KINGSTON UNIVERSITY

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ISBN No. 1-872058-98-1
Kingston University
Date: November 2006

Kingston Business School
Occasional Paper Series No 60

20:09:96.orconf.4

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ABSTRACT

Although it is central business policy, structure is subordinated to other decision variables in strategic theory. The paper develops new a theory of structure. Transferable rents created by synergies or stand alone values, are summarized by an organization matrix, which applies both to private and public sector firms. Developing ideas from activity analysis, and co-operative games, conditions for efficient structures are set out. The theory is applied to current issues in corporate restructuring and internal pricing and to the rationale and scope of firms.

Key words: organization matrix, core businesses, competencies, Shapely value, and co-operative games.

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Revised nov2006

20:09:96.orconf.4

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INTRODUCTION

Traditionally, structure has been a backdrop, which is redesigned as a result of what happens to other strategic variables: *"Structure follows strategy...Few aphorisms have penetrated Western business thinking as deeply...."* (Bartlett and Ghoshal, 1994). However, interdependence between strategy and structure, rather than precedence of one over the other, is evident in business decisions in European and North American firms; Xerox, IBM, ICI, ABB, Matsushita, and Siemens, for example, have undertaken radical structural change, as part of their strategy. The pivots of business process engineering (Hammer and Champy, 1993), are structural changes, in the form of defining activities, and grouping them into effective coalitions. The rhetoric of structure, and reengineering, has also influenced the public, and voluntary sectors: the testimony is in the spread of techniques such as compulsory tendering, outsourcing, and internal markets.

The evolution of corporate structure has been analysed (Chandler, 1962; Sloan 1962; Williamson, 1975 and 1985): structural differentiation among divisionalised firms has been described (Prahalad and Bettis, 1986: Goold and Campbell, 1987; Goold, Campbell and Alexander, 1994); restructuring has been studied extensively (Bowman and Singh, 1993); yet no general theory of organisational structure exists. Hence there is an important gap in the approach to corporate strategy is founded. In this paper, corporate structure is in the foreground, and other sources of competitive advantage (Porter, 1980, 1985) make up a backdrop of ceteris paribus conditions.

Corporate structure is analyzed through the metaphor of forming and reconstructing coalitions in a co-operative game (N, z), with N players or activities, with transferable payoffs, z, and enforceable agreements. Transferable payoffs are surpluses, that can be seen equivalently, as economic rent, or value added: over time, rent is created, and destroyed, by factors internal and external to the organization, and perhaps, recreated. The connection between structure and coalition formation is consistent with "*reshuffling the organisational deck to deal a better hand"* (Moss Kanter, 1989), or *"the total of ways in which labour can be divided into distinct tasks and co-ordination achieved,"*

(Mintzberg, 1989). The building blocks of an organisation are activities. Structure describes how they are assembled, linked together, or partitioned into coalitions (divisions, product groups, or reporting centres). Justification for building, acquiring or retaining businesses has shifted from portfolio management theories, to emphasis on the importance of core businesses and competencies (Hamel and Prahalad, 1994). These are usually defined in terms of metaphors: the paper provides an operational definition.

The emphasis is on team production (Alchian 1982; Alchian and Demsetz, 1972; Radner, 1986). Ultimately organisations justify their existence (Coase, 1937), only if they add more value as an interdependent whole, (by an amount which exceeds the costs of organising team members), than as a set of independent parts, linked by markets. If production of B uses two co-operating inputs, b^i and b^j (so that $\partial^2 B/\partial b^i\ \partial b^i \neq 0$), it is difficult to determine individual contributions, and hence to monitor performance, and understand how value added can be increased (Alchian and Demsetz, 1972). The paper adapts and develops Arrow's abstract treatment of social cost (Arrow 1969, Starrett 1972), to the context of organizations. Businesses or activities are viewed as producers of joint outputs: the value they add is the sum of their contribution as stand alone entities, plus their contribution to other businesses in the organization. Treating interdependence in this way, means that S shaped relationships in cost and the organization matrix, which has linear properties, can summarize production functions, (brought about by positive and negative synergies).

Although the organisation matrix represents a novel approach, it has three related foundations: i. the theory of the firm, (Holmstrom and Tirole 1989; Tirole, 1990; Conner, 1991), especially the relationship between firms and markets (Coase, 1937; Penrose, 1959; Richardson 1996), and transactions cost (Williamson, 1975; Hart, 1996); ii. linear economic models (Koopmans, 1951, Samuelson Dorfmann and Solow, 1958; Gale, 1960); and iii. co-operative games, (von Neumann and Morgenstern, 1944; Myerson, 1991; Moulin, 1988; Aumann, 1985).

Aims of the paper

The aims of the paper are: i. to set out a general theory of corporate structure and the conditions for efficient structures; ii. to define core businesses and their relationship to structural efficiency; iii. to examine the implications of the model, summarized by the organization matrix, for some aspects of current corporate strategy, especially focus on core activities, and reengineering. The organization matrix (as presented here) is essentially a static concept, a snapshot at a moment in

time, which describes potential rather than actual payoffs. The problem of realization of payoffs is discussed elsewhere[1]. However attempts are made to link concepts in the paper with current scholarship in management science, especially through the application, to organizations, of Shapely values, and the core of co-operative games.

Plan of the paper

The paper is arranged into four parts. In the first, the organization matrix is introduced, together with the notion of a convex structure. The second sets out the conditions for structural efficiency in two kinds of organization; integrated structures, in which every activity is linked; and partitioned structures that are made up of quasi independent coalitions, divisions, or other groupings. The existence of efficient structures, and problem of imputing payoffs, or rentals to players in the organization, is discussed in the third section. The paper concludes with a brief summary, and remarks on the significance of some of the issues in the paper[2].

