appendices.

2 Overeducation

The measurement of overeducation and credentialism involves many difficulties, both conceptual and empirical.[1] There is strong evidence, however, that the incidence of overeducation is substantial. An influential study by Sicherman (1991), for instance, reports that 40 percent of US workers are overeducated in the sense that they had more education than required to get their current job; Hersch (1991) finds overeducation figures ranging from 28 to 78 percent for different groups of workers in a sample from Oregon. In the UK, several studies indicate that about 30 percent of all respondents were overeducated and that the figure may be above 40 percent among those possessing more than the lowest level of qualifications (Sloane et al (1999), Dolton and Vignoles (2000),

[1] Green et al (1999) and Hartog (2000) discuss some of the issues involved.

Rigg et al (1990)). Summarizing the evidence, Green et al (1999, p.15) suggest that "overeducation is a widespread phenomenon both in Europe and the United States of America". Undereducation - workers who report having less education than required to get the job - also exists. Quantitatively, most studies indicate that about 10-20 percent of all workers are undereducated. The existence of undereducation on this scale could reflect unmeasured heterogeneity. Alternatively, however, it could indicate credentialism: although employers may want workers with the "required education", this level may not be needed to do the job.

From the perspective of the present paper, it is not merely the levels, but also the changes in the incidence of overeducation and credentialism that are critical. With induced overeducation, a rise in unemployment tends to increase overeducation. When it comes to short run fluctuations, however, this effect may be offset by the effects of differential labour hoarding.[2] Like induced overeducation, differential labour hoarding implies that low-skill workers are affected disproportionately by unemployment, but the underlying mechanism and the effects on measured overeducation will be different. Induced overeducation focuses on the effects on different groups of workers of proportional changes in the number of high- and low-skill jobs; differential labour hoarding, on the other hand, suggests that temporary changes in demand will lead to non-proportional changes in the number of jobs, and when high-skill workers in low-skill jobs are laid off as a result of differential labour hoarding, there is a tendency for overeducation to decrease.[3] A priori it is difficult to say which of these effects will dominate in the short run.[4] In the medium term, however, differential hoarding ceases to be important and we would expect a negative correlation between employment and overeducation.

In the UK, evidence suggests that the incidence of overeducation increased strongly between the 1970s and 1980s (a period of rising unemployment) but may have stabilized since the late 1980s (Green et al (1999)). Robinson and Manacorda (1997, p. 3) find that

[2]It is well-known that because of differential labour hoarding, fluctuations in aggregate activity can lead to fluctuations in relative labour demand and wage inequality. Empirical work supports the extension of the adverse effects of unemployment on income distribution to time-scales beyond short-run fluctuations; e.g. Blinder and Esaki (1978) and Jäntti (1994).

[3]The measure of overeducation may be biassed in a downturn, however. Doeringer and Piore (1971) report the widespread use of "bumping": large American firms with well-developed internal labor markets, they argue, respond to a temporary decline in demand by laying off unskilled workers and letting their skilled workers take over unskilled tasks.

[4]There is some evidence that differential labour hoarding may dominate in the short run. Thus, using Dutch data from the 1990s, Gautier (2000) reports that the proportion of high-skill workers in low-skill positions falls in a recession.

in the UK between 1984 and 1994 "the increase in the supply of better educated labour has allowed firms to indulge in 'credentialism', employing more highly qualified staff to do jobs which previously were done by less qualified staff". Rigg et al (1990) present evidence to the effect that in the late 1980s, 25 percent of UK employers had substituted graduates for non-graduates; only about a third of these jobs had been upgraded in terms of content. Furthermore, in the UK an index of required qualifications rose between 1986 and 1992, but then fell slightly during the period of falling unemployment from 1992 to 1997 (Green et al (2000)). More generally, Hartog's (2000) survey of the literature reports an increasing incidence of overeducation (and decreasing undereducation) since the 1970s in a number of countries.[5] In the US, the evidence is ambiguous. In a longer-term perspective, Wolff (2000, p. 27) concludes that between 1950 and 1990 there has been a growing mismatch "between skill requirements of the workplace and the educational attainment of the workforce, with the latter increasing much more rapidly than the former". Daly et al. (2000), on the other hand, find a decline in overeducation between 1976 and 1985. With a rapid rise in average years of schooling, however, overeducation may increasingly take the form of a discrepancy between actual and required quality of education; a focus on years of schooling will fail to register any overeducation if, for instance, MIT graduates accept jobs which otherwise could and would have been filled by graduates from local colleges.[6]

Overall, the evidence may not be conclusive but it is strong enough that the possibility and potential effects of induced overeducation should be taken seriously.

[5]In their meta-analysis of 25 studies of overeducation, Groot and Maassen van den Brink (2000, p. 153) suggest that the "incidence of overeducation appears to have declined". This conclusion, based on raw averages, is contradicted by their own regression results, which control for some of the differences across studies with respect to, inter alia, the definition of overeducation (Table 3). Groot and Maassen van den Brink also suggest (Table 4) that the incidence of overeducation is unrelated to unemployment. A simple cross-section analysis of the results obtained in studies from a number of different countries says little, however, about the time-series effects of changes in unemployment.

[6]Dolton and Vignoles (2000) point out this problem and report that, in the UK, "graduates with a higher quality education, i.e. those who attended universities (rather than polytechnics - see Appendix B) and those having better degree grades were less likely to be overeducated" when overeducation is defined as a graduate working in a job that does not require a graduate degree (Dolton and Vignoles (2000, p. 183)).

3 The standard model

For simplicity, assume that there are only two kinds of workers, high-skill and low-skill, and that the supplies of these two types of labour are kept constant at H and L, respectively. Furthermore, let the production function of the representative firm exhibit constant returns to scale with respect to these two labour inputs[7],

$$Y = AF(N_H, N_L) \tag{1}$$

where Y is output; N_H and N_L denote input of high- and low-skill labour, respectively, and where changes in the multiplicative constant A describe Hicks-neutral technical change. If firms face given wage rates, the first order conditions for profit maximization imply that

$$\frac{W_H}{P} = mAF_1 = mAf^H(\frac{N_H}{N_L}); f^{H\prime} < 0 \tag{2}$$

$$\frac{W_L}{P} = mAF_2 = mAf^L(\frac{N_H}{N_L}); f^{L\prime} > 0 \tag{3}$$

where F_1 and F_2 are the partial derivatives of F. The proportionality factor m is given by $m = 1 + \frac{d \log P}{d \log Y} \leq 1$ which reduces to $m = 1$ in the simple case with perfectly competitive product markets. Using (2) and (3), the relative wage can be written

$$\frac{W_H}{W_L} = \frac{f^H}{f^L} = h(\frac{N_H}{N_L}); h' < 0 \tag{4}$$

Equations (1) to (4) describe the demand for labour. Using (4) it is readily seen that, in the absence of skill-biased technical change, relative employment and relative wages cannot move in the same direction in this standard set-up. Thus, one needs to introduce shifts in the h-function in (4) in order to obtain a pattern that fits the stylized facts of a rise in both W_H/W_L and N_H/N_L.

[7] The assumptions of linear homogeneity and the absence of other inputs can be relaxed. Consider a general specification

$$Y = A\psi(N_H, N_L, Z)$$

where Z is a vector of other inputs. The analysis would go through substantially unchanged on the weaker assumptions that

(i) ψ is separable in (N_H, N_L) and Z so that the production function can be rewritten

$$Y = A\varphi\left(\theta(N_H, N_L), Z\right)$$

and

(ii) the function θ, which aggregates the two types of labour input, is homothetic.

4 Skill asymmetries and induced overeducation

4.1 A simple model

The asymmetry between the options of high- and low-skill workers can be captured by a reformulation of the standard model. As before, let the constant supplies of high- and low-skill workers be H and L, and assume that production requires the performance of both high- and low-skill tasks. High-skill workers, however, may now be employed in either high- or low-skill jobs. Algebraically,

$$H = N_H + N_{LH} + U_H = N_{HT} + U_H \tag{5}$$

$$L = N_{LL} + U_L \tag{6}$$

$$N_L = N_{LL} + N_{LH} \tag{7}$$

where N_H and N_{LH} are the employment of high-skill workers in high- and low-skill jobs, respectively, and $N_{HT} = N_H + N_{LH}$ is total employment of high-skill workers; N_{LL} is the employment of low-skill workers in low-skill jobs, and U_H and U_L denote unemployment of high- and low-skill workers. With these definitions, the specification of the production function, equation (1), remains valid.

Overeducation can be modeled using a variety of different approaches. Theoretical models using a matching approach, for instance, have been presented by McKenna (1996), Muysken and Weel (2000) and Deding (2000)[8] while Skott (2003a) shows that overeducation may emerge naturally in a framework with efficiency wages. The structural models are complex, however, and the precise implications for induced overeducation are sensitive to a range of assumptions, including assumptions about the values of parameters that are hard to measure or estimate. Unless one feels very confident about the empirical relevance and adequacy of a particular structural model, it may therefore be preferable to focus directly on the implications of simple specifications that, although not as theoretically satisfying, still capture the essence of induced overeducation. This, indeed, will be our approach in the present paper.

We shall assume, first, that (a sufficient number of) firms prefer high- to low-skill workers, even when filling low-skill jobs. This ranking may arise in different ways. One

[8] Van Ours and Ridder (1995) also formalise the process of job competition as a matching model; they test the model on Dutch data for 1981-88 and find evidence for job competition at higher but not at lower levels of education; Groot and Hoek (2000) discuss some weaknesses of the study by Van Ours and Ridder.

simple story runs as follows.[9] Fairness considerations dictate that all workers in low-skill jobs be paid the same wage since attempts to differentiate would lead to worker resentment and shirking. High-skill workers, however, may be (slightly) more productive in these jobs. This assumption is in line with Büchel's (2002) finding that "overeducated workers are generally more productive than others" and that this is why "firms hire overeducated workers in large numbers". For present purposes, the productivity difference can be arbitrarily small. If all workers in low-skill jobs must be paid the same wage, firms will prefer high-skill workers as long as there is any productivity difference. To simplify the analysis we focus on the limiting case with the productivity difference going to zero.

Employed workers, secondly, can and do engage in job search, and working below one's formal qualification may send a better signal to prospective employers than (prolonged) unemployment. Thus, a high-skill worker in a low-skill job may have the same probability of getting a high-skill job as an unemployed high-skill worker. It is rational, therefore, for those high-skill workers that get relatively low disutility from a low-skill job and relatively high disutility from unemployment to accept a low-skill job rather than become unemployed. We shall assume that workers differ in their evaluation of the disutilities associated with unemployment and low-skill jobs. To be more specific, the proportion λ prefers low-skill jobs and the proportion $(1-\lambda)$ unemployment. The parameter λ is taken to be independent of the wage in low-skill jobs. This simplification can be rationalized in various ways; the simplest way, perhaps, is to assume that unemployment benefits are proportional to low-skill wages.

Combining the two assumptions about firms' and workers' preferences, we get a simple functional relation between N_{LH} and N_H:

$$N_{LH} = \lambda \left(H - N_H \right); 0 \leq \lambda \leq 1 \tag{8}$$

The parameter λ represents a measure of the strength of induced overeducation; one extreme, $\lambda = 0$, corresponding to the complete absence of overeducation and the other, $\lambda = 1$, to the case where all the high-skill workers that fail to get high-skill jobs move into a low-skill job.

What is the likely range of λ-values? If it is assumed - in line with the evidence - that at least 30 percent of all workers are overeducated, the linear specification in (8) implies a λ-value of over 0.8 for plausible values of the employment rates. Thus, using

[9] See Skott (2003a) for an alternative approach to the joint determination of wages, employment rates and the degree of overeducation in a shirking model.

(8), the condition

$$\frac{N_{LH}}{N_{HT}+N_{LL}} \geq 0.3$$

can be rewritten

$$\frac{\lambda}{1-\lambda} \geq 0.3 \left(\frac{N_{HT}+N_{LL}}{H-N_{HT}} \right)$$

The employment rate for high-skill workers, N_{HT}/H is unlikely to be less than 0.9. It follows that λ will exceed 0.73 even if $N_{LL}=0$. If only 10 percent of all workers are overeducated, this lower bound on λ (associated with $N_{LL}=0$) drops to 0.47. Thus, even if one believes that overeducation is usually overestimated, this argument suggests that $\lambda-$values below 0.5 are unlikely.

Turning now to the demand for labour, equations (2)-(4) still hold if W_H denotes the wage rate paid for high-skill work, as opposed to the average wage received by high-skill workers; the latter now is given by

$$W_{HA} = \frac{W_H N_H + W_L N_{LH}}{N_{HT}} \tag{9}$$

Wage inequality in this set-up takes two forms. Between-group inequality between high- and low-skill workers is captured by the ratio W_{HA}/W_L while the within-group dispersion of wages among high-skill workers can be described by

$$\sigma = \sqrt{\frac{N_H}{N_{HT}} \left(\frac{W_H - W_{HA}}{W_{HA}} \right)^2 + \frac{N_{LH}}{N_{HT}} \left(\frac{W_L - W_{HA}}{W_{HA}} \right)^2} \tag{10}$$

Empirical evidence on inequality and unemployment almost invariably relate to the wage and unemployment rates for different skill categories of workers. Thus we have good information on N_{HT} and N_{LL} (but not N_H and N_L) and on W_{HA}/W_L (but not W_H/W_L). Equations (2)-(3) and (5)-(10), however, enable us to derive the implications of changes in N_{HT} and N_{LL} for the unobserved composition of jobs (N_H, N_L) as well as for observable wage inequality $(W_{HA}/W_L, \sigma)$. These implications are summarized in Proposition 1.

Proposition 1 *Equations (2)-(3), and (5)-(10) imply the following expressions for* $d\log N_H$*,* $d\log N_L$*,* $d\log \frac{W_{HA}}{W_L}$ *and* $d\log\sigma$ *:*

$$d\log N_H = \frac{N_{HT}}{N_{HT}-\lambda H} d\log N_{HT} \tag{11}$$

$$d\log N_L = \frac{1}{N_{LL} + \frac{\lambda}{1-\lambda}(H - N_{HT})} \left[N_{LL} d\log N_{LL} - \frac{\lambda}{1-\lambda} N_{HT} d\log N_{HT} \right] \tag{12}$$

$$d\log\frac{W_{HA}}{W_L} = \frac{\frac{W_H}{W_L}N_H}{\frac{W_H-W_L}{W_L}N_H + N_{HT}}\left[\eta d\log N_L + \left(-\eta + \frac{W_H - W_L}{W_H}\frac{\lambda H}{N_{HT}}\right)d\log N_H\right] \quad (13)$$

$$d\log\sigma = \frac{W_L}{W_{HA}-W_L}d\log\frac{W_{HA}}{W_L} - \frac{1}{2}\frac{H}{H-N_H}d\log N_H \quad (14)$$

where

$$\eta = -\frac{d\log\frac{W_H}{W_L}}{d\log\frac{N_H}{N_L}} = -\frac{d\log h\left(\frac{N_H}{N_L}\right)}{d\log\frac{N_H}{N_L}} > 0 \quad (15)$$

is the inverse of the elasticity of substitution.

The key result is that with the introduction of a skill asymmetry and the assumption that some high-skill workers have low-skill jobs, the ratios N_H/N_L and N_{HT}/N_{LL} may move in opposite directions. A proportional increase in N_{HT} and N_{LL}, for instance, will cause N_H to go up (equation (11)) while N_L will decline if $\frac{\lambda}{1-\lambda}N_{HT}$ exceeds N_{LL} (equation (12)). This decoupling of movements in the job ratio N_H/N_L from movements in the employment ratio N_{HT}/N_{LL} implies that a skill bias may no longer be needed to explain the simultaneous increase in relative employment and relative wages. The somewhat complicated expression for $d\log W_{HA}/W_L$ in Proposition 1 reflects this basic result.

4.2 Numerical examples

Consider a case which combines a 3 percent decline in $\frac{N_{HT}}{H}$ with a 20 percent decline in $\frac{N_{LL}}{L}$. Assume, furthermore, that the initial values of the employment rates are $\frac{N_{HT}}{H} = 0.94$ and $\frac{N_{LL}}{L} = 0.85$, that the supplies of high and low skill are roughly equal, and that the initial value of W_{HA}/W_L is 2. If we look at the figures for men and take low skill to mean "lower secondary education or less" and high skill as "college level or higher", these numerical values fit the US evidence for the period 1970-1990 (OECD 1994, Tables 1.6, 1.10 and 1.16, Chart 5.2). Using equations (11)-(12), they imply that

$$d\log N_H = -\frac{0.94}{0.94-\lambda}0.03 \quad (16)$$

$$d\log N_L = d\log\left(N_{LL}+N_{LH}\right) = \frac{0.85}{0.85+\frac{\lambda}{1-\lambda}0.06}\left(-0.20 + \frac{\lambda}{1-\lambda}\frac{0.94}{0.85}0.03\right) \quad (17)$$

As indicated by Table 1, λ-values above 0.7 imply that N_H/N_L will decline despite the observed increase in $\frac{N_{HT}}{H}/\frac{N_{LL}}{L}$. It is the ratio N_H/N_L which - via equation (4) -

determines the change in W_H/W_L. Since some high-skill workers have moved into low-skill jobs, an increase in W_H/W_L does not necessarily imply that the average wage of high-skill workers will have gone up relative to the low-skill wage. Equations (13)-(14), however, in combination with the results in Table 1 allow us to calculate the predicted changes in W_{HA}/W_L and σ for different values of λ and η. As argued above, we expect λ to lie between 0.5 and one. The elasticity of substitution between low- and high-skill tasks is likely to be below unity, and we consider values of η - the inverse of the elasticity of substitution - from 1 to 4.[10]

Table 1: Changes in $d\log N_H$ and $d\log N_L$
using stylized US data and different values of λ

$d\log N \ \backslash \ \lambda$	0.5	0.6	0.7	0.8	0.9
$d\log N_H$	-0.064	-0.083	-0.118	-0.201	-0.705
$d\log N_L$	-0.156	-0.136	-0.105	-0.052	0.060

The actual US increase in W_{HA}/W_L was about 20 percent. Table 2 shows that for $\lambda = 0.8$ this observed increase can be fully accounted for by the observed changes in employment if η is just above 2.[11] The predicted proportional change in the wage dispersion for high-skill workers then becomes about 60 percent. This figure fits the observed increase in the "within education and experience" dispersion of wages quite well (Welch 1999). Thus, with these parameter values, there would be no need to invoke skill-biases or other explanations. Lower values of λ and η reduce the fraction of the observed change that is accounted for.

[10] All estimates of the elasticity of substitution between different skill categories in Card et al (1999) are very low (and in some cases negative). Murphy and Welch (1992) present estimates of the elasticities of complementarity (which are closely related to η_H and η_L) which suggest η-values in the 1-3 range between high school and college graduates. Since they are based on the assumption that $N_{HT} = N_H$ and $N_{LL} = N_L$, however, these estimates may be biased if there is significant overeducation, as modelled in this paper.

[11] In the absence of overeducation, a 17 percent increase in the employment ratio would lead to a 34 percent fall in the relative wage of high-skill workers if $\eta = 2$. Thus, although $\lambda-$values below 0.7 may fail to reverse the change in relative wages, small values of λ will still help towards resolving the paradox.

Table 2: $d\log\frac{W_{HA}}{W_L}$ and $d\log\sigma$ using stylized US data
and different values of λ and η

	$d\log\frac{W_{HA}}{W_L}$					$d\log\sigma$			
$\lambda\backslash\eta$	1	2	3	4	$\lambda\backslash\eta$	1	2	3	4
0.5	-0.105	-0.194	-0.283	-0.371	0.5	0.154	0.060	-0.035	-0.130
0.6	-0.075	-0.126	-0.176	-0.226	0.6	0.193	0.138	0.082	0.026
0.7	-0.029	-0.018	-0.006	0.005	0.7	0.260	0.273	0.286	0.299
0.8	0.054	0.181	0.308	0.435	0.8	0.408	0.579	0.750	0.920
0.9	0.255	0.712	1.169	1.626	0.9	1.188	2.261	3.335	4.409

As a second example, we use $N_{HT}/H = 0.96$, $N_{LL}/L = 0.91$ and $W_{HA}/W_L = 2$ as the initial values, assume $H = L$, and consider the effects of a combination of a 3 percent decline in high-skill employment N_{HT}/H and a 12 percent decline in low-skill employment N_{LL}/L. Using the same skill delineations, these figures fit the German experience between 1978 and 1987 (OECD 1994, Tables 1.6, 1.10, 1.16).

Table 3: $d\log\frac{W_{HA}}{W_L}$ and $d\log\sigma$ using stylized German data
and different values of λ and η

	$d\log\frac{W_{HA}}{W_L}$					$d\log\sigma$			
$\lambda\backslash\eta$	1	2	3	4	$\lambda\backslash\eta$	1	2	3	4
0.5	-0.038	-0.059	-0.080	-0.102	0.5	0.352	0.330	0.307	0.285
0.6	-0.013	-0.001	0.011	0.022	0.6	0.387	0.399	0.411	0.424
0.7	0.027	0.092	0.158	0.223	0.7	0.445	0.518	0.590	0.663
0.8	0.101	0.269	0.438	0.607	0.8	0.571	0.774	0.976	1.178
0.9	0.287	0.747	1.207	1.667	0.9	1.059	1.795	2.531	3.268

In contrast to the US case, German high-skill workers did not experience a rise in their relative wage but a modest 3 percent decline over the period 1978-87. Table 3 shows that, using $\eta = 2$, a perfect fit with the observed change is obtained for a λ-value between 0.5 and 0.6. The proportional change in the dispersion of wages for high-skill workers in this case is about 35 percent. A lower value of λ for Germany than the US may be explained by a more rigid labour market and a smaller proportion of overeducated workers. Thus, Daly et al. (2000, p. 172) report that "German men are about half as likely to be overeducated and about 60% less likely to be undereducated than working men in the United States. The same pattern holds for German women".

In the case of both the US and Germany, according to these calculations, one may be able to account for the observed changes in relative wages and employment without recourse to biased technical progress, using what appear to be plausible parameter values. But clearly the numbers are crude and there are many limitations.[12] Thus, one should not read too much into the surprisingly good fit; certainly, it is not our claim in this paper that the mechanism of induced overeducation gives an adequate explanation of the observed movements in wage inequality and that other influences on wage inequality, including institutional changes and skill biases, can be ignored. The numerical examples, however, do indicate that the effects of skill asymmetries and induced changes in overeducation can be quantitatively important.

5 Endogenizing changes in employment

5.1 Wage curves

In section 4 we took the changes in the employment of high- and low-skill workers, N_{HT} and N_{LL} as exogenous. We now specify the 'supply side' of the labour market in order to endogenize these changes.

A standard specification of the wage curves suggests that

$$\frac{W_H}{P} = Bg^H(N_H); g^{H\prime} > 0 \tag{18}$$

$$\frac{W_L}{P} = Bg^L(N_L); g^{L\prime} > 0 \tag{19}$$

The skill asymmetry and the presence of overeduction, however, imply that these wage curves need to be modified to allow for the fact that some high-skill workers have low-skill jobs. Hence, in place of (18)-(19) we shall use

$$\frac{W_H}{P} = g^H(\frac{N_H}{H}, \frac{N_{LH}}{H}); g_1^H > g_2^H \geq 0 \tag{20}$$

$$\frac{W_L}{P} = g^L(\frac{N_{LL}}{L}); g^{L\prime} > 0 \tag{21}$$

[12] Empirically, for example, there have been trend increases in both in the demand and supply of skills. Common trends in the demand and supply of skills are irrelevant to the argument concerning a recent increase in the skill bias. To simplify the exposition we have therefore assumed a constant skill composition of the work force (thus removing the trend increase in the supply of skills) and Hicks-neutral technical change (thus removing the trend increase in the demand for skills).

A simple efficiency-wage argument for equations (20)-(21) runs as follows. Assume that there are two possible effort levels, 0 and 1, and that a worker will shirk if the wage is below the 'fair wage' for the job. Firms have an incentive to pay exactly the fair wage and, as in Akerlof and Yellen (1990), it is assumed that the fair wage depends on market conditions. Since only high-skill workers can fill high-skill jobs, the market conditions for these jobs are summarised in (20) by high-skill workers' employment rates in high- and low-skill jobs. Low-skill jobs can be filled by both high- and low-skill workers but unemployed high-skill workers, by assumption, do not want a low-skill job (cf. p. 7 above). Hence, it seems reasonable to suppose that market conditions for low-skill jobs can be described by the employment rate of low-skill workers, as in (21).

Equation (20) can be simplified since, using (8), both N_H and N_{LH} can be expressed as functions of the (observable) total employment rate for high-skill workers. We have $N_H/H = (N_{HT}/H - \lambda)/(1-\lambda)$ and $N_{LH}/H = (\lambda/(1-\lambda))(1-N_{HT}/H)$. Hence, equation (20) implies that[13]

$$\frac{W_H}{P} = \bar{g}(\frac{N_{HT}}{H}); \bar{g}' > 0 \tag{22}$$

Equations (2)-(3), (8) and (21)-(22) imply that neutral shifts in the aggregate demand for labour may cause the ratios W_{HA}/W_L and N_{HT}/N_{LL} to move in the same direction. A decline in A, for instance, may lead to a rise in both the relative wage and the relative employment rate of high-skill workers. This possibility is readily demonstrated in the special case in which the wage equations (22) and (21) take the following form[14]

$$\frac{W_H}{P} = BN_{HT}^{\mu} \tag{23}$$

$$\frac{W_L}{P} = BN_{LL}^{\mu} \tag{24}$$

Straightforward but cumbersome calculations (see Appendix B) show that:

Proposition 2 *Equations (2)-(3), (8) and (23)-(24) imply that*

$$\frac{d\log\frac{N_H}{N_L}}{d\log\frac{Am}{B}} = \frac{\lambda H\left(N_{HT}+N_{LL}\right)}{\left(\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L + \left(\eta - \eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL} + \eta_H\lambda H\right)N_{HT}} > 0 \tag{25}$$

$$\frac{d\log\frac{W_H}{W_L}}{d\log\frac{Am}{B}} = -\eta\frac{d\log\frac{N_H}{N_L}}{d\log\frac{Am}{B}} = \frac{-\eta\lambda H\left(N_{HT}+N_{LL}\right)}{\left(\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L + \left(\eta - \eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL} + \eta_H\lambda H\right)N_{HT}} < 0 \tag{26}$$

[13] The sign of the derivative of $\bar{g}$ follows from the restrictions $g_1^H > g_2^H \geq 0$.

[14] This specification is used by Card et al. (1999). Since they do not consider overeducation, however, $N_{HT} = N_H$ and $N_{LL} = N_L$ in their setup.

where

$$\eta = -\frac{d\log\frac{W_H}{W_L}}{d\log\frac{N_H}{N_L}} = -\frac{d\log h\left(\frac{N_H}{N_L}\right)}{d\log\frac{N_H}{N_L}} = \eta_H + \eta_L > 0 \tag{27}$$

$$\eta_H = -\frac{d\log f^H}{d\log\frac{N_H}{N_L}} > 0, \eta_L = \frac{d\log f^L}{d\log\frac{N_H}{N_L}} > 0 \tag{28}$$

Furthermore,

$$\frac{d\log\frac{N_{HT}}{N_{LL}}}{d\log\frac{Am}{B}} = -\frac{\eta}{\mu}\frac{d\log\frac{N_H}{N_L}}{d\log\frac{Am}{B}} = \frac{-\eta\lambda H\left(N_{HT}+N_{LL}\right)}{\mu\left(\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L + \left(\eta - \eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL} + \eta_H\lambda H\right)N_{HT}} < 0 \tag{29}$$

and

$$\frac{d\log\frac{W_{HA}}{W_L}}{d\log\frac{Am}{B}} = K\left(\frac{W_H - W_L}{W_H}\left[\frac{\eta}{\mu}N_{LL} + N_L\right] - \eta\left(N_{HT}+N_{LL}\right)\right) \tag{30}$$

where

$$K = \frac{\lambda\frac{W_H}{W_L}N_H H}{\left[N_H\frac{W_H - W_L}{W_L} + N_{HT}\right]\left[\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L + \left(\eta - \eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL} + \eta_H\lambda H\right]N_{HT}} > 0 \tag{31}$$

The first point to note in Proposition 2 is the reversal of the sign of $d\log\frac{N_{HT}}{N_{LL}}/d\log\frac{Am}{B}$ compared to $d\log\frac{N_H}{N_L}/d\log\frac{Am}{B}$. An increase in Am/B raises the ratio of high-skill to low-skill *jobs* but reduces the ratio of employment of high- to low-skill *workers* (compare (25) and (29)). Putting it differently, the employment ratio N_{HT}/N_{LL} and the wage ratio W_H/W_L move together (compare (26) and (29)). Secondly, the effects of changes in Am/B on the more interesting wage ratio $\frac{W_{HA}}{W_L}$ are ambiguous (equation (30)). If η is small - that is, if there is a high elasticity of substitution between high- and low-skill inputs - then the expression will be positive and wage ratio w_{HA}/w_L and the employment ratio N_{HT}/N_{LL} will move in opposite directions. High values of η and μ, on the other hand, will ensure that $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$ becomes negative, and in this case a decline in Am/B will produce an outcome which fits the observed pattern, at least in a qualitative sense: the position of low-skill workers will deteriorate with respect to both relative wages and relative employment.

5.2 Numerical examples

The mere possibility that both W_{HA}/W_L and N_{HT}/N_{LL} may increase as the result of a fall in Am/B does not establish the empirical relevance of the argument. The parameter conditions may be restrictive or the magnitude of the effects could be negligible for empirically relevant parameter values.

The new parameter in this section is the elasticity, μ, of the real wage with respect to employment. Empirical estimates of this elasticity vary widely but a plausible range for μ runs from 0.5 to 4.[15] Using this range for μ in combination with λ-values from 0.5 to 0.9 and η-values from 1 to 4 we get the results in Tables 4-5. Tables 4a-4b give the values of $d\log\frac{N_{HT}}{N_{LL}}/d\log\frac{Am}{B}$ and $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$ for different combinations of λ and μ. In these tables the inverse of the elasticity of substitution is assigned the value $\eta = 2$. Tables 5a-5b fix μ at a benchmark level of 2 and allow λ and η to vary. Both Tables 4 and 5 use US-calibrated values for the initial values of $N_{HT}/H = 0.94, N_{LL}/L = 0.85$ and $W_H/W_L = 2$ and it is assumed that $\eta_H = \eta/2$.

Table 4: The values of $d\log\frac{N_{HT}}{N_{LL}}/d\log\frac{Am}{B}$ and $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$ for $\eta = 2, \eta_H = 1, W_H/W_L = 2$ and different values of λ and μ.

	4a: $d\log\frac{N_{HT}}{N_{LL}}/d\log\frac{Am}{B}$					4b: $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$			
$\lambda\backslash\mu$	0.5	1	2	4	$\lambda\backslash\mu$	0.5	1	2	4
0.5	-1.94	-0.88	-0.37	-0.14	0.5	-0.37	-0.54	-0.53	-0.43
0.6	-2.37	-1.09	-0.47	-0.18	0.6	-0.44	-0.65	-0.67	-0.56
0.7	-2.82	-1.32	-0.59	-0.24	0.7	-0.50	-0.76	-0.80	-0.71
0.8	-3.28	-1.57	-0.72	-0.31	0.8	-0.52	-0.82	-0.90	-0.85
0.9	-3.78	-1.86	-0.90	-0.42	0.9	-0.37	-0.63	-0.74	-0.76

[15] OECD (1994, Table 5.2) gives the semi-elasticity of real wages with respect to unemployment rates for the aggregate labour market in a number of countries. The estimates range from -1 to -10, with estimates for the US and UK close to -1 and those for most continental European countries and Japan between -3 and -5. The semi-elasticity is related to the elasticity of real wages with respect to employment: if $u = \frac{L-N}{L}$ is the unemployment rate then

$$du = -(1-u)(d\log N - d\log L)$$

The relatively low OECD estimate for the semi-elasticity is contradicted by other studies. Card et al (1999) estimate supply elasticities in the 0.2-0.4 range in a disaggregated setup, implying μ-values between 2.5 and 5. Blanchard and Katz (1997) suggest that in the macro data there may be no long-term relation between the unemployment rate and the level of real wage - corresponding to a vertical wage curve or, algebraically, $\mu \to \infty$ in (26)-(27) - while disaggregated data for the US states imply that the absolute value of the long run semi-elasticity of the real wage with respect to the unemployment rate is well above 5.

Table 5: The value of $d\log\frac{N_{HT}}{N_{LL}}/d\log\frac{Am}{B}$ and $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$ for $\mu=2, \eta_H=\eta/2, W_H/W_L=2$ and different values of λ and η.

5a: $d\log\frac{N_{HT}}{N_{LL}}/d\log\frac{Am}{B}$

$\lambda\backslash\eta$	1	2	3	4
0.5	-0.28	-0.37	-0.41	-0.44
0.6	-0.38	-0.47	-0.52	-0.54
0.7	-0.48	-0.59	-0.63	-0.66
0.8	-0.63	-0.72	-0.76	-0.79
0.9	-0.85	-0.90	-0.92	-0.93

5b: $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$

$\lambda\backslash\eta$	1	2	3	4
0.5	-0.33	-0.53	-0.63	-0.69
0.6	-0.43	-0.67	-0.78	-0.84
0.7	-0.53	-0.80	-0.92	-0.99
0.8	-0.62	-0.90	-1.02	-1.08
0.9	-0.50	-0.74	-0.82	-0.87

The tables show that

- N_{HT}/N_{LL} and W_{HA}/W_L move in the same direction for all parameter combinations in the two tables.
- using the values $\lambda = 0.8$ and $\eta = 2$, which in section 4 gave a perfect fit for the US when changes in employment were taken as exogenous, the rough equality in the empirical data between $d\log N_{HT}/N_{LL}$ and $d\log W_{HT}/W_L$ is obtained for a μ-value just under 2.
- when $\lambda = 0.8$, $\eta = 2$ and $\mu = 2$, a 20-25 percent fall in Am/B could account for the observed changes in N_{HT}/N_{LL} and W_{HA}/W_L in the US.

A drop of 20-25 percent in Am/B may seem large. Annual labour productivity growth in the US, however, was down 1.2 percentage points in 1973-96 compared with 1950-73 while real wage growth for men dropped by about 3 percentage points (Maddison (1997, Table 11), Gottschalk (1997, p. 25)). The cumulative effects of these changes in the wage trend are large. Thus, the real wage would have been almost twice as high in 1996 if the trend had been unchanged. If wage aspirations (as measured by the time path for B) adjust slowly, the decline in real-wage growth could therefore cause a significant drop in Am/B .[16]

[16]The existence of a link between the fall in productivity growth and increased unemployment has been suggested by a number of studies (e.g. Stiglitz (1997), Blanchard and Wolfers (2000)). Workers, having become accustomed to high annual rates of wage increase, demanded a continuation of this trend. With a decline in productivity growth, this continuation became impossible and unemployment, which dampens wage demands, or inflation was the result. Skott (2003b) shows that a decline in productivity growth may lead to a permanent rise in the unemployment even if wage aspirations adjust endogenously.

The German case, in which low-skill workers experienced a deterioration of their relative employment position but a slight improvement in the relative wage, is harder to explain. With exogenous employment changes, a good fit was obtained in section 4 for $\eta = 2$ and a λ-value between 0.5 and 0.6. Given the specification in (23)-(24) of the supply side of the labour market, however, the changes in employment and relative wages cannot both be accounted for by a decline in Am/B.[17] Using $\eta = 2, \mu = 2$ and $\lambda = 0.5$ (reflecting the less flexible labour market), a fall of 20-25 percent in Am/B would generate a rise of just under 10 percent in N_{HT}/N_{LL}, which is in line with the evidence, but an increase of over 10 percent in W_{HA}/W_L.

6 Conclusions

It is commonly believed that skill-biases in technical change and/or changes in the relative supplies are needed in order to produce a pattern in which high-skill workers do better than low-skill workers in terms of both employment and wages. The argument in this paper challenges this view.

The US pattern is particularly striking and, not surprisingly, the US experience has been the main focus of debate. In this paper we have argued that using plausible parameter values, the broad US pattern may even be consistent with the absence of skill-biases and an unchanged pattern of labour supply (or, more realistically, with unchanged trend increases in both skill requirements and the supplies of skill). The mechanism behind our argument is exceedingly simple: some high-skill workers who fail to get high-skill jobs will move into low-skill jobs. Once this possibility is recognized, the observed changes in the employment rates of high and low-skill workers can generate increasing wage inequality, both within and between skill categories. The mechanism, moreover, is consistent with the observation that the change in relative employment has occurred within industries and does not reflect a structural shift away from low-skill industries.

The changes in employment in turn may be related, at least in part, to the productivity slowdown that occurred around 1970. Thus, our argument links changes in unemployment and wage inequality to productivity movements in a highly parsimonious way: a single (unexplained) change - the productivity slowdown - is used to account for the observed increase in both unemployment and inequality between 1970 and the early 1990s. Furthermore, the model correctly predicts that recent increases in productivity

[17] Tables 4-5 use US observations for the initial values of N_{HT}/H, N_{LL}/L and W_H/W_L. A recalculation based on German initial values has only minor effects and does not change the qualititative picture.

growth will be associated with reductions in unemployment and inequality.

One may question the magnitude and durability of the effects of changes in productivity growth on the ratio Am/B. It should be noted, however, that reservations in this respect concern the specification of the supply side of the labour market (basically the adjustment of the parameter B in the wage curves to changes in the productivity parameter A) and that, as such, these reservations have no bearing on the argument in section 4. Given the extent to which high-skill workers accept low-skill jobs (the value of λ) and the elasticity of substitution (the inverse of η) it is possible to trace the implied effects of exogenous employment changes on the relative wage and, as shown in section 4, the changes in relative wages are consistent with the observed employment changes for plausible values of the critical parameters λ and η.

The effects highlighted in this paper do not exclude other influences on inequality. International competition, skill-biases in technical progress and changes in labour market institutions may all have contributed to the observed trends in wage inequality and in the level and composition of unemployment. In fact, our results implicitly support the view that institutional differences on the supply side of the labour market play an important role. Thus, in section 5 the German evidence could not be explained using a labour supply specification that seemed to work reasonably well for the US. If differences in wage setting institutions are important from a cross-section perspective then changes in these institutions should play a role in a time-series perspective. Many countries have experienced significant changes in their labour market institutions since 1970 and it would be surprising if these developments had left no mark on unemployment or inequality.

The model leaves out many complicating factors, quite aside from the possible, complementary influences of skill-biases, international trade and wage-setting institutions. Changes in the (trend increase of) supply of skill and in the quality of formal education, for instance, may well have contributed to the observed cross-sectional and time-series patterns for relative wages and unemployment. The assumption of only two skill categories clearly involves another drastic simplification as does the assumption of homogeneity of all workers within a skill category. This latter assumption precludes the possibility that measured overeducation reflects unobserved quality differences among workers. Yet another set of limitations is our neglect of undereducation and the use of a simple "job-competition model" in which low-skill workers cannot perform high-skill jobs while high- and low-skill workers are perfect substitutes in low-skill jobs. The stylized facts underlying the numerical examples are crude, finally, and the numerical examples are merely indicative.

These limitations notwithstanding, the analysis highlights a mechanism which has received little attention in the recent literature on changes in wage inequality. Using a formalization which deliberately left out skill-biases, international competition and institutional changes, we have shown that the effects of skill asymmetries, induced overeducation and credentialism may be very substantial and that they deserve further examination.

7 Appendices

7.1 Appendix A: Proof of proposition 1

Equation (8) and the definitions in (5)-(7) imply that

$$N_H = \frac{N_{HT} - \lambda H}{1-\lambda} \tag{A1}$$

$$N_L = N_{LL} + \frac{\lambda}{1-\lambda}(H - N_{HT}) \tag{A2}$$

Equations (11)-(12) can be derived by logarithmic differentiation of (A1)-(A2), noting that by assumption the labour supply H is constant.

Turning to (13), the definition of the relative wage is given by

$$\frac{W_{HA}}{W_L} = \frac{W_H N_H + W_L N_{LH}}{N_{HT} W_L} = \frac{W_H N_H + W_L \lambda (H - N_H)}{(N_H + \lambda (H - N_H)) W_L} = \frac{\omega N_L + \lambda (H - N_H)}{N_H + \lambda (H - N_H)} \tag{A3}$$

where

$$\omega = \frac{W_H N_H}{W_L N_L} \tag{A4}$$

The ratio ω represents the relative income shares of high and low-skill inputs and we have

$$\begin{aligned} d\log\omega &= \frac{d\log\omega}{d\log\frac{N_H}{N_L}} d\log\frac{N_H}{N_L} \\ &= (-\eta + 1) d\log\frac{N_H}{N_L} \end{aligned} \tag{A5}$$

Taking logs and differentiating (A3) we get

$$\begin{aligned}
d\log\frac{W_{HA}}{W_L} &= d\log\left(\omega N_L+\lambda\left(H-N_H\right)\right)-d\log\left(N_H+\lambda\left(H-N_H\right)\right)\\
&= \frac{1}{D}\left[\omega N_L\left(d\log\omega+d\log N_L\right)-\lambda N_H d\log N_H\right]\\
&\quad -\frac{1}{N_{HT}}N_H\left(1-\lambda\right)d\log N_H\\
&= \frac{\omega N_L}{D}\left[\begin{array}{c}\left(-\eta+1\right)d\log\frac{N_H}{N_L}+d\log N_L-\frac{\lambda N_H}{\omega N_L}d\log N_H\\ -\frac{D}{N_{HT}}\frac{N_H}{\omega N_L}\left(1-\lambda\right)d\log N_H\end{array}\right]\\
&= \frac{\omega N_L}{D}\left[\eta d\log N_L+\left(1-\eta-\lambda\frac{W_L}{W_H}-\frac{D}{N_{HT}}\frac{W_L}{W_H}\left(1-\lambda\right)\right)d\log N_H\right]\\
&= \frac{\omega N_L}{D}\left[\begin{array}{c}\eta d\log N_L+\\ \left(-\eta+\frac{\left(1-\lambda\frac{W_L}{W_H}\right)N_{HT}-(1-\lambda)\frac{W_L}{W_H}D}{N_{HT}}\right)d\log N_H\end{array}\right]\\
&= \frac{\omega N_L}{D}\left[\eta d\log N_L+\left(-\eta+\frac{W_H-W_L}{W_H}\frac{\lambda H}{N_{HT}}\right)d\log N_H\right]\\
&= \frac{\frac{W_H}{W_L}N_H}{\frac{W_H-W_L}{W_L}N_H+N_{HT}}\left[\begin{array}{c}\eta d\log N_L+\\ \left(-\eta+\frac{W_H-W_L}{W_H}\frac{\lambda H}{N_{HT}}\right)d\log N_H\end{array}\right] \qquad \text{(A6)}
\end{aligned}$$

where

$$D=\omega N_L+\lambda\left(H-N_H\right)=\frac{W_H-W_L}{W_L}N_H+N_{HT}>0 \qquad \text{(A7)}$$

In order to derive (14) we note first that the expression for σ can be rewritten using the definitions of W_{HA}, N_{LH} and N_{HT}:

$$\begin{aligned}
\sigma &= \frac{W_{HA}-W_L}{W_{HA}}\sqrt{\frac{N_H}{N_{HT}}\left(\frac{W_H-W_{HA}}{W_{HA}-W_L}\right)^2+\frac{N_{LH}}{N_{HT}}}\\
&= \left(1-\frac{W_L}{W_{HA}}\right)\sqrt{\frac{N_H}{N_{HT}}\left(\frac{W_H-\frac{N_H}{N_{HT}}W_H-\frac{N_{LH}}{N_{HT}}W_L}{\frac{N_H}{N_{HT}}W_H+\frac{N_{LH}}{N_{HT}}W_L-W_L}\right)^2+\frac{N_{LH}}{N_{HT}}}\\
&= \left(1-\frac{W_L}{W_{HA}}\right)\sqrt{\frac{N_H}{N_{HT}}\left(\frac{N_{LH}}{N_H}\right)^2+\frac{N_{LH}}{N_{HT}}}\\
&= \left(1-\frac{W_L}{W_{HA}}\right)\sqrt{\frac{N_{LH}}{N_H}}\\
&= \left(1-\frac{W_L}{W_{HA}}\right)\sqrt{\frac{\lambda\left(H-N_H\right)}{N_H}} \qquad \text{(A8)}
\end{aligned}$$

Hence,

$$\begin{aligned}
d\log\sigma &= d\log\left(1-\frac{W_L}{W_{HA}}\right)+\frac{1}{2}\left[d\log\left(\lambda\left(H-N_H\right)\right)-d\log N_H\right] \\
&= \frac{W_{HA}}{W_{HA}-W_L}\left(-\frac{W_L}{W_{HA}}\right)d\log\frac{W_L}{W_{HA}}+\frac{1}{2}\left[\frac{-\lambda N_H}{\lambda\left(H-N_H\right)}d\log N_H-d\log N_H\right] \\
&= \frac{W_L}{W_{HA}-W_L}d\log\frac{W_{HA}}{W_L}-\frac{1}{2}\frac{H}{H-N_H}d\log N_H \qquad \text{(A9)}
\end{aligned}$$

7.2 Appendix B: Proof of proposition 2

7.2.1 Derivation of the expression for $d\log\frac{N_H}{N_L}/d\log\frac{Am}{B}$

Combining (2)-(3) and (23)-(24) we get

$$N_{HT}^{\mu}=\frac{Am}{B}f^H\left(\frac{N_H}{N_L}\right) \qquad \text{(B1)}$$

$$N_{LL}^{\mu}=\frac{Am}{B}f^L\left(\frac{N_H}{N_L}\right) \qquad \text{(B2)}$$

Taking logs and differentiating, we get

$$\mu d\log N_{HT}=d\log\frac{Am}{B}+\frac{d\log f^H}{d\log\frac{N_H}{N_L}}\left(d\log N_H-d\log N_L\right) \qquad \text{(B3)}$$

$$\mu d\log N_{LL}=d\log\frac{Am}{B}+\frac{d\log f^L}{d\log\frac{N_H}{N_L}}\left(d\log N_H-d\log N_L\right) \qquad \text{(B4)}$$

Using equations (7 and (8) and the definitions of η, η_L and η_H, equations (B3)–(B4) can be rewritten

$$\left(\mu\frac{\left(1-\lambda\right)N_H}{N_{HT}}+\eta_H\right)d\log N_H=d\log\frac{Am}{B}+\eta_H d\log N_L \qquad \text{(B5)}$$

$$\frac{\mu}{N_{LL}}\left(N_L d\log N_L+\lambda N_H d\log N_H\right)=d\log\frac{Am}{B}+\eta_L\left(d\log N_H-d\log N_L\right) \qquad \text{(B6)}$$

Solving (B5) for $d\log N_H$ and substituting into (B6), we get

$$\begin{aligned}
&\left(\frac{\mu N_L+\frac{\eta_H}{\mu\frac{(1-\lambda)N_H}{N_{HT}}+\eta_H}\mu\lambda N_H}{N_{LL}}+\frac{\mu\frac{(1-\lambda)N_H}{N_{HT}}}{\mu\frac{(1-\lambda)N_H}{N_{HT}}+\eta_H}\eta_L\right)d\log N_L \\
&= d\log\frac{Am}{B}\left(1+\frac{\eta_L}{\mu\frac{(1-\lambda)N_H}{N_{HT}}+\eta_H}-\frac{\frac{\mu}{\mu\frac{(1-\lambda)N_H}{N_{HT}}+\eta_H}\lambda N_H}{N_{LL}}\right) \qquad \text{(B7)}
\end{aligned}$$

or

$$d\log N_L = \frac{\eta N_{LL} + \mu N_L - \mu\lambda H \frac{N_{HT}+N_{LL}}{N_{HT}}}{\mu\left(\left(\eta - \eta_L \frac{\lambda H}{N_{HT}}\right) N_{LL} + \mu \frac{(1-\lambda)N_H}{N_{HT}} N_L + \lambda\eta_H H\right)} d\log \frac{Am}{B} \tag{B8}$$

Equation (D5) can be rewritten

$$\left(\mu \frac{(1-\lambda)\, N_H}{N_{HT}} + \eta_H\right)(d\log N_H - d\log N_L) = d\log \frac{Am}{B} - \mu \frac{(1-\lambda)\, N_H}{N_{HT}} d\log N_L \tag{B9}$$

or

$$d\log \frac{N_H}{N_L} = \frac{1}{\mu \frac{(1-\lambda)N_H}{N_{HT}} + \eta_H} d\log \frac{Am}{B} - \frac{\mu \frac{(1-\lambda)N_H}{N_{HT}}}{\mu \frac{(1-\lambda)N_H}{N_{HT}} + \eta_H} d\log N_L \tag{B10}$$

Substituting the expression for $d\log N_L$ - equation (B8) - into (B10) we now get

$$d\log \frac{N_H}{N_L} = \frac{\lambda H\,(N_{HT} + N_{LL})\, d\log \frac{Am}{B}}{\left(\left(\eta - \eta_L \frac{\lambda H}{N_{HT}}\right) N_{LL} + \mu \frac{(1-\lambda)N_H}{N_{HT}} N_L + \lambda\eta_H H\right) N_{HT}} \tag{B11}$$

Equation (B11) can be rewritten as (25) in the proposition.

7.2.2 Derivation of the expression for $d\log \frac{W_H}{W_L}/d\log \frac{Am}{B}$ and $d\log \frac{N_{HT}}{N_{LL}}/d\log \frac{Am}{B}$

To find $d\log \frac{W_H}{W_L}/d\log \frac{Am}{B}$ and $d\log \frac{N_{HT}}{N_{LL}}/d\log \frac{Am}{B}$, use (4) and (23)-(24), respectively, in combination with (B11) to get

$$\begin{aligned} d\log \frac{W_H}{W_L} &= d\log h\left(\frac{N_H}{N_L}\right) = -\eta d\log \frac{N_H}{N_L} \\ &= \frac{-\eta\lambda H\,(N_{HT} + N_{LL})\, d\log \frac{Am}{B}}{\left(\left(\eta - \eta_L \frac{\lambda H}{N_{HT}}\right) N_{LL} + \mu \frac{(1-\lambda)N_H}{N_{HT}} N_L + \lambda\eta_H H\right) N_{HT}} \end{aligned} \tag{B12}$$

and

$$\begin{aligned} d\log \frac{N_{HT}}{N_{LL}} &= \frac{1}{\mu} d\log \frac{W_H}{W_L} \\ &= \frac{-\eta\lambda H\,(N_{HT} + N_{LL})\, d\log \frac{Am}{B}}{\mu\left(\left(\eta - \eta_L \frac{\lambda H}{N_{HT}}\right) N_{LL} + \mu \frac{(1-\lambda)N_H}{N_{HT}} N_L + \lambda\eta_H H\right) N_{HT}} \end{aligned} \tag{B13}$$

7.2.3 Derivation of $d\log\frac{W_{HA}}{W_L}/d\log\frac{Am}{B}$

Equation (13) in Proposition 1 gives an expression for $d\log\frac{W_{HA}}{W_L}$ in terms of $d\log N_H$ and $d\log N_L$. Using this equation in combination with (B8) and (B11) we get

$$\begin{aligned}
\frac{d\log\frac{W_{HA}}{W_L}}{d\log\frac{Am}{B}} &= \frac{\frac{W_H}{W_L}N_H}{\frac{W_H-W_L}{W_L}N_H+N_{HT}}\left[\begin{array}{c}\left(-\eta+\frac{W_H-W_L}{W_H}\frac{\lambda H}{N_{HT}}\right)\frac{d\log\frac{N_H}{N_L}}{d\log\frac{Am}{B}}\\ +\frac{W_H-W_L}{W_H}\frac{\lambda H}{N_{HT}}\frac{d\log N_L}{d\log\frac{Am}{B}}\end{array}\right]\\
&= M\left[\begin{array}{c}\left(-\eta+\frac{W_H-W_L}{W_H}\frac{\lambda H}{N_{HT}}\right)\frac{\lambda H(N_{HT}+N_{LL})}{\left(\left(\eta-\eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL}+\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L+\lambda\eta_H H\right)N_{HT}}\\ +\frac{W_H-W_L}{W_H}\frac{\lambda H}{N_{HT}}\frac{\eta N_{LL}+\mu N_L-\mu\lambda H\frac{N_{HT}+N_{LL}}{N_{HT}}}{\mu\left(\left(\eta-\eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL}+\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L+\lambda\eta_H H\right)}\end{array}\right]\\
&= M\frac{\lambda H\left[\frac{W_H-W_L}{W_H}\left(\frac{\eta}{\mu}N_{LL}+N_L\right)-\eta\left(N_{HT}+N_{LL}\right)\right]}{\left(\left(\eta-\eta_L\frac{\lambda H}{N_{HT}}\right)N_{LL}+\mu\frac{(1-\lambda)N_H}{N_{HT}}N_L+\lambda\eta_H H\right)N_{HT}} \qquad \text{(B14)}
\end{aligned}$$

where

$$M=\frac{\frac{W_H}{W_L}N_H}{\frac{W_H-W_L}{W_L}N_H+N_{HT}}$$

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