Information and Ambiguity: Herd and Contrarian Behaviour in Financial Markets*

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Abstract

The paper studies the impact of informational ambiguity on behalf of informed traders on history-dependent price behaviour in a model of sequential trading in financial markets. Following Chateauneuf, Eichberger and Grant (2006), we use neo-additive capacities to model ambiguity. Such ambiguity and attitudes to it can engender herd and contrarian behaviour, and also cause the market to break down. The latter, herd and contrarian behaviour, can be reduced by the existence of a bid-ask spread.

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1 Introduction

Herding in financial markets has been explored extensively in recent years.[1] The main focus has been on forms of rational herding: informational herding, reputation-based and compensation-based herding. In this paper we confine our attention to informational herding in financial markets. We demonstrate that ambiguity can cause the occurrence of herding, as well as contrarian behaviour.

Ambiguity, first defined by Knight (1921), refers to situations where subjective probabilities are not perfectly known or even unknown. Ambiguity arises from sources such as missing information or disagreement of expert opinions or the lack of confidence on the information quality. Cumulative evidence from laboratory experiments has suggested that behaviour under ambiguity is different from behaviour under risk, such as the Ellsberg Paradox (1961): see, for example, Camerer and Weber (1992). It has been argued that financial market is a likely candidate for considering the effects of ambiguity. In this paper, we consider ambiguity concerning an asset's trading value. As suggested in Hirshleifer (2001), such ambiguity could be either about the asset's fundamental volatility or lack of confidence in the quality of information regarding the value of the asset. In this paper, we model investors' preference when facing such ambiguity and suggest a non-neutral attitude to such ambiguity as a cause of herding and contrarian behaviour.

The market structure of our theoretical framework, focuses on the acquisition of new information by informed traders: through the prevailing market price of the asset and by a private signal about the values of the asset that is being traded.[2] We modify the existing paradigm by introducing the notion of uncertainty or ambiguity, but where uncertainty is captured by non-additive beliefs (Schmeidler (1989)). In effect, informed traders are assumed to be uncertain about the probabilities (as in Knight (1921)) governing the realisation of the value of the asset that is being traded.

[1] Hirshleifer and Teoh (2003) provide an overview of the recent theoretical and empirical research on this topic.

[2] Our model follows that developed by Bikhchandani, Hirshleifer and Welch (1992), based on the work of Glosten and Milgrom (1985), which itself is a derivative of that of Copeland and Galai (1983). This type of framework has featured in one form or another in much of the subsequent literature, including the influential paper by Avery and Zemsky (1998).

The model of decision-making under uncertainty we use is Choquet expected utility (henceforth CEU) which is due to Schmeidler (1989). In this model, individuals' beliefs are represented as capacities (non-additive beliefs).

Experiments on decision-making under risk (i.e. with known probabilities) have shown that individuals tend to overweight both high and low probability events. This can be explained by insensitivity of perception in the middle of the range on probabilities. For instance, difference from 0.55 to 0.60 is not perceived to be as great as that between 0 and 0.05. It has been argued that the decision weights assigned to events are an inverse S-shaped function over the "given" probabilities of occurrence of those events (see, Gonzalez and Wu (1999), Abdellaoui (2000), and Bleichrodt and Pinto (2000)). That function can be approximated by the simple piece wise linear function,

$$\phi(P) = \begin{cases} 1 & \text{for} \quad P = 1 \\ \delta\alpha + (1-\delta) \cdot P & \text{for} \quad 0 < P < 1 \\ 0 & \text{for} \quad P = 0 \end{cases} \tag{1}$$

where $\phi(P)$ denotes the weight the event is given in decisions and $(\alpha, \delta) \in [0, 1]$.

If probabilities are not known, then we have a problem of decision-making under ambiguity. In this case a similar phenomena has been found (see, Kilka and Weber (2001)). Individuals do not assign subjective probabilities to events. Instead they overweight both highly likely and highly unlikely events. (In this case the likelihood of events is subjective.) Chateauneuf, Eichberger and Grant (2008) axiomatised such decision-making under ambiguity. They show that preferences may be represented as a Choquet integral with respect to a neo-additive capacity. A neo-additive capacity is analogous to the piecewise linear function in equation (1) but applies to uncertainty rather than risk.

We shall make the assumption that is dominant in the literature on sequential trading models that a given agent is risk-neutral, so that the function which maps outcomes into utility is linear. Together with the assumption of CEU preferences with beliefs represented as neo-additive capacities, this implies that preferences may be represented as a weighted average of the expected value of utility, the maximum value of utility and the minimum value of utility. This is expressed as,

$$\alpha\delta Max\,(w) + \delta(1-\alpha)Min\,(w) + (1-\delta)\,\mathbf{E}_{\pi}w, \tag{2}$$

where $Max(w)$ (resp. $Min(w)$) is the maximum (resp. minimum) value of, say, trading an asset, in our model, and $\mathbf{E}_{\pi}w$ is the expected value of trading an asset given the set of relevant probabilities, π.[3] The neo in the capacity epitomises the fact it is a *n*on-*e*xtreme-*o*utcome additive capacity.

Knight (1921) maintained that agents differ in their attitudes to ambiguity. The majority of people are ambiguity-averse, behaving more cautiously when probabilities are undefined, while a significant minority of individuals appear to be the opposite, being ambiguity-loving (see the experimental evidence in Camerer and Weber (1992)). In CEU, agents are ambiguity-averse if they put more weight on "bad" outcomes than do EU maximisers, while they are ambiguity-loving if they put more weight on "good" outcomes. We define ambiguity-averse behaviour as *pessimism* (*optimism*) when they place more weight on the possibly low (high) value of an asset.

As our focus is the acquisition of new information and its impact on prior (public or market) beliefs about the value of the asset, it is necessary to formulate a process by which those beliefs are up-dated. We view ambiguity-attitude (α) as a characteristic of the individual, while beliefs and perceived ambiguity about beliefs are factors which can depend on the environment. Thus we believe an updating rule should preserve ambiguity-attitude (α), while revising beliefs and ambiguity as new information is received. Other desirable features of an updating rule are that it should coincide Bayesian updating for additive beliefs and that it should preserve dynamic consistency.

There are a number of proposals which extend Bayesian updating to ambiguous beliefs. These include the optimistic rule, the Dempster-Shafer rule (Gilboa and Schmeidler (1993)); the Generalised Bayesian Update (henceforth, GBU), Eichberger, Grant and Kelsey (2007); and recursive multiple priors (Epstein and Schneider (2003)). When applied to neo-additive capacities both the Optimistic and Dempster-Shafer updates changed ambiguity-attitude (α) as well as beliefs. In contrast, the GBU does not change ambiguity attitude and just revises beliefs. For this reason we use the GBU. Of the rules described, only recursive multiple priors satisfies full dynamic consistency. However as we shall argue in the conclusions, given our market framework, these potential dynamic inconsistencies do not create any problems for our

[3] The parameters α and δ may be measured experimentally, see Kilka and Weber (2001).

analysis.[4]

Using neo-additive capacities to capture the uncertainty of the informed trader about information, we are able to demonstrate that herd and contrarian behaviour can occur. That behaviour only deviates from SEU to allow for ambiguity; and so contrasts with the claims of others (such as, Shleifer and Summers (1990 and Kirman (1993)). Indeed, there is a range of market (public) expectations of the value of the asset being traded over which such behaviour can occur. Furthermore, herd and contrarian behaviour arise under the same set of information about the alternative values that the asset can take and the arrival of private signals of informed traders about the probabilities of those values. Therefore, we do not require different informational frameworks to see the possibility of either kind of behaviour, as do, for example, Avery and Zemsky (1998).

The remainder of the paper is organized as follows. In Section 2 we provide a simple example to illustrate some of the points made above and to provide a context for our paper. Section 3 outlines the Glosten-Milgrom market micro structure that we use and the informational properties that we consider in our analysis of herd and contrarian behaviour. In section 4, we note some basic principles of ambiguity theory, and present the neo-additive capacity and its updating. Section 5 provides (conventional) definitions of herding/contrarian behaviour. We demonstrate the possibility of contrarian and herd behaviour. Section 6 provides some illustrative observations on the path of prices in a phase of herd selling and how it can be ended by an informed trader with a high signal and a high degree of optimism or attitude to ambiguity. Section 7 contains some concluding observations, in the course of which we comment on the absence of money pumps and dynamic inconsistency in our framework.

2 A Simplified Example

We consider a market in which an asset is traded whose intrinsic value (w) is unknown, but is known to be either 1 or 0. (This is a maintained assumption throughout the paper.) Let there

[4] We do not consider recursive multiple priors in this paper since it was developed for a different framework; one with an infinite horizon rather than two periods and multiple priors rather than CEU preferences. However, we do not believe that these differences are crucial and it would be possible to prove similar results to ours using the recursive multiple priors model.

be informed traders in the market who receive private signals (x), insider information, about which of those two values is correct. They receive those signals, "high" and "low", with the probabilities given in Table 1. So, the probability of either signal occurring and conveying the intrinsic worth of the asset is p (where it is assumed that $p > 1/2$).

Table 1

Signal probabilities

	$P(x = h\|w)$	$P(x = l\|w)$
$w = 1$	p	$1 - p$
$w = 0$	$1 - p$	p

Suppose that from the past history of trades and prices on the market, the "market" expectation of the value of the asset is π, namely, the probability that the value is 1. Suppose also that in the absence of a signal the (potentially informed) trader will take that belief as the basis of his decision to purchase or sell the asset given the bid and ask prices of the market maker. Let the next trader on the market receive a high signal and suppose that he decides whether to buy or sell the asset, or not to trade, according to his *expected utility* of the asset; his actual utility being a linear function of the asset's value. So, his expected utility then is his estimate of the probability that the asset's value is 1. Assume for the sake of argument that the market maker sets his price at π (hence we ignore bid and ask prices at this stage). Let the informed trader apply this extra information to revise the market's probability that the asset's value is 1 by means of Bayes' up-dating rule. For him, this probability will become

$$\pi^{'} = \frac{p\pi}{p\pi + (1-p)(1-\pi)} \tag{3}$$

.

Then $\pi^{'}$ will exceed π, so that the asset will be purchased. The informed trader acts according to his signal, whatever has been happening to the recent history of the market price of the asset.

By contrast, now assume that the trader is uncertain about the up-dated probability defined in equation (3), perhaps because he does not have full confidence in, feels ambiguous about, the inside information that he has acquired. He will then also lack confidence in the updated probability π'. Suppose preferences can be represented by

$$\alpha\delta + (1-\delta)\,\mathbf{E}_{\pi'} w,$$

which is in the spirit of the *CEU* representation in equation (2). It is immediate that this revised expected value (utility) might be lower than π; to take the simplest example, as $\alpha \to 0$ and $\delta \to 1$, expected utility $\to 0$. In this situation the trader ignores his high signal and does not purchase the asset: in fact, he sells the asset to the market maker. The sufficient condition for such an outcome is: $\pi > 1 + \delta(\alpha - 1)$. The same occurrence can arise if the informed trader accepts p intrinsically, but has doubts about π. These doubts (or ambiguities) are represented by the *CEU*, equation (2). Then again, for example, as $\alpha \to 0$ and $\delta \to 1$, the greater is the chance that the *CEU* will be lower than π, for any value of the latter.

Such overriding of his private signals due to informational ambiguity by the informed trader is the generator of herd and contrarian behaviour. However, the above is only illustrative, since it lacks a rigorous formal framework. Importantly, it relies upon an ad hoc evaluation of what are, in terms of our observations in the Section 1, *unknown probabilities* and, therefore, of subjective features of the trader. The aim of the remainder of the paper is to rectify these deficiencies.

3 Market Micro Structure

Market Mechanism. As in Glosten and Milgrom (1985) a risky asset is exchanged for money among market makers and two types of traders, informed and uninformed. The true value of the asset is $w \in \{0, 1\}$. Trading occurs sequentially and one trader (who can be of either type) is randomly selected in each period. There is an infinite sequence of traders indexed by $t = 0, 1, 2, ...$ This sequence can be thought of as one of time periods. MMs set the prices at which they will trade at the beginning of each trading period, when the selected trader can buy or sell a unit of the asset or not trade, after which he must exit the market.

Traders. Informed traders receive a private signal concerning the value of the asset. In the mainstream literature they base their trading decision on their expected utility (value) of the asset, whereas here they do so according to their CEU of the asset. The trading decisions of the uninformed traders do not concern us and we use traders as a shorthand for informed traders.

The private signals (x), noted above, that are received by informed traders are identical to those proposed by Bikhchandani, Hirshleifer and Welch (1992), being two in number: $x \in \{h, l\}$; indicating that the value of the asset is "high" (h) or "low" (l). Their probabilities are given in Table 1, and we recall that it is assumed that $p > 1/2$. Intuitively, signal l is more likely when $w = 0$ and it can be interpreted as a "Bearish" signal. Similarly, $x = h$ can be interpreted as a "Bullish" signal. We have these expected values of the asset: $\mathbf{E}[w|x = l] < \mathbf{E}[w] < \mathbf{E}[w|x = h]$. These signals have been labelled "value uncertainty" by Avery and Zemsky (1998): their "monotonic signals". They, effectively, can be seen as defining signals that cannot produce herd or contrarian behaviour given the assumed objective of the informed trader.[5]

Market Makers. As argued in Chari and Kehoe (2003), including market makers is a convenient way of modelling trade between informed and uninformed traders. We assume, in accord with the literature upon which we are drawing, that there is finite number of long-lived risk-neutral market makers under Bertrand competition (Easley and O'Hara (1987)). Market makers make money from the uninformed traders but lose money to the informed traders, but due to Betrand competition they make zero expected profit (utility). They base their prices on the market's belief as to the value of the asset.

Market belief concerning the asset's value. Traders act sequentially and observe H_t, the history of actions (trades and their type) and prices (bid, ask), up until time t (the appearance of the market of trader t). We define $\pi_1^t = P(w = 1|H_t)$ as the market belief at time t, the probability in effect, that the value is high, conditional on the market history H_t. This is the expectation, taken at the end of trader $(t-1)$'s trading; of the value of the asset that will underlie the trading environment when trader t comes to the market. For any given trader's action $a \in \{s, b, n\}$, where s represents selling, b buying, and n no trading, the market's belief

[5] Avery and Zemsky (1998) introduced a further layer of uncertainty, what they call "event uncertainty" following the finance literature where this was first introduced by Easley and O'Hara (1987) (see, also Easley and O'Hara (1992)). They combine the two types of uncertainty to produce non-monotonic signals and thereby herding behaviour. They also introduce the notion of "composition uncertainty" which when combined with value uncertainty can lead to contrarian behaviour.

updates according to the Bayes' rule. Should there be a positive (negative) price history, then, $\pi_1^{t=1} > \pi_1^0$ ($\pi_1^{t=1} < \pi_1^0$). Those prices histories, of course, could last for longer periods so that, in general, for a positive history, $\pi_1^{t=n} > \pi_1^{t=n-1} > \pi_1^{t=n-2} > ....$; and, for a negative history, ($\pi_1^{t=n} < \pi_1^{t=n-1} < \pi_1^{t=n-2} < ...$). We refer, more appropriately, to such histories again under *the market makers' belief and price-setting* below.

Private belief. An informed trader at given t receives a private signal concerning the value of the asset in addition to the public information π_1^t. He then updates the probabilities that the value is high (π_1^t) or low ($1 - \pi_1^t$), employing the standard Bayesian rule on conditional probabilities; so that in the former instance, the up-dated probability is given by equation (3). However the trader lacks confidence in these updated beliefs. We use neo-additive capacities to model this lack of confidence. He then decides upon his trading strategy on the basis of the (signal-updated) CEU of the asset, given the market maker's bid and ask prices.[6]

Market Makers' belief and price-setting. We follow most (but not all of the literature) in assuming that the MMs do not receive private signals about the value of the asset and have only the same information as that in the public domain. In the sequential trading model, as Glosten and Milgrom (1985) demonstrated, the stochastic prices at which transactions take place follow a Martingale process, with respect to the market maker's information. Consequently, the market's expected value of the asset for some period (trader) t, will equal the price at which the asset has just been traded on the market, its current price. That is, $\pi_1^t = P(w = 1|H_t)$, will be set at that current price. However, in accord with most of the literature, it is assumed that they and market participants know that a private signal will always have been received by any informed trader who is selected to trade at each point in time. Then they must set a bid-ask spread around π_1^t (see, for example, Glosten and Milgrom (1985)). Nevertheless, the spread is frequently overlooked or ignored in the literature; with authors taking π_1^t, the trader's expected value of the asset, as the *price* of the asset, or taking it to be approximately so. We do not adopt that procedure and consider bid and ask prices: in this way the possibilities of no trade thereby are highlighted.

[6] We note that we follow the published literature and assume that there are no transactions costs. Additionally, we note that papers by Romano (2006) and Cipriani and Guarino (2006) consider aspects of transactions costs in the Glosten-Milgrom model; though not in our framework.

Bid and Ask prices. The ask price is the price at which the market maker will sell the asset to a potential buyer. In the absence of ambiguity the market maker is concerned solely with maximising expected profit (or its expected utility). This implies that he will set the ask price so that it equals his expected value (utility) of the asset, should the next trader be a purchaser of the asset. In that way he can make up for the fact that he will gain on average from the uninformed traders but will lose out on average to the informed traders. Consequently, given Bertrand competition, the expected profit (or utility) from a sale or a purchase will be zero (Easley and O'Hara (1987)). Since all market maker's have the same market information and hence prior belief, the market makers can calculate each other's optimal prices: consequently, they must also quote the same ask and bid prices. The market maker's expected value of the asset must turn out to be what he sold it for, or what he paid for it. If we assume to begin with that he is concerned with the expected value of the asset then he will set the ask price (A^t) equal to his expectation of the asset's value consequent upon a purchase by the next trader. That is:

$$A^t = \mathbf{E}[w_t \mid b] = w_l P[w = w_l \mid b] + w_h P[w = w_h \mid b] \ . \tag{4}$$

Here: w_l and w_h are the two, low and high, values of the asset; P denotes probability; and b denotes the decision to buy by the trader t. Given that $w_l = 0$, its component in A^t is otiose here. Of course, $P[w = w_h \mid b]$ is the probability that the value of the asset is high if the market maker's trade in the next period should be a sale.

From Bayes' Rule:

$$P[w = 1 \mid b] = \frac{P[w = 1]P[b \mid w = 1]}{P[w = 1]P[b \mid w = 1] + P[w = 0]P[b \mid w = 0]} \ . \tag{5}$$

The conditional probabilities in (5) depend obviously (O'Hara (1997)) on these probabilities: (i) that the trader will be informed or uniformed ; (ii) that the informed trader will have received a signal; (iii) of any high (low) signal, p and $(1 - p)$, respectively, should a signal have been received; (iv) that the informed trader will buy given the receipt of either signal; and, (v) that the uninformed trader will buy a unit of the asset at the ask price when the value is high or low.

Assumptions made about the probability that the trader is informed vary. One frequent supposition, which we adopt here, is that the market maker knows, somehow, through the previous history of trading, the proportion of traders who are informed (say, ξ); accordingly, that ratio is used as the probability that the next trader will be informed. As noted above, we make the largely standard assumption that everyone knows that any informed trader will have received a signal of either type. In that situation, for example, the probability that the next trader will be an informed trader who will buy the asset under a high signal, will be ξp and the probability that he will purchase the asset when he has received a low signal will equal $\xi(1-p)$. We assume that the uninformed trader will buy with probability θ and sell with probability μ. As also noted above, the market maker possesses no finer set of information than the market, so that his priors for the value of the asset as trader t comes to the market must be $P[w = w_h = 1 \mid H_t] = \pi_1^t$, and $P[w = w_l = 0 \mid H_t] = 1 - \pi_1^t$; so we can write the ask price that is set for t as:

$$A^t = \left[\frac{\pi_1^t(\xi p + \theta(1-\xi))}{\pi_1^t(\xi p + \theta(1-\xi)) + (1-\pi_1^t)(\xi(1-p) + \theta(1-\xi))}\right]. \tag{6}$$

On similar assumptions, the bid price counterpart to equation (6) for the situation in which the market trader determines his expectation of the asset's value consequent upon his observing a sale by the next trader is:

$$B^t = \left[\frac{\pi_1^t((1-p)\xi + \mu(1-\xi))}{\pi_1^t((1-p)\xi + \mu(1-\xi)) + (1-\pi_1^t)(\xi p + \mu(1-\xi))}\right]. \tag{7}$$

The market maker's expected profit from a sale or a purchase must be zero by construction. Thus, the arrival of a purchaser of the asset on the market at t of, for example, will give:

$$\mathbf{E}[(A^t - w)] = (A^t - E[w \mid b_t])P(b_t) = 0. \tag{8}$$

Further, it is immediate that the ask (bid) price will increase (fall) when the proportion of informed traders in the market (ξ) increases. Should the market maker, for example, make a sale of the asset to trader t he will take A^t as his prior for the next trader that the value of the asset is high (1) and so his new prior that the value is low (0) will be equal to $1 - A^t$.[7]

4 Modelling Ambiguity and CEU

We now outline single person decisions when there is ambiguity. Ambiguity is modelled by non-additive beliefs and preferences are represented as a Choquet integral with respect to these

[7] Then: $A^t = \left[\frac{\pi_1^t(\xi p+\theta(1-\xi))^2}{\pi_1^t(\xi p+\theta(1-\xi))^2+(1-\pi_1^t)(\xi(1-p)+\theta(1-\xi))^2}\right]$.

beliefs, as in Schmeidler (1989). We focus on neo-additive capacities and on CEU based upon them. Throughout we use the following notation:

Notation We consider a finite set of states of nature S. The set of events is taken to be the set of all subsets of S, which we denote by Σ. The set of possible outcomes or consequences is denoted by X. An act is a function from S to X. The set of all acts is denoted by $A(S)$.

4.1 Capacities and the Choquet Integral

A capacity generalises the notion of probability and assigns non-additive weights to events. We use a special case of the Schmeidler model axiomatised by Chateauneuf, Eichberger and Grant (2008), in which beliefs are represented as neo-additive capacities.

Definition 4.1 *For* $(\alpha, \delta) \in [0,1]$ *and given an additive probability* π *on* S*, define a neo-additive-capacity* ν *by* $\nu(A) = \delta\alpha + (1-\delta)\pi(A), \emptyset \subsetneqq A \subsetneqq S; \nu(\varnothing) = 0; \nu(S) = 1.$

Chateauneuf, Eichberger and Grant (2008), show that the Choquet expected value of a function $f : S \to \mathbb{R}$ with respect to the neo-additive capacity ν is given by:

$$\mathrm{CEU}(v) = \int f dv = \delta\alpha \cdot \sup_{s\in S}(f) + \delta(1-\alpha) \cdot \inf_{s\in S}(f) + (1-\delta)\,\mathbf{E}_\pi(f).$$

The Choquet integral is like an expectation as it is a weighted sum of utilities. The weight assigned to a state depends on how the outcome is "ranked".[8] For a neo-additive capacity, the Choquet integral is a weighted average of the highest payoff, the lowest payoff and the expected payoff. The parameter δ is a measure of ambiguity; and the parameter α measures the individual's attitude to ambiguity. The neo-additive capacity is consistent with the observation of Kilka and Weber (2001) that individuals tend to overweight highly likely and highly unlikely events in their decision-making. It also accommodates both optimistic and pessimistic attitudes to ambiguity. We can take optimism (pessimism) to prevail when the individual over-weights the favourable (unfavourable) outcome: so here, pure optimism (pessimism) holds if $\alpha = 1$ $(\alpha = 0)$.

[8] Gilboa (1987), Schmeidler (1989) and Sarin and Wakker (1992) provide axiomatisations for CEU preferences. Wakker (2001) characterises capacities representing ambiguity-averse or pessimistic attitudes of a decision maker.

Intuitively, these preferences describe a situation in which agents have an underlying additive belief. However, they lack confidence in the latter. We can interpret the additive part of CEU, $\mathbf{E}_{\pi}(f)$, as the agent's belief and $(1-\delta)$ as his degree of confidence in that belief.[9]

4.2 Up-dating Neo-additive Capacities

To apply CEU with neo-additive beliefs to a dynamic process, it is necessary to model how agents update their beliefs upon the arrival of new information; and thence their CEU. Consider first the up-dating of the neo-additive capacity. We define the capacity for any event, A given the occurrence of any event F, using the Generalised Bayesian Updating rule axiomatised by Eichberger, Grant and Kelsey (2007).[10]

Definition 4.2 *Let $v = \delta\alpha + (1-\delta)\pi$ be a neo-additive capacity and let F be a subset of S. Then if A is a non-empty subset of F we define the updated neo-additive capacity $\nu_F(A)$ by*

$$\nu_F(A) = \begin{cases} \tau_F \delta\alpha + (1-\tau_F\delta)\pi_F(A) & \text{if } A \subsetneq F, \\ 1 & \text{if } A = F, \end{cases}$$

$$\text{where } \tau_F = \frac{1}{(1-\delta)\pi(F)+\delta} \text{ and } \pi_F(A) = \begin{cases} \pi(A)/\pi(F) & \text{if } \pi(F) > 0, \\ 0 & \text{if } \pi(F) = 0. \end{cases}$$

This rule has the advantage that the updated preferences can again be represented as a Choquet integral with respect to a neo-additive capacity. Thus we remain within the original class of preferences. A second advantage is that the updated capacity is itself a neo additive capacity with the same α. Thus the updating rule does not alter the decision-maker's ambiguity attitude but only updates beliefs.[11]

Lemma 4.1 *The Choquet expected utility with respect to a conditional neo-additive capacity is,* $\mathrm{CEU}(v_F) = [1-\tau_F\cdot\delta]\,\mathbf{E}_{\pi|F}(f) + \tau_F(\delta\alpha\cdot\sup f + \delta(1-\alpha)\cdot\inf f)$.

[9] When $\delta = 1$, these preferences coincide with the Hurwicz (1951) criterion (axiomatised in Arrow and Hurwicz (1972)).

[10] This up-dating rule is: $v_F(A) = \frac{v(A\cap F)}{v(A\cap F)+\overline{v}(A^c\cap F)}$ where: $\overline{v}(A^c \cap F) = 1 - v(F^c \cup A)$; given the conjugate capacity, $\overline{v}$, defined as: $\overline{v}(F) = 1 - v(F^c)$.

[11] Neither the Dempster-Shafer rule nor the Optimistic update shares this property.

The proof is immediate. Note, the Choquet utility (here also expected value) of a random variable with respect to any conditional neo-additive capacity is well defined even if the conditioning event is an ex ante zero probability event, provided $\delta > 0$. More generally, the more unlikely (in terms of the additive 'prior' π) is the event, the less confidence (the lower is $1-\tau_F\delta$) the individual has in the 'additive part of the theory' and the more weight (the greater τ_F is) he places on 'extreme' outcomes (depending upon his degree of ambiguity and his attitude to it). A consistent signal ($F = E$) reduces confidence less than does an inconsistent signal ($F \neq E$).

5 Herd and Contrarian Behaviour

5.1 Definitions

We adopt essentially the same definition of herding and contrarian behaviour as Avery and Zemsky (1998). However, we cannot state the definitions in exactly the same way because we work with the actual trading prices (bid and ask) of the market trader, rather than the market's expectation of the value of the asset. Formally, we define herd behaviour as:

Definition 5.1

1. *Given a positive history of prices, so that* $\pi_1^t > \pi_1^0$*, should trader* t *have received a low private signal* $x = l$*, he will engage in a herd buy if* $\mathbf{E}_{T,x=l}^t(w) > \mathbf{E}(A^t)$*.*

2. *Given a negative history of trades,* $\pi_1^t < \pi_1^0$*, should trader* t *have received a high private signal* $x = h$*, he will engage in a herd sell if* $\mathbf{E}_{T,x=h}^t(w) < \mathbf{E}(B^t)$*.*

Here, for example: $\mathbf{E}_{T,x=l}^t(w)$ represents the informed trader's expected value of the asset, or its EU_T or CEU_T, whichever happens to constitute his decision rule, at t given the signal $x = l$. Concomitantly, we define contrarian behaviour as:

Definition 5.2

1. *Given a positive history of trades,* $\pi_1^t > \pi_1^0$*, should trader* t *have received a high private signal* $x = h$*, he will engage in contrarian selling if* $\mathbf{E}_{T,x=h}^t(w) < B^t$*.*

2. *Given a negative history of trades,* $\pi_1^t < \pi_1^0$*, should trader* t *have received a low private signal* $x = l$*, he will engage in contrarian buying if* $\mathbf{E}_{T,x=l}^t(w) > A^t$*.*

In effect, an informed trader will engage in herd behaviour when, for whatever reason he, as it were, overrides what his private signal is indicating about the value of the asset, and trades in line with market sentiment. Such a trader will engage in contrarian behaviour when he discounts his private signal to trade against the market trend.[12] We note that this overriding of the private signal is the definition of *an informational cascade*.

5.2 Analysis of Trading with CEU

We now turn to study in detail how differences in the information, beliefs, and concomitant assessments of the asset's value between the MM and informed traders can generate different trading behaviour and prices. It will be recalled that the market maker possesses the same information set as is publicly available (the past history of trading prices, be they ask or bid prices, plus the knowledge that informed traders exist and receive high and low signals). Consequently, to reiterate this, the market expectation of the value of the asset at time t is the probability that the value of the asset will be 1, namely, π_1^t.

Any informed trader will update that price/market expected value of the asset, consequent upon his receiving any private signal. From Bayes' rule the updated belief about that value, namely, the probability that $w = 1$ (given that we can ignore the other value of $w = 0$) conditional on the type of signal is: [13]

$$\pi(w|x) = \begin{cases} \pi_{x_h}(w) = \pi(w = 1|x = h) = \frac{p\pi_1^t}{p\pi_1^t + (1-p)\left(1-\pi_1^t\right)} \\ \pi_{x_l}(w) = \pi(w = 1|x = l) = \frac{(1-p)\pi_1^t}{(1-p)\pi_1^t + p\left(1-\pi_1^t\right)}. \end{cases} \quad (9)$$

If the trader's objective is to maximise the *expected utility of the asset*, then that utility or value will be given by $\pi_{x_h}(w)$, for a high signal and $\pi_{x_l}(w)$ for a low signal in equation (9).

[12] Such behaviour has been labelled by Chari and Kehoe (2003) as generating one of "waves of optimism and pessimism" rather than of herd behaviour. However, logically our definitions do, indeed, define herd and contrarian behaviour by a given trader; if it happens that a sequence of informed traders all appear over a period and they have all receive high signals then should the circumstances delineated in (i) of Definition (5.1), this will coincide with a boom in prices and what seems like a wave of optimism. But it is not necessarily the case that the latter is prevalent in the market from the observed trades; and, under (i) of Definition (5.1) there will not be optimism in the market in the usual sense, since low signals have been received by the traders.

[13] As noted, when $w = 0$, there is no need to figure out $\pi_1(0)$ and $\pi_0(0)$, since they do not appear in any of the expected value (utility) formulations.

However, should the trader's objective be to maximise his *Choquet expected value of the asset*, we need to formulate his $CEU_T(w)$ based on his up-dated neo-additive beliefs. Denote by $v_x(w)$ the conditional neo-additive belief of w given x, then it follows from Definition 4.2 and Lemma 4.1 that the informed trader's CEU is:

$$CEU_T(v_x) = (1 - \tau_x \delta)\,\mathbf{E}_{\pi|x}(f) + \tau_x(\delta\alpha \cdot \sup f + \delta(1-\alpha)\cdot \inf f), \tag{10}$$

where, in our framework, $\sup f = 1$ and $\inf f = 0$. Here, for example: $\mathbf{E}_{\pi|x}(f)$ is the upper expression in (9) when the high signal is received ($x = 1$); τ_x is given by τ_F in Definition 4.2 where, of course, $x = F$; and, τ_x is the same for both the high and low signals since they have identical probabilities of occurrence (Table 1).

We show how the perceived uncertainty of informed traders can generate contrarian and herd behaviour in the market.

(1) *With a private signal but no ambiguity in the perceptions of informed traders there is neither herd nor contrarian behaviour.*

The trader's action now depends upon his EUs or the expected values of the asset, given by the first and second expressions in equation (9) for the "Bullish" signal and the "Bearish" signal, respectively. Those expressions are, respectively, concave and convex with respect to π_1^t.[14] We observe that they have identical magnitudes at, respectively, $\pi_1^t = 0$ and $\pi_1^t = 1$. Now, consider how $EU_{T,h}(w)$ and $EU_{T,l}(w)$ relate, respectively, to A^t and B^t. Take, for example, the relationship between $EU_{T,h}(w)$ (equation (9) and A^t (equation (6)). They have identical values at, respectively, $\pi_1^t = 0$ and $\pi_1^t = 1$;and $EU_{T,h}(w)$ must lie above A^t at all intermediate values of π_1^t since:

$$EU_{T_{x=h}}(w) \ > A^t \text{ as } 1 < 2p \tag{11}$$

and the right hand inequality must hold since $p > 1/2$. The bid price (equation (7)) must lie entirely above that of $EU_{T,l}(w)$ on that same condition, since:

$$B^t > EU_{T_{x=l}}(w) \text{ as } : 1 < 2p. \tag{12}$$

Accordingly, given the receipt of a high (low) signal by the informed trader he will always buy (sell) the asset. Even supposing that the market's expectations (π_1) over time have fallen

[14]Thus, for example, $\frac{\partial EU_{T,h}(w)}{\partial \pi_1^t} = \frac{p(1-p)}{z} > 0$; $\frac{\partial}{\partial \pi_1^t}(\frac{\partial EU_{T,h}(w)}{\partial \pi_1^t}) = \frac{-2p(1-p)(2p-1)}{z^3} < 0$, where: $z \equiv p\pi_1^t + (1-p)(1-\pi_1^t) > 0; p > 1/2$.

(risen) consistently, the informed trader will not ignore his high (low) signal and engage in herd selling (buying); neither will he engage in contrarian behaviour.

In effect, when $\mathbf{E}[w|x=l] < B^t < \mathbf{E}(w_t) = \pi_1^t < A^t < \mathbf{E}[w|x=h]$, we have a situation where *the private signals are monotonic*, as defined by Avery and Zemsky (1998). With monotonic signals the market has complete information about the probability of (its own) expected value of the asset, since the market can deduce the probability of the two possible values of the asset, π_1^0, for $w = 1$; and $1-\pi_1^0$, for $w = 0$.

(2) *With a private signal and informed trader ambiguity: Herd and Contrarian behaviour.*

For the informed trader the choice of action on the market is given by his $CEU(w)$ under the receipt of either signal compared with the ask (equation (6)) and bid (equation (7)) prices. For the "Bullish" signal "Bearish" signal, respectively, these are:

$$CEU_{T,x_h}(w) = [1 - \tau_x \delta] \cdot \mathbf{E}_{\pi|x_h} + \tau_x \delta \alpha. \tag{13}$$

$$CEU_{T,x_l}(w) = [1 - \tau_x \delta] \cdot \mathbf{E}_{\pi|x_l} + \tau_x \delta \alpha. \tag{14}$$

In equations (13) and (14), we reiterate that the conditional expectations are given in equation (9), and τ_x is given by Definition 4.2, and it takes the same value independently of which signal is received, since the probability of either signal is p (from Table 1).

We can state this Proposition on *herd behaviour*:

For given δ, α, for informed traders there exist q^*, q^{**} such that:

Proposition 5.1 *If* $\pi \in [0, q^*]$, *with a recent history of rising* π *over that range, as a consequence of a sequence of rising transactions prices, an informed trader who, having received a low signal, is selected to trade on the market at* t, *will herd buy with positive probability. Respectively, If* $\pi \in [q^{**}, 1]$, *with a recent history of falling* π *over that range, as a consequence of a sequence of falling transactions prices, an informed trader who, having received a high signal, is selected to trade on the market at* t, *will herd sell with positive probability.*

Proof. Consider (1). Note that potentially the actions of traders, uninformed as well as informed, with the latter in receipt of perhaps varying signals, can move the price, and hence, π for any t, to anywhere in the range 0 to 1. In addition, as demonstrated above, with $p > 1/2$, that $A^t > \pi > B^t$. Now, assume that prices and the market's expectation of the value of the asset have been raising over the stated range in recent trades. Then, when a low signal is received, the trader will buy the asset provided that equation (14) lies above A^t (equation (6)) over the range of π from 0 up to q^*, at which value latter value it intersects A^t. That outcome must arise because: (i) the value of equation (14) at $\pi_1^t = 0$ is $\tau_x \delta \alpha$, whilst that of $A^t = 0$;(ii) A^t is concave, whilst equation (14) is convex, in π_1^t; and, (iii) the value of equation (14) at $\pi_1^t = 1$ is $1 - \tau_x \delta(1-\alpha) = [(1-\delta)p + \alpha\delta]/[(1-\delta)p + \delta]$, which is lower than 1, the value of A^t. Accordingly, equation (14) must lie above A^t for some "low" values of π_1^t, below it for some "high" values of π_1^t, and hence intersect at some "intermediate" value of π_1^t. Confirmation of (ii), recalling that $p > 1/2$, is provided by:

$$\frac{\partial A^t}{\partial \pi_1^t} = \frac{kr}{(k\pi_1^t + r(1-\pi_1^t))^2} > 0; \frac{\partial}{\partial \pi_1^t}\left(\frac{\partial A^t}{\partial \pi_1^t}\right) = \frac{-2kr(k-r)}{(k\pi_1^t + r(1-\pi_1^t))^3} < 0; \tag{15}$$

$$k = \xi p + \theta(1-p) > 0; r = \xi(1-p) + \theta(1-\xi); k > r \tag{16}$$

$$\frac{\partial CEU_{T,x_l}(w)}{\partial \pi_1^t} = [1 - \tau_x \delta].\frac{p(1-p)}{[(1-p)\pi_1^t + p(1-\pi_1^t)]^2} > 0. \tag{17}$$

$$\frac{\partial}{\partial \pi_1^t}\left(\frac{\partial CEU_{T,x_l}(w)}{\partial \pi_1^t}\right) = [1 - \tau_x \delta].\frac{2p(1-p)(2p-1)}{[(1-p)\pi_1^t + p(1-\pi_1^t)]^3} > 0. \tag{18}$$

Now consider (2). To prove this, it must be possible for the trader's CEU consequent upon receipt of a high signal, equation (13), to lie below the market maker's bid price, B^t, over the stated price range. So that at some at $\pi = q^{**}$they must intersect, and as the price rises towards 1, equation (13) must lie below B^t. This is possible because: (i) as with equation (14), equation (13) has a value at $\pi_1^t = 0$ of $\tau_x \delta \alpha$, whilst B^t has a value of 0, like A^t;(ii) at $\pi_1^t = 1$, equation (14) shares the same value as equation (13), $1 - \tau_x \delta(1-\alpha)$,which is lower than 1, the value of B^t. Consequently, equation (13) and B^t must intersect at some value of π_1^tbetween 0 and 1, since both equations are continuous in π_1^t. B^t is convex, and equation (13) is concave, in π_1^t:

$$\frac{\partial B^t}{\partial \pi_1^t} = \frac{ab}{(a\pi_1^t + b(1-\pi_1^t))^2} > 0; \frac{\partial}{\partial \pi_1^t}\left(\frac{\partial B^t}{\partial \pi_1^t}\right) = \frac{-2ab(a-b)}{(a\pi_1^t + b(1-\pi_1^t))^3} > 0. \tag{19}$$

$$a = (1-p)\xi + \mu(1-\xi); b = \xi p + \mu(1-\xi); a < b$$

$$\frac{\partial CEU_{T,x_h}(w)}{\partial \pi_1^t} = [1 - \tau_x \delta].\frac{p(1-p)}{[(1-p)\pi_1^t + p(1-\pi_1^t)]^2} > 0. \quad (20)$$

$$\frac{\partial}{\partial \pi_1^t}\left(\frac{\partial CEU_{T,x_h}(w)}{\partial \pi_1^t}\right) = [1 - \tau_x \delta].\frac{2p(p-1)}{[(1-p)\pi_1^t + p(1-\pi_1^t)]^3} < 0 \quad (21)$$

■

Proposition 5.1 is illustrated in Figure 1, where π is given by the 45° line. In constructing Figure 1 these parameter values have been assumed for the Choquet Expected Utility, bid and ask equations: $\alpha = 0.5; \delta = 0.6; \xi = 0.5; \theta = \mu = 1/3$ (so implying that there is probability of 1/3 that the uninformed trader will not trade), and $p = 0.7$.[15] Equation (14) intersects A^t at $\pi = q^* = 0.288848$; and, equation (13) and B^t intersect at $\pi = q^{**} = 0.743409$. The probability values $n^*$and n^{**} are referred to in the following text.

Proposition 5.1 gives rise to several corollaries, of which the following are the most pertinent:

Corollary 5.1 *Price ranges will exist for the high and low signal over which the informed trader will not trade.*

Proof. Assume that a high signal has been received. There will be a price range over which the trader will neither sell nor buy the asset. That price range must lie between the intersection of equation (13) with A^t (say, n^{**}) and B^t (the value q^{**}, already established above). Given that $A^t > B^t$, except at $\pi_1^t = 0$ or 1, and the properties of equation (13) established in Proposition 5.2, it follows that equation (13) intersects both A^t at a lower value that it does B^t. So, equation (13) lies below A^t and above B^t for $\pi_1^t \in [n^{**}, q^{**}]$.

Now, let a low signal be received. It follows from the proof of Proposition 5.2, since equation (14) intersects A^t, at q^*, it must intersect B^t, and at some $n^* = \pi_1^t > q^*$.Therefore, for $\pi_1^t \in [q^*, n^*]$, the Choquet Expected Utility of the trader, given a low signal, lies below the ask price and above the bid price. ■

[15] We note that these values of α and δ are compatible with the experimental results of Kilka and Weber (2001).

Corollary 5.2 *A ceteris paribus increase in the informed trader's ambiguity-aversion (i.e a decrease in α) will increase (reduce) the range of prices over which he will engage in herd selling (buying).*

Proof. Our focus is the values of q^* and q^{**}. From equations (13) and (14) we note that a reduction in α will lower the (identical) value of $CEU_{T,x_h}(w)$ and $CEU_{T,x_l}(w)$ at $\pi_1^t = 1$ and also at $\pi_1^t = 0$: since at those respective values of π_1^t, the Choquet Expected Utilities are,$1 - \tau_x\delta(1-\alpha)$ and $\tau_x\alpha\delta$. Consequently, $CEU_{T,x_h}(w)$ must intersect B^t at a lower value of π_1^t than it did previously (q^{**}); and, likewise, $CEU_{T,x_l}(w)$ intersect A^t, at a lower value of π_1^t than hitherto (q^*). Therefore, the price range over which herd selling will occur increase, whilst that over which herd buying will occur will fall. ■

Corollary 5.3 *A ceteris paribus increase in the informed trader's degree of ambiguity (δ) will increase the range of prices over which he will engage in herd selling and buying.*

Proof. An increase in the value of δ will increase the (identical) value of $CEU_{T,x_h}(w)$ and $CEU_{T,x_l}(w)$ at $\pi_1^t = 0$, and lower it at $\pi_1^t = 1$.At $\pi_1^t = 0$, $CEU = \delta\alpha[(1-\delta)p + \delta]^{-1}$. Hence: $\partial CEU/\partial\delta = p\alpha[(1-\delta)p+\delta]^{-2} > 0$. At $\pi_1^t = 1, CEU = [(1-p)+\delta\alpha]/[(1-\delta)p+\delta]$; hence, $\partial CEU/\partial\delta = p(\alpha-1)[(1-\delta)p+\delta]^{-2} < 0$, given $(\alpha,\delta) \in [0,1]$. Therefore, $CEU_{T,x_h}(w)$ must intersect B^t at a lower value of π_1^t than it did previously; whilst, $CEU_{T,x_l}(w)$ must intersect A^t at a higher value. Hence, in terms of Proposition (5.2), q^{**} declines and q^*increases. The price range over which herd selling and buying can occur will increase. ■

Corollary 5.4 *In the absence of a bid-ask spread, so that the market maker sets the bid and ask prices at π_1^t, the range of prices over which herd selling and buying will occur will be increased.*

Proof. At any given value of $\pi_1^t, A^t > \pi_1^t > B^t$.Consequently, when these three prices are identical, it must follow that after the receipt of a high signal, $CEU_{T,x_h}(w)$ must intersect π_1^t, the new price, at a lower value of π_1^t than that at which it intersects B^t.The q^{**} of Proposition (5.2) will fall. When a low signal is received, $CEU_{T,x_l}(w)$ must intersect π_1^t, the new bid price at a higher value π_1^t than that at which it intersected A^t. The range over which the informed trader will engage in herd buying has increased. The $q^*$of Proposition (5.2) will increase. ■

Consequent upon Proposition 5.2 *contrarian behaviour* can also arise under ambiguity and CEU preferences, and we can state the following Proposition:

Proposition 5.2 *For given δ, α, for informed traders there exist q^*, q^{**} such that:*

1. *If $\pi \in [q^{**}, 1]$, with a recent history of rising π over that range, and with a high signal, contrarian selling occurs with positive probability.*

2. *If $\pi \in [0, q^*]$, with a recent history of falling π over that range, and with a low signal, contrarian buying occurs with positive probability.*

Proof: This follows directly from that for Proposition 5.2 with price histories reversed. ■

Proposition 5.2 obviously admits of three companion corollaries to Corollary 5.2, Corollary 5.3 and Corollary 5.4, with proofs clearly identical, mutatis mutandis, to those of the three earlier corollaries:

Corollary 5.5 *A ceteris paribus increase in the informed trader's ambiguity-aversion (i.e a decrease in α) will increase (reduce) the range of prices over which he will engage in contrarian selling (buying).*

Corollary 5.6 *A ceteris paribus increase in the informed trader's degree of ambiguity (δ) will increase the range of prices over which he will engage in contrarian selling and buying.*

Corollary 5.7 *In the absence of a bid-ask spread, contrarian buying and selling will occur over a wider price range.*

The propositions demonstrate that informational ambiguity can be the cause of herding and contrarian behaviour. The corollaries give consistent and supportive claims that herding and contrarian behaviour are more likely to occur when there is greater ambiguity about information received. Greater optimism (larger α) encourages herd/contrarian buying and greater pessimism (larger $1 - \alpha$) encourages herd/contrarian selling.

6 Heuristics on: Asset Prices and Reversing Herd Selling

It is not our objective here to attempt to track over time (traders), bid, ask, and the transactions prices of the asset. However, using just two further traders, we set out an illustrative sequence of those prices, to show, herd sell behaviour (an asset price "crash") could be terminated, and by an informed trader, with a "high" level of optimism (α). We assume an uninformed trader appears next on the market and is followed by an informed trader.

Thus, take as starting point, the market and trading situation that occurs at time t, as set-out in the second part of Proposition 5.2, when trader t decides to engage in herd selling. Accordingly, in our framework, this means that the market maker will set his new expected value of the asset (effectively, that it will be 1) at B^t, which will be his prior for setting his bid and ask prices ready for $t+1$; with $(1-B^t)$ being his prior that the expected value of the asset will be 0. It follows from equation(7) that:

$$B^{t+1} = \left[\frac{\pi_1^t a^2}{\pi_1^t a^2 + (1-\pi_1^t)b^2}\right]; a = (1-p)\xi + \mu(1-\xi); b = \xi p + \mu(1-\xi). \tag{22}$$

Using the new prior and equation(6), the new ask price becomes:

$$A^{t+1} = \left[\frac{ka\pi_1^t}{ka\pi_1^t + rb(1-\pi_1^t)}\right]; k = \xi p - \theta(1-\xi); r = \xi(1-p) + \theta(1-\xi). \tag{23}$$

In accord with our assumption about the arrival on the market of a specific type of trader, trader $t+1$ is an uninformed trader. Assume that for whatever reason, perhaps because he views the market as being in a general decline, that he also sells the asset. The transaction price (in, we may say, at time $t+1$) will then again be the bid price. Up-dating equation(22) and equation(23), we have, the following prices that confront trader $t+2$ when he arrives on the market :

$$B^{t+2} = \left[\frac{\pi_1^t a^3}{\pi_1^t a^3 + (1-\pi_1^t)b^3}\right] \tag{24}$$

$$A^{t+2} = \left[\frac{ka^2\pi_1^t}{ka^2\pi_1^t + rb^2(1-\pi_1^t)}\right]. \tag{25}$$

In the circumstances the rational market maker has reduced both his ask and bid prices. That this is clear for the ask price from a comparison of equations(6), (23), and (25), since that $b > a$; and, given the latter, for the bid price from a comparison of equations (7), (22), and (24).

Trader $t+2$ is an informed trader. The market's prior probability that the asset's value will be 1, will be equal to the B^{t+1}, the transaction price for $t+1$. He will up-date that by one or other of the expressions in equation(9), depending upon the type of signal he has received. Suppose that the latter is the high signal, then the informed trader's expected value of the asset is:

$$\frac{p\pi_1^t a^2}{p\pi_1^t a^2 + (1-p)(1-\pi_1^t)b^2}. \tag{26}$$

Then the decision to trade depends upon the ambiguity (α) and the attitude to it (δ) of the trader. As $\alpha \to 1$, and $\delta \to 0$, $ap > b(1-p)$, the probability increases that the informed trader's CEU will exceed A^{t+2}. Thus, using equation(13) and replacing $\mathbf{E}_{\pi|x_l}$ with equation(26), we have to compare these two magnitudes:

$$\frac{ka^2\pi_1^t}{ka^2\pi_1^t + rb^2(1-\pi_1^t)} \quad \text{and } \epsilon \left[\frac{p\pi_1^t a^2}{p\pi_1^t a^2 + (1-p)(1-\pi_1^t)b^2}\right] + \eta \tag{27}$$

$$\text{where } \epsilon = \frac{(1-\delta)p}{(1-\delta)p+\delta}; \eta = \frac{\delta\alpha}{(1-\delta)p+\delta}$$

The parameters a, b, k and r, are defined in equation(22) and equation(23). Following the numerical example upon which Figure 1 was based, we let $k = b$ and $r = a$. Then:

$$\frac{ba^2\pi_1^t}{ba^2\pi_1^t + ab^2(1-\pi_1^t)} < \frac{p\pi_1^t a^2}{p\pi_1^t a^2 + (1-p)(1-\pi_1^t)b^2} \Rightarrow \tag{28}$$

$$\frac{1 + \left(\frac{b(1-\pi_1^t)}{a\pi_1^t}\right)}{1 + \left(\frac{b}{a}\right)^2 \left(\frac{(1-p)(1-\pi_1^t)}{p\pi_1^t}\right)} \Rightarrow ap > b(1-p)$$

Hence, as $\delta \to 0$, $\varepsilon \to 1$, it is probable that the informed trader's CEU will exceed A^{t+2}. Take the values that we have assumed in our numerical example for the parameters that determine a

and b: $\xi = 0.5; \theta = \mu = 1/3$. Then: $a = r = (19/60); b = k = (31/60)$. We assume, in line with that example, that the initial π_1^tmust exceed 0.743409, given by the point q^{**} on Figure 1. So let it be 0.75: then, $A^{t+2} = 0.647727$, and the trader's CEU will be:

$$CEU_{T,x_h} = \frac{(1-\delta)p.(0.724484) + \delta\alpha}{(1-\delta)p + \delta} \to 0.724484 : as \quad \delta \to 0 \tag{29}$$

Recalling that in our numerical example, $p = 0.7$, $A^{t+2} =$ 0.647727. Therefore, $CEU_{T,x_h} > A^{t+2}$ when: $0.0537299 > \delta(0.70147 - \alpha)$. So, no matter what the value of δ happens to be, if $\alpha \geq 0.70147$, so that the informed trader's ambiguity attitude is high, in excess of our informed trader who herd sold at the "initial" price of $\pi_1^t = 0.75$, whose ambiguity parameter was 0.5. The price of the asset has been falling consistently for several periods (over the appearance of several traders) and is down to 0.529843 (B^{t+1}at the given parameter values, a, b, $p = 0.7$, and $\pi_1^t = 0.75$) before the $t+2$ trader arrives on the market. That trader does not "discount" his high signal to an extent which stops him from following that high view about the asset's value. The downslide in the market has been halted.

7 Concluding observations: Money Pumps and Dynamic Consistency

We have re-examined herding behaviour in a financial market where trade is sequential and prices of assets are endogenously determined. To investigate the effects of ambiguity in financial markets, we modelled agents' beliefs as neo-additive capacities and their preference as CEU. We have demonstrated that herd and contrarian behaviour can be rational for informed traders facing ambiguity and with ambiguity attitudes that condition their trading strategy on the market. Such contrarian behaviour, for example, can terminate a price bubble. Additionally, there is greater scope for situations where informed traders can be seen, rationally, not to trade, so causing the market to break down.

The approach that we take in this paper to the modelling of informational ambiguity and attitudes to it is one that has support from the literature on the psychology of decision-making and associated laboratory experiments. It is an approach that does not need the various informational structures that are used in the influential study of Avery and Zemsky (1998). Of itself, it formally only requires the notion which underlies all the literature in this field, that of value uncertainty.

The finance literature offers support for our approach, even though empirical studies of herding and contrarian behaviour have not provided clear support for any of the competing rationalisations of such behaviour. For example, in a recent paper Zhang (2006) found that the level of ambiguity (either uncertainty about the fundamental volatility or about the quality of information) perceived about a firm by an investor is positively related to the return of herd tradings on its stocks in US markets. Also, in a substantial earlier study, we note that Lakonishok et al (1994) found that a contrarian trading strategy performs better for smaller firms than for larger firms. Information about smaller firms and their prospects/stock returns is likely to be less in quantity and quality (more uncertain/ambiguous) than that which is available to investors in larger firms.

Now, it is often argued that individuals who deviate from subjective EU are subject to so called 'money pumps'. In other words such individuals will persistently lose money in financial markets and hence their influence will can be neglected in the long run. These arguments do not apply to the traders in our model. We note that CEU preferences satisfy all the standard microeconomic rationality conditions, i.e. they are complete, reflexive, transitive, and respect state-wise dominance. Thus individuals with such preferences cannot be disadvantaged in a static context for the usual reasons. Any inconsistencies must arise from the interaction of the dynamic aspects of the model and the way in which preferences are updated as new information is received.

We consider individuals who have CEU preferences and use the GBU updating rule. Necessary and sufficient conditions for dynamic consistency in this context have been found in Eichberger, Grant and Kelsey (2005). These conditions are not satisfied by neo-additive capacities, hence in principle violations of dynamic consistency are possible in our model. However, these potential dynamic inconsistencies are not important in practice.

Consider the traders. Dynamic consistency issues are not relevant for the uninformed traders, since they only trade once and do not update their beliefs. To be subject to a money pump an individual must make at least two trades. Each informed trader also only trades once and thus cannot be dynamically inconsistent in the sense of losing money or of choosing a dominated option. It is possible that such traders may be dynamically inconsistent in the weaker sense that they may make a trade which they would not have chosen at time zero. There is no a priori reason why traders who exhibit this weak kind of dynamic inconsistency should not exist in markets since they will not tend to lose money even in the long run.

Consider the market makers. They are the only repeated traders in the market. Such individuals are potentially vulnerable to a money pump. However since the market makers have standard additive beliefs in our model, money pumps are not possible in practice. This is the reason that we assume that the decisions of market-makers are not affected by ambiguity.

We believe that it would be possible to extend our framework to a situation in which agents execute multiple trades over time, whist retaining dynamic consistency. In a recent paper Epstein and Schneider (2003) have axiomatised recursive multiple prior preferences, which are dynamically consistent and compatible with ambiguity aversion. Hannay and Klibanoff (2007) suggest an alternative way to extend multiple prior preferences to an intertemporal context. Their preferences are dynamically consistent but violate consequentalism. We conjecture that a more complex model of herding could be developed with either of these models. However, we believe that our result that herding is more likely with ambiguous signals would remain true. Moreover, in such a model, traders would be dynamically consistent in the strong sense of always implementing their initial plans.

References

ARROW, K. J., AND L. HURWICZ (1972): "An Optimality Criterion for Decision-making under Ignorance," in *Uncertainty and Expectations in Economics: Essays in Honour of G. L. S. Shackle*, eds. (CARTER, C. F., AND J. L. FORD), Oxford: Basil Blackwell, 1-11.

ABDELLAOUI, M. (2000): "Parameter-Free Elicitation of Utility and Probability Weighting Functions," *Management Science*, 46, 1497-1512.

AVERY, C., AND P. ZEMSKY (1998): "Multidimensional Uncertainty and Herd Behaviour in Financial Markets," *The American Economic Review*, 88, 724-748.

BIKHCHANDANI, S., D. HIRSHLEIFER, AND I. WELCH (1992): "A Theory of Fads, Fashion, Custom and Cultural Change as Informational Cascades," *Journal of Political Economy*, 5, 992-1027.

BIKHCHANDANI, S., AND S. SHARMA (2001): "Herd Behaviour in Financial Markets," *IMF Staff Papers*, Vol. 47.

BLEICHRODT, H., AND J. L. PINTO (2000): "A Parameter-Free Elicitation of the Probability Weighting Function in Medical Decision Analysis," *Management Science*, 46, 1485-1496.

CAMERER, C., AND M. WEBER (1992): "Recent Developments in Modelling Preferences: Uncertainty and Ambiguity," *Journal of Risk and Uncertainty,* 5, 325-370

CHARI, V., AND P. KEHOE (2003): "Financial Crises as Herds: Overturning the Critiques," *NBER Working Paper Series No. W*9658.

CHATEAUNEUF, A., J. EICHBERGER, AND S. GRANT (2008): "Choice under Uncertainty with the Best and Worst in Mind: Neo-Additive Capacities," *Journal of Economic Theory* 137, 538-567.

CHOQUET, G. (1953-4): "Theory of Capacities," *Annales Institut Fourier,* 5, 131-295.

CIPRIANI, M., AND A. GUARINO (2006): "Transaction Costs and Informational Cascades in Financial Markets: Theory and Experimental Evidence," *Working Paper*, ELSE, University College London.

COPELAND, T., AND D. GALAI (1983): "Information effects on the bid ask spread," *Journal of Finance*, 38, 1457-1469.

DOW, J., AND S. R. C. WERLANG (1992): "Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio," *Econometrica*, 60, 197-204.

EASLEY, D., AND M. O'HARA (1987): "Price, Trade Size, and Information in Securities Markets," *Journal of Financial Economics,* 19, 69-90.

EASLEY, D. AND M. O'HARA (1992): "Time and the Process of Security Price Adjustment," *The Journal of Finance,* 47, 577-605.

EICHBERGER, J., S. GRANT, AND D. KELSEY (2005): "CEU Preferences and Dynamic Consistency," *Mathematical Social Sciences*, 49, 143-151.

EICHBERGER, J., S. GRANT, AND D. KELSEY (2007): "Updating Choquet Beliefs," *Journal of Mathematical Economics.* 43, 888-899.

EICHBERGER, J., AND D. KELSEY (1996): "Uncertainty Aversion and Dynamic Consistency," *International Economic Review,* 37, 625-640.

ELLSBERG, D. (1961): "Risk, Ambiguity and the Savage Axioms," *Quarterly Journal of Economics*, 75, 643-669.

EPSTEIN, L. G., AND M. SCHNEIDER (2003): "Recursive Multiple-Priors," *Journal of Economic Theory*, 113, 1-31.

GILBOA, I. (1987): "Expected Utility with Purely Subjective Non-additive Probabilities," *Journal of Mathematical Economics,* 16, 65-88.

GILBOA, I., AND D. SCHMEIDLER (1993): "Updating Ambiguous Beliefs," *Journal of Economic Theory,* 59, 33-49.

GLOSTEN, L. R., AND P. R. MILGROM (1985): "Bid, Ask and Transaction Prices in a Specialist Market with Heterogeneously Informed Traders," *Journal of Financial Economics,* 14, 71-100.

GONZALES, R., AND G. WU (1999): "On the Shape of the Probability Weighting Function," *Cognitive Psychology*, 38, 129-166.

HANANY, E., AND P. KLIBANOFF (2007): "Updating Preferences with Multiple Priors," *Theoretical Economics*, 2, 261-298.

HIRSHLEIFER, D. (2001): "Investor Psychology and Asset Pricing," *Journal of Finance*, 56, 1533-1597.

HIRSHLEIFER, D., AND S. H. TEOH (2003): "Herd Behaviour and Cascading in Capital Markets: A Review and Synthesis," *European Financial Management,* 9, 25-66.

HURWICZ, L. (1951): "Some Specification Problems and Applications to Econometric Models," *Econometrica,* 19, 343-344.

KELSEY, D. (1995): "Dutch Book Arguments and Learning in a Non-Expected Utility Framework," *International Economic Review,* 36, 187-206.

KILKA, M., AND M. WEBER (2001): "What Determines the Shape of the Probability Weighting Function under Uncertainty?," *Management Science,* 47, 1712-1726.

KIRMAN, A. (1993): "Ants, Rationality and Recruitment," *Quarterly Journal of Economics*, 108, 137-156.

KNIGHT, F. H. (1921): *Risk, Uncertainty, and Profit,* Houghton Mifflin, New York.

LAKONISHOK, J., SHLEIFER, A., AND R.W.VISHNY (1994): "Contrarian Investment, Extrapolation and Risk, " *Journal of Finance*, 49, 1541-1578.

O'HARA, M. (1997), *Market microstructure theory*, Blackwell, Oxford.

ROMANO, G. R. (2006): "Learning, Cascades and Transaction Costs," *Working Paper*, no. 123, Department of Economic Science, Università Degli Studi Di Salerno.

SARIN, R., AND P. WAKKER (1992): "A Simple Axiomatization of Non-Additive Expected Utility," *Econometrica,* 60, 1255-1272.

SCHMEIDLER, D. (1989): "Subjective Probability and Expected Utility without Additivity," *Econometrica,* 57, 571-587.

SHILLER, R. J. (2001): *Irrational Exuberance*, Broadway Books, New York.

SHLEIFER, A., AND L.H SUMMERS (1990): "The Noise Trader Approach to Finance," *Journal of Economic Perspectives*, 4, 19-32.

TALLON, J.-M. (1998): "Asymmetric Information, Nonadditive Expected Utility, and the Information Revealed by Prices: An Example," *International Economic Review,* 39, 329-342.

VIVES, X. (1996): "Social Learning and Rational Expectations," *European Economic Review,* 40-3/5, 589-601.

WAKKER, P. (2001): "Testing and Characterizing Properties of Nonadditive Measures Through Violations of the Sure Thing Principle,"*Econometrica,* 69, 1039-1060.

ZHANG, X. FRANK. (2006): "Information Uncertainty and Stock Returns," Journal of Finance, 61, 105-136.