Foundations

Consider an organisation made up of a set of N activities. Value is added by them in two ways; (i) by stand alone entities, $z(b_{ij})$, (i = j), and (ii) by a set of potentially synergistic relationships, $z(b_{ij})$ (I ≠ j). The problem of interdependence is simplified by treating synergies as separate products (or services). The value of the organisation as a whole, Z(B), becomes the sum of these contributions:

$$Z(B) \leq \sum_{(i,j \in N)} z(b_{ij}) \qquad (1)$$

Variables $z(b_{ij})$ represent the value added, or more precisely the (economic) rent of activities, a surplus of returns over cost. Inequality (1) represents a snapshot at a point in time: coalitional structure is in the foreground, against the backdrop of other strategic variables. Among other variables are; size in terms of the number of value adding activities; the preferences and motivations of the multiple decision makers in the organization (reflecting corporate culture, mission and intent); their decisions about products, technologies and pricing; and uncontrollable variables in the organizational environment. The matrix summarizes the *state* of the organization, as determined by strategic variables and decisions of the past: it is the imprint of history. Payoffs are viewed as

[1] Problems of realisation of payoffs, the dynamic properties of the matrix, the growth of firms and the distribution of payoffs, are discussed in other chapters and in other papers (Matthews, 1985, 1994)

[2] Especially with reference to the ruthless economy. See Jeff Madrick (19…)

potential rather than actual values: hence the inequality [(1)].

The term activity is a general category: it may refer to an operating company, as in Newsview, or a store in a retail chain; a reporting or profit centre, a functional area, or a strategic business unit that co-ordinates and controls production in a divisionalised corporation. It corresponds to a discrete set of value chains, which can be jointly managed, on a more or less independent basis, and linked horizontally, and vertically, to other parts of the organisation. Activities are more or less equivalent to processes, defined as *"a collection of linked activities that take one or more input and creates an output of value to the customer"* (Hammer and Champy, 1993). The problem of corporate structure can be divided into two parts: that of defining core businesses and forming coalitions.

The properties of the organization matrix can be summarized as follows:

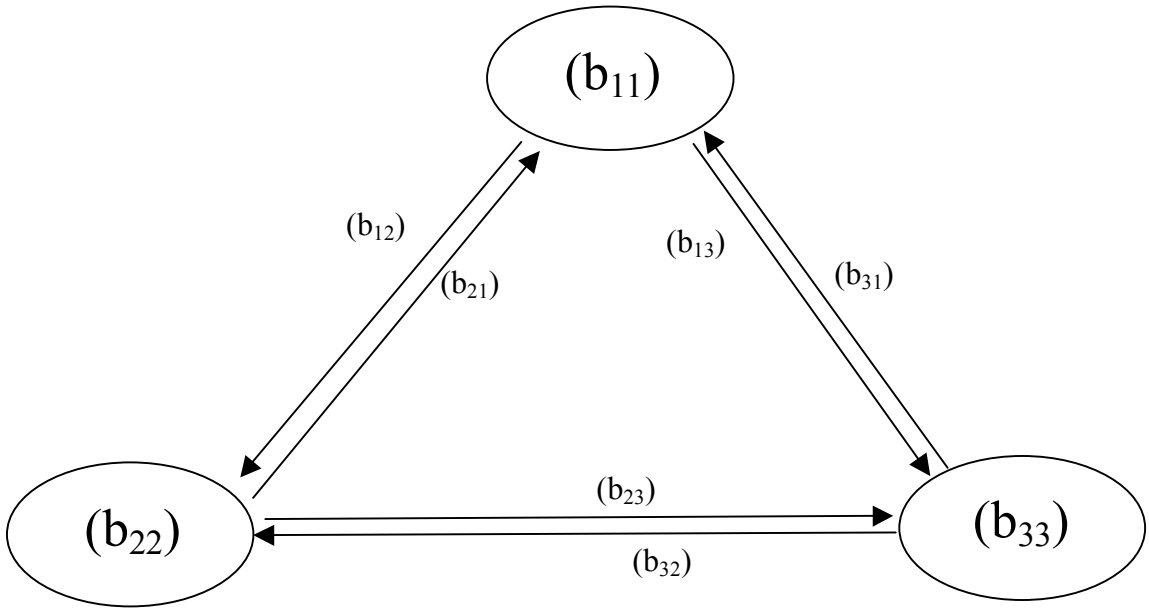
> *Definition 1. The properties of the organization matrix are: i. symmetry; ii. null player, iii. Linearity in stand alone activities* [(2)].

Only the contribution of each business counts towards overall value added, not the particular number associated with it, (*symmetry*). If a business adds nothing to a coalition, it has zero value, (*null player*). The significance of treating synergies as joint outputs should also be noted. Suppose every activity or coalition of activities is increased by a scalar, $\lambda > 0$, then its rental is increased by λ Z(S): there are constant returns to each coalition (*linearity*). Equally, when activities or coalitions are linked, their may be increasing or diminishing returns to co-operation. Following Arrow's treatment of externalities (1969), the organization matrix encompasses a subtle distinction, between constant returns (additivity and divisibility of payoffs treated individually), to particular activities or coalitions, and the possibility of non constant returns, or superadditivity (if synergies are positive or negative), when they are combined.

Figure 1 provides alternative illustrations of an organization made up of three activities. In (a) and (b) it is set out in matrix terms. Figure 1(c) describes, in a simplified way, the pattern of interdependence that existed in Newsview [3] between three activities, background research, a library

[3] Newsview is an alias for a media company concerned with the issue of (shadow) pricing an activity (a library) which added value only through synergies.

storing news, and production of stories. The library added value to research and stories, $z(b_{12})$, and $z(b_{13})$ respectively, and stored their outputs, $z(b_{21})$, and $z(b_{31})$, for further use. A proposal was made to shut the library because its value, $z(b_{11})$, as a stand alone entity was negligible, possibly negative. Application of the model resulted in identification and quantification of synergies: value added by the library turned out to be greater than the underlying or replacement value of the assets, and the option of a sell off was rejected, since to the activity was worth to Newsview its disposal value.



(3) Newsview

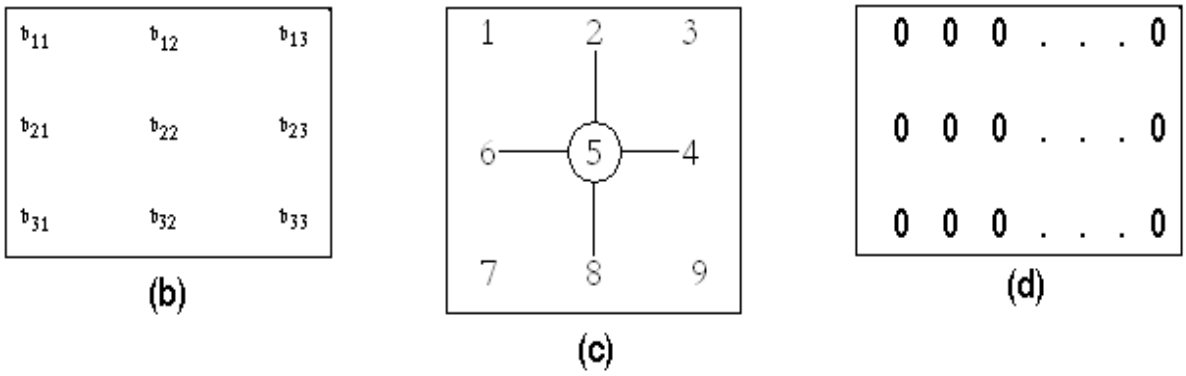


Figure 1

Treating corporate structure in terms of synergies in the organisation matrix naturally suggests an interpretation of organizations as co-operative games, in which enforceable agreements can be made between coalitions. In Figure 1(b), payoffs to coalitions are z{1,2,3}=45, z{1,2}=14, z{1,3}=20, z{2,3}=26, z{1}=1, z{2}=5, z{3}=9. The complexity of the problem of corporate restructuring can easily be demonstrated: given an organisation of N businesses, 2^N-1 coalitions can be formed so if there are 15 businesses, the number of possible coalitions is of the order of 10^{12}. The organization matrix simplifies the problem, by summarising the complex possibilities facing each player in a game, with a single value, Z(B).

Three alternative ways of partitioning an organisation are illustrated in Figure 2. Z(N) is the value of an integrated structure, or grand coalition, Z(C) relates to a structure of more or less independent coalitions (in the limiting case it is a set of independent or stand-alone activities, made up entirely of diagonal elements). Z(I) describes an organisation which is attempting to reconstruct interdependence: schematically, it perhaps represents the post-entrepreneurial organisation described by Moss Kanter (1989), or the multidimensional pattern of a transitional (Bartlett, 1986; Bartlett and Ghoshal, 1989) seeking selectively to combine integration, linkage, and co-ordination of coalitions.

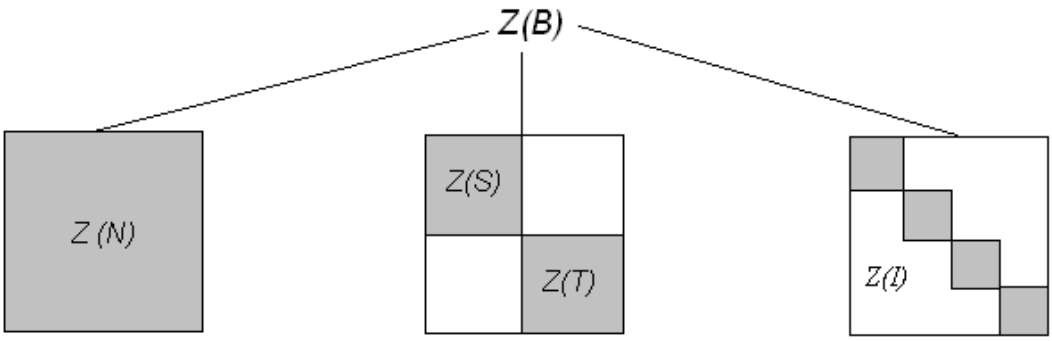


Figure 2

Synergies

The of diagonal elements of the organization matrix correspond to economic value added (EVA) that are used by companies like Coca-Cola, AT&T and Quaker Oats (Fortune, 1993). They are economic rents encompassing all net cash flows, including the weighted average cost of capital (net of tax deductions and adjusted for risk). Value added is an economic surplus or rent and can be expressed either as a present value or as an annual net cash flow. It may be enhanced by such managerial diversification that improves on the risk return trade-off achievable by shareholders (Campbell and Luchs, 1992; Lyubkin, 1989), or by exploiting tax shields.

Interdependence is a two way process: one activity potentially creates value in others; similarly it permits others to create value. This might be described as the yin and yang of business synergy: neither would be possible, if the organisation were unbundled, or partitioned, in such a way those activities were effectively isolated. In Figure 1(b), inward synergies to activity 2 are illustrated by the vertical lines, (6+4), and outward synergies by the horizontal, (2+8).

> *Definition 2: synergy or complementarity. In the organization matrix synergies are off diagonal elements ($k \neq i$, $k \neq j$). Inward synergies, σ_{ik}, are the contributions to value added in activity, k, by other activities in the organization and outward synergies, σ_{kj} the contribution of k to other activities. Total synergies in k are the sum of the two:*
>
> $$\sigma^k = \sigma_{ik} + \sigma_{kj} \qquad (2).$$

British Airways, for example, valued synergy or off diagonal benefits of their alliance with US Air, in 1993/1994, at $15 million. This rose to $105 million in 1994/1995. The main areas of value added were revenue enhancement at travel agents, through sharing codes and reservation screens.

The marginal value of an activity is the contribution it makes to economic rent of a particular coalition: it is the difference made to the rental of a coalition, if it were withdrawn from that coalition. In the three member coalitions in Figure 1, the marginal contribution of each activity is, respectively, (z{1,2,3} - z{2,3}), (z{1,2,3}- z{1,3}), and (z{1,2,3}- z{1,2}), that is 19, 25, and 31.

Definition 3: marginal contribution. The marginal contribution of a business, k, to a coalition S, is its value as a stand-alone unit, σ_{kk}, plus the value of inward and outward synergies. Generally, if S is any coalition of size s, in N, the marginal value of k is

$$m(b^k) = Z(S \cup \{k\}) - Z(S) \quad all\ S \subseteq N\ and\ k = s+1 \qquad (3).$$

Organizational matrices are convex, if they satisfy the condition, that the bigger the coalition joined

by an activity, the greater the marginal contribution: convexity means that there is increasing returns to co-operation.

Definition 4: convexity. An organization matrix is convex if it satisfies the condition:

$$(a)\ [S \subset T] \Rightarrow [\ Z(S \cup \{k\}) - Z(S) \leq Z(T \cup \{k\} - Z(T)],$$

$$all\ k \in N\ and\ all\ S,\ T \subset N \setminus \{k\}^{(4)} \qquad (4).$$

Interdependence may mean that the sum of marginal contributions is greater than the value of the organisation as a whole. This is not necessarily a problem: if we are interested in valuing the organisation, Z(B) is the appropriate benchmark; when valuing an individual business, $m(b^k)$ is appropriate. Positive synergies imply that the sum of marginal values of businesses exceeds the value of an organization as a whole, and this may be significant for mergers. Acquiring managers have at least to recreate existing synergies, before they can begin to add value to new businesses. Thus in 1(b), there is an upward bias by the organization which owns business 2: its marginal (and

hence reservation) value to them is 25, in comparison to potential acquirers, who receive only stand alone values, and must recreate the difference (25-5), even to begin to add value. This upward bias may explain overpayment by acquiring companies, and also favour management buyouts, since existing managers *" might be expected to have a better idea of the value of businesses and the value of synergies than new companies or Management buy-out financiers* [4]

The role of the centre

Entropy within organizations together with competitive pressures causes the organization matrix to degenerate to the pattern of zeros pictured in Figure 1(d). The centre is viewed as the source through which competitive advantage is created and recreated. It encompasses entrepreneurial and administrative functions (Goold and Campbell, 1987, 1994; Chandler 1991), and the capacity to decentralise, provide motivation, control, and incentives. Qualities of leadership, vision, and creation of an appropriate culture, are also often attributed to the centre, which is thought of, rather as the source of managerial competence, generating and sustaining competitive advantage (or failing to do so), than a physical entity, such as a head office, or boardroom, or a cohort of senior managers.

In this paper, the centre is the name given to the capital, unobservable in it self, that is the source of economic and strategic competence and dynamic capability (Teece and Pisano 1994). In a sense, it is the heart of an organization. Its impact but not its essence is observed through roles: i. enhancing the value of activities; ii. creating synergies or relatedness between activities, and between coalitions; iii. designing efficient structures, and restructuring, in response to changes in the underlying *state* variables; iv. determining the distribution of payoffs, or rents, between stakeholders. This section is primarily concerned with the first two issues. Efficient structures are discussed in the next two sections and some comments are made on the distribution of payoffs in the concluding remarks.

Enhancement describes the contribution of existing centre management to the rent of an organization: this is as the difference between its current value, Z(B), and disposal value, to the highest bidder, D(B). Gains from relatedness can be measured as the difference between the current value of an organization (activity), and the replacement cost of the underlying assets, viewed as stand alone entities.

[4] Financial Times, May 18th 1995

Definition 4: measurement in the organization matrix.

(i). Enhancement by centre management, H[Z(B)], can be measured as the difference between the value of assets under current management, Z(B), and their highest disposal value D(B).

So H[Z(B)] = Z(B) - D(B) *(5).*

(ii). Relatedness, Z(R), is the difference between the value of assets under current management, Z(B), and the replacement cost of businesses as stand alone assets, R(B).

So Z(R)= Z(B) - R(B) *(6).*

Gains from relatedness G(R) can be expressed alternatively as the sum of inward synergies $Z(\sigma_{ST})$, and outward synergies $Z(\sigma_{ST})$, (v(S) and v(T) respectively) between coalition S and other coalitions, T, (or between business k and the rest of the organization in which case S=k and T= N\k). This implies an alternative definition of convexity in the organization matrix;

Definition 3(a): convexity.

The organization matrix can be described as convex if gains from relatedness G(R) between any two coalitions S,T $\subseteq$ N, are

$$G(R) = Z(\sigma_{ST}) + Z(\sigma_{TS}) = \sum_{i\in s}\sum_{j\in t}\ \zeta(b_{ij}) + \sum_{i\in T}\sum_{j\in S}\ \zeta(b_{ij})$$

$$= S \cap T, \qquad (7)$$

an alternative definition of convexity in an organization matrix is that all gains from relatedness are positive,

$G(R) \geq 0$ *(4a)*

These ideas are summarised in Figure 3. Gains from relatedness between two coalitions S and T, (boxes 1 and 3), are illustrated in boxes 2 and 4.

$Z(S) = \sum_{ij\varepsilon s} z(b_{ij})$ 1	$Z(\sigma_{ST}) = \sum_{i\varepsilon Sj\varepsilon T} z(b_{ij})$ 2
3 $Z(\sigma_{TS}) = \sum_{i\varepsilon Tj\varepsilon S} z(b_{ij})$	4 $Z(T) = \sum_{ij\varepsilon T} z(b_{ij})$

Figure 3

The remarks on measuring relatedness, and enhancement, crucial to the practical significance of the organizational matrix, illustrate observations by Wittington (1989), and Edwards, Kay, and Mayer (1987): they maintain that pessimism, about the usefulness of accounting data, for strategic decisions, is misplaced. Replacement values provide a basis for assessing synergy or relatedness: disposal values of assets, can be used to measure enhancement, and the relative efficiency of managers. Deficiencies of accounting theories, and methods have been stressed. Since they reflect interdependence inadequately, it is argued, they are of limited use, either in value chain analysis, or restructuring. The problem according to Edwards, Kay, and Mayer, is not with accounting methods as such, but with the absence of a framework, capable of linking information to strategy. The organisational matrix provides such a framework.

Efficient Structures

Core businesses, and core competencies, are at the root of much current thinking in strategy; the resource based approach for example (Wernerfelt, 1984; Barney, 1991; Connor, 1991; Hamel and Prahalad, 1994). They are also the basis of some of the most prevalent current fashions in corporate consulting; business process reengineering for example. Confusion arises in the literature because writers lack a (non circular) definition of core businesses[5]. This section provides two

[5] We need an independent way of evaluating whether a business is core or not.

related definitions of core businesses, based on synergy: essentially they are based on satisficing and a recognition the no overall optimum is likely to exist. The first, which is intuitively plausible, may result in inefficient organizations: the second, a weaker condition, performs better, but is an inadequate framew[6]ork for corporate restructuring. Neither provides an unambiguous route for competitive advantage, for two reasons, that are outlined in the section: focusing only on current core businesses, and competencies, may rule out adaptation, by developing new, unrelated activities that create rent, and concentration on individual businesses, lead to neglect of efficiency in the organization as a whole.

viability

An organisation is viable if two conditions are met: firstly, positive added value; and secondly, it must be worth more than the sum of the constituent activities, working independently.

> *Proposition 1: viability[7]. An organization is viable if it adds value, and the whole exceeds the value of stand alone businesses:* $Z(B) \geq 0$ *and* $Z(B) \geq \sum_{k \in N} z(b_{kk})$.
>
> *Taken together with value creation, viability implies generating positive synergies:*
>
> $\sum_{k \in N} z(\sigma^k) \geq 0$ *(8).*

Viability, according to Proposition 1, emphasises interdependence and the role of external competition: negative rents imply that an organization is not covering costs, including the risk adjusted cost of equity and debt, and hence is threatened with bankruptcy, and negative gains from relatedness, suggest that, eventually, the organization must be split up.

Core businesses

Core businesses are rarely defined except in terms of metaphors, like *"stick to the knitting"* (Peters and Waterman, 1982), tautologies, or vagaries: *core businesses are the set of activities best fitted to give the firm competitive advantage*; or they reflect a *dominant general management logic*. According to Hamel and Prahalad (1994), *"[a] core competence is a bundle of skills and technologies that enables a company to provide a particular benefit to customers. At Sony it is "pocketability", and the core competence is miniaturisation... The commitment a firm makes to*

[6] See chapters 2 and 3 above for a discussion of this. Approximate measures of efficiency fit with Simon's view of bounded rationality and satisficing.

building a new core competence is a commitment to creating or further perfecting a class of customer benefits, not commitment to creating a specific product market opportunity." But how might a manager identify a core competence? The reasoning seems circular. Core competencies provide competitive advantage presumably by providing benefit to consumers. *How do we recognise the presence of core competencies? Though the evidence of competitive advantage (positive rental), and the provision of benefit to consumers (whatever that may mean).* What is needed is a way of identifying core competencies, that is not just a list of the supposed attributes of an apparently successful company. A clearer definition, identifying core businesses with the markets, in which *"the firm can employ its distinctive capabilities",* is given by Kay (1993). But these still leaves open the questions as to which markets are relevant? And what the distinct competencies might be? An operational definition of core businesses, in terms of marginal values, which encompasses Kay's observations, seems appropriate.

Definition 5: core businesses or activities (strong condition). Core businesses make a positive marginal contribution to the organization as a whole Z(N).

$m(b^k) = \sigma_{ik} + \sigma_{kj} + \sigma_{kk} \geq 0$ *(8).*

These accords with the perception that core businesses add value and generate synergies: but satisfying the strong core condition, although intuitively plausible, as a definition of core activities, does not guarantee the viability of an organization. The reason for this is clear, when it is remembered that the strong core condition takes into account, only the marginal contribution of an activity to the grand coalition, N.

Proposition 2: the strong core condition does not ensure viability.

Using core business, in the strong sense, is unsatisfactory: i. an organization may contain non-core businesses, and yet add value, when viewed as a whole; ii. it may not be viable, although it is made up entirely of core businesses. If the stand alone values of businesses are all positive, satisfying the strong core condition, guarantees value added. But this gives only qualified support for concentrating on basic businesses, since it may not insure against negative synergies[1].

Figure 4 demonstrates various aspects of Proposition 2. A viable organisation containing non-core

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[7] Essentially this definition coincides with superadditivity.

businesses is illustrated in (a): (b) is an organisation made up entirely of core businesses, which is not viable; the organisation in (c) is not viable, although it satisfies the strong core condition; (d) illustrates the condition that if all core businesses have positive stand-alone values they add value but may not be viable. Spinning off single core businesses increases value added, b^3 in (a). In (c), shutting down altogether retrieves a loss-making situation, even though all the businesses concerned are core.

2	8	-10
14	14	-10
6	-6	12

(a)

3	1	4	1
-4	-8	2	5
-3	3	-14	6
-1	7	3	-20

(b)

-21	12
12	-22

(c)

24	-11
-11	24

(d)

Figure 4

Failure to add value, because of negative stand-alone values is a limiting case only if one holds a particularly benign view of the operation of stock markets. Given this view, stock market competition would presumably eliminate quoted companies that fall into this category. Such competitive pressures are absent in the case of private, or state owned companies and in inefficient markets. The problem of non-viability might also arise in decentralised firms, with significant transfers between businesses. If the quasi-market condition is imposed, that each business should make a positive contribution (satisfy the strong core condition), the absence of an external market still leaves open the possibility that the organisation is not viable. An example of this may be the British National Health Service [(7)], formerly a vertically integrated organisation, which has adopted a division between purchasers and providers and imposes the quasi-market condition upon

constituent businesses (hospitals or regions), that they add value, in terms of internal transactions, but there is no external market to check on the value of the organization as a whole.

Since the core business condition does not ensure viability, an alternative is suggested, which relates to a distinction (Ginsberg and Baum, 1994), between adaptation by extending existing competencies and learning new ones:

> *Definition 6: the weak core condition (WCC). A business is core if makes a positive marginal contribution; that is, it creates positive rent for some coalition of activities in an organization:*
> $z(b^k) \geq 0, k \in S, \text{ and } S \subseteq N$ *(9).*

Activities that make a positive contribution to existing coalitions, ($S \subseteq N$), develop relatedness incrementally. Creating new activities (that is new single member coalitions, S=1), corresponds to developing new organizational traits, or expanding boundaries, S>N): the weak core condition encompasses enhancing rents by diversifying into such novel activities. It recognises the ability of imagination and creativity to create something, out of nothing.

As well as creating new activities, enhancement may refer to increasing existing rents (either of stand alone activities, or by increasing actual or potential synergy[8], but, in so far as relatedness points to linking existing routines, competence at this may be antithetical to creating new activities and attributes, processes, or procedures, and lead to maladaptive specialisation. *Competency traps* (Levitt and March, 1988; Levinthal, 1994), occur when success from existing activities, inhibits acceptance of superior, but novel, procedures: such traps however emanate from a confusion between competence viewed as a source, and competence expressed in terms of existing activities.

Structural efficiency

Given the sources of competitive advantage and their expression in the organization matrix, what are the characteristics of an efficient structure? How is this related to core businesses? These questions are now considered.

> *Definition 7: efficient structures. A structure is efficient if, given the organization*

[8] Issues of exploration and exploitation respectively. See chapters 2 and 4.

matrix, it is feasible, $\Sigma_{(k\in N)} z(b^k) = Z(N)$*, and if value added or rent cannot be increased merely by forming new coalitions,* $\Sigma_{(k\in S)} z(b^k) \geq Z(S)$*, all* $S \subset N$.

So defined, an efficient structure can be seen as an application of the idea of the core of a co-operative game to organizations: i. feasibility means that an efficient structure absorbs no more than the rent available given the organization matrix, and ii. in an efficient structure, it is not possible to increase the rent attributable to any activity merely by reassembling existing coalitions. The core of an organization, considered as a co-operative game is a set of allocations or imputations of rent to activities that cannot currently be improved upon[9]. The second feature is of course a restatement of Pareto efficiency: Pareto efficiency being allocations or imputations of rent that it is impossible for all players to improve upon. The first is more problematic: if a structure is efficient only when the organization is partitioned into quasi-independent coalitions, it does not satisfy the feasibility condition, because in exceeds what is available to the grand coalition.

The equivalence between the core of an organization matrix and an efficient structure can easily be verified.

Definition 8: the core of the organization matrix. This is defined as a vector of payoffs to activities that is feasible, and that no coalition can improve upon:

$\square_{(k\in N)} z(b^k) = Z(N)$ *and* $\square_{(k\in S)} z(b^k) \geq Z(S)$*. all* $S \subseteq N$ *(10).*

Thus efficient structures exist if and only if the core of the organization matrix is not empty.

Unless increased co-operation, by forming ever-larger coalitions adds value, a structure cannot be efficient, so convexity and the efficiency of organizational structures are linked.

Proposition 3: in an efficient organization all constituent coalitions, (S, T $\subseteq$ *N) are convex.*

Clearly unless the grand coalition N is convex (S = N), the organization matrix, as distinct from its proper coalitions (S $\subset$ N), is not convex. Considering (7) and Definition (3a), if the gains from

[9] Given potential payoffs existing at a point of time. This assumes a given state of knowledge. See chapters 2 and

relatedness, G(R), are expressed as the sequence of marginal contributions, to a coalition, S, as it becomes larger, in a convex structure, this sequence is increasing. Proposition 3 can be restated equivalently in terms of relatedness.

> *Proposition 3(a): a fully integrated organization consisting of N businesses is efficient if: i. $Z(B) \geq 0$, and ii. gains from relatedness are positive, $G(R) \geq 0$.*

Proof. *Establishing necessity is straightforward. By definition, (i) must hold. Suppose an organization has an efficient structure, when it is partitioned into two independent coalitions, S and T, ($S, T \subset N$), and suppose the gains from relatedness are positive, $G(R) \geq 0$. This is a contradiction. For sufficiency, suppose $G(R) \leq 0$, for any two coalitions S and T ($S, T \subset N$) in an integrated organization, breaking the organization down into independent coalitions can increase value.*

> *Proposition 3(b): a partitioned organization, Z(N) is efficient if for any two coalitions S and T, if 3 conditions hold: i. Z(S), and $Z(T) \geq 0$, (all S and $T \subseteq N$), and ii. all gains from relatedness, $G(R) = Z(\sigma_{ST}) + Z(\sigma_{TS})$, within a coalition are positive, and iii. all G(R), excluded by partitioning are negative.*

Proof. *By definition (i) must hold. If (ii) is not satisfied, break-up of existing divisions will increase rent. If condition (iii) does not hold, further integration will increase the value of the organization. If (ii) and (iii) hold, neither integration, nor disintegration can increase value.*

Core businesses and structural efficiency

The implications of the argument for core businesses can now be stated.

> *Proposition 4: focusing on core businesses is not a sufficient condition for structural efficiency. It may not even be necessary.*

Proposition 4 is significant: it demonstrates that one of the foundations of the strategy of organizations, focus on core businesses, is deficient. Not surprisingly, creating efficient structures requires consideration of organizations as a whole, rather than just individual activities. Given the

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3 above.

possibility of a competency trap, concentrating on core businesses may inhibit the creation of viable organizations.

Solution Concepts

In this section, two issues are examined. The first concerns the *evaluation* of the marginal contribution of an activity, when it can contribute to many different coalitions in an organization: in this connection, the Shapely value is discussed. The *existence* of efficient structures is the second issue, and the notion of the core of a co-operative game is relevant here. Existence has a specialised meaning: *feasibility* in the sense that payoffs to activities use up no more than the total rent available to the grand coalition; and *efficiency* which describes a coalitional structure, which is such that it is impossible to increase the rent imputed to one activity, without reducing that to another (given the *state* variables of the matrix). Both issues relate to potential, rather than actual circumstances: ideal distributions and structures are goals that firm work towards, in dynamic circumstances, possessing limited powers of cognition, and calculation[10]. However, if the goal is a mirage, in that it does not exist, even in principle, it does not deserve pursuit. Hence existence, in the specialised sense, has practical significance.

The deficiency of core businesses as a foundation for strategic decisions has been outlined. This section outlines another flaw, which surfaces from inquiry into the core. There may be many distributions of value added, associated with efficient structures. But, given the limitation that coalitions are allowed no more than is available to the grand coalition (feasibility), the core may be empty, and an efficient coalitional structure might not exist: this happens, as the explained in the section, when the organization matrix is not balanced. Thus the role of the centre consists, both of defining activities, and setting the limits on their links with the rest of the organization. An application of the core, is the equivalence principle, which states that corresponding to the core is a set of competitive prices, that allows decisions about structure to be decentralised, provided that each individual agent is insignificant. Here, this means that individual activities are *atomless*, in the sense that they generate a small proportion of the total rent of the organization. But organizations are not made up of atomless entities. So the role of the centre in arranging efficient structures cannot be decentralised and even budgetary arrangements for control and incentive mechanisms are

[10] Essentially the observations about efficiency in this section refer to the following conditions: convex landscapes and continuous (or upper semi continuous objective functions. It may be that if everyone knows that such conditions are unlikely to exist, that establishing the a priori conditions for the existence of efficient structures is irrelevant. However everyone does not seem to know and for the last 30 years or so governments

problematic. When the core is empty, there is no set of internal prices associated with an efficient structure. Thus whether or not the efficient structures exist, a further element of conventional wisdom of modern corporate strategy is challenged; reliance on internal markets as a way of stimulating efficiency[11].

The properties of the organization matrix set out in definition 1 ensure that it is always possible to impute a value to an activity; the Shapley value. It is independent of whether there is an efficient structure or not.

Payoffs to activities

The contribution of an activity is related to its marginal contribution. There are many ways in which a business can be marginal. Which is to count? Using the Shapely value (Shapely, 1953) a unique value can be assigned to each activity in the organization matrix.

Definition 9: the Shapely value, k_V. This allocates value added by the grand coalition Z(N) among all k businesses as follows:

$$v_k = \Sigma_{(0 \leq s \leq N-1)} [s!\,(N-s-1)!][N!]^{-1} \Sigma_{(S \subset N \backslash k,\ |S|=s)} [Z(S \cup \{k\}) - Z(S)]$$

with 0! =1, $Z(\phi) = 0$, and $\Sigma_{(\tilde{k}N)} v_k = Z(N)$ **(11).**

Expression (10) is the expected marginal contribution of an activity, b^k, to a coalition S consisting of s activities **(S=s).** The right hand term is the marginal contribution of the activity to the appropriate coalition. On the left is the probability of the coalition occurring randomly, $\sum_{(0 \leq S \leq N-1)}$ $[s!\,(N-s-1)!][N!]^{-1}$. A random coalition is got by figuratively arranging all N businesses in a line: there are N! such orderings, each assumed to be equally likely. The probability weighting is the number of ways that coalition S can precede b^k in the ordering as a proportion of the total number of orderings [(4)].

Following Shapely, there is exactly one function, corresponding to the properties of the organisation matrix which attributes a unique payoff to each player or business in the corporate structure game: this is the Shapely value, associating each business, k, given the underlying

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have have bee pursuing market solutions to problems on the assumption that the conditions exist.

[11] A more robust argument, in terms of information is made in earlier chapters.

organisational matrix, with a unique payoff. The sum of these unique payoffs, is the value of the organization matrix Z(B). It provides a means of (i). attributing value to businesses in the presence of synergies, and (ii). rewarding them in a way which corresponds to their power to create synergies(5). The Shapely value of activity 1, v_1 in the three-activity game in Figure 1(b) is calculated as follows:

$v_1 = 1/3z(1) + 1/6[z(1,2) -z(2) + z(1,3) -z(3)] + 1/3[(Z(N) - z(2,3)]$

$v_1 = 1/3(1) + 1/6[14 - 5 + 20 - 9] + 1/3[45 -26] = 10.$

Similarly v_2= 15 and v_3 = 20, giving $\sum_{(k=1,2,3)} v_k = Z(N) = 45$.

The properties of the organization matrix set out in the earlier section fulfil the three axioms set out in Shapley's original approach: *symmetry, null player*, and *linearity* (Hart, 1996; Myerson, 1991).

> *Proposition 5. Given the properties of the organization matrix, the Shapely value exists, is unique, and represents an efficient allocation of payoffs to activities* (7).

The intuitive interpretation of this result is that, given the properties of the organization matrix, which correspond to Shapley's axioms, forming coalitions is described by 2^{N-1} independent linear equations which give a unique value to each coalition. These equations are subject to a linear mapping, (from R^{2N-1} into R^N), by the probability of the coalition occurring randomly, into a set k= N of unique payoffs to each activity.

The existence of efficient structures

Does an efficient structure exist in the sense that there are feasible payoffs to activities that it is impossible to improve upon? Given the definition of efficient structures, this is equivalent to asking whether the core of the organization matrix itself exists. Conditions for the existence of the core of an organization matrix are:

> *Proposition 7: the existence of efficient structures in an organization matrix. The core of an organization matrix exists if and only if it is a convex structure. Otherwise the core is empty.*

To verify the existence of the core of an organization matrix first note that only if it is a convex

structure is it superadditive. Superadditivity means that for any pair of coalitions, S and T as

$$Z(S \cup T) \geq Z(S) + Z(T) \qquad (12).$$

where $S \cap T = \phi$. Convex structures are superadditive since all gains from relatedness are positive.

The Bonderava - Shapely proofs of the non-emptiness of the core depend upon balancedenss which can be looked upon as strong superadditivity (Aumann, 1985). Suppose the possibility for every activity of forming part time coalitions is introduced. The proportion of time spent by each activity in a coalition may be split into λ different parts so that for every coalition S that includes k, $\Sigma\lambda_{(S:\check{k}S)} = 1$. Value added by this part time coalition is $\lambda_S Z(S)$.

Definition 10: balancedness. For any set of balanced weights, λ_S, with

$\Sigma_S\lambda_S = 1, S \subset N \qquad (13),$

$\Sigma_{(S:k\in S)} \lambda_S Z(S) \leq Z(N) \qquad (14).$

The summation $\Sigma_{(S:\check{k}S)}$ denotes all proper coalitions containing k.

The condition for a three-activity game to have a non-empty core is that it should be balanced which means that

$z(1) + z(2) + z(3) \leq Z(N)$,

$z(1) + z(2,3)$, $z(2) + z(1,3)$, $z(3) + z(1,2)y \leq Z(N)$, and

$1/2[z(1,2) + z(1,3) + z(2,3)] \leq Z(N)$.

Note that the matrix described in Figure 1(c) is balanced and that in 4(a) is not.

Since the existence of efficient structures is linked to the core, an efficient structure exists if the core exists and this in turn depends on whether the organization matrix is balanced. The following proposition, essential to the existence of the core can now be stated:

Proposition 8: an organization matrix is balanced, if and only if, it is convex.

Proof of Proposition 8. *In particular convexity means that $Z(S) \leq Z(N)$ for all $S \subset N$. The organization matrix is linear so $\Sigma_{(S:k\in S)} \lambda_S Z(S) \leq \Sigma_{(S:k\in S)} \lambda_S Z(N)$. Summing over all k activities*

gives $\Sigma_{(\text{all } k\in N)}\Sigma_{(S:k\in S)}\ \lambda_S Z(S) \leq \Sigma_{(\text{all } k\in N)}\Sigma_{(S:k\in S)}\ \lambda_S Z(N) = Z(N)$. *Conversely, an organization matrix is not balanced if* $Z(S) > Z(N)$.

The proof of Proposition 7 can now be stated. *An organization matrix is balanced, if and only if, it is convex, so suppose* $z(b^k)$ *is an allocation in the core of (N,z) and* λ_S *is a set of balanced weights. For all* $S \subset N$,

$\square_{(k\in N)}\ z(b^k) \geq Z(S) \Rightarrow\ \lambda_S\,[\square_{(k\in S)}\ z(b^k)] \geq \lambda_S\, Z(S).$

Remembering (13), summing these inequalities gives,

$\square_{(S\subset N)}\lambda_S Z(S)\ \leq\ \square_{(S\subset N)}\square_{(k\in S)}\lambda_S z(b^k) = \square_{(k\in N)}\square_{(S:k\in S)}\ \lambda_S\ z(b^k) = \sum\ _{(k\in N)} z(b^k) =$

$Z(N).$

So if the organization matrix is balanced and the core exists, the allocation or imputation of rent is feasible. Conversely if the core is empty, there is a hyperplane, $\sum_{(K\in N)}\ z(b^k) = Z(N)$, *denoting payoffs to the grand coalition, that is disjoint from the convex non-empty subset of* R^N ***defined by***

$\sum_{(K\in S)}\ z(b^k) \geq Z(S)$, ***for all*** $S \square N$.

These are payoffs to partitioning the organization matrix. By the standard separation theorem, this implies the existence of non negative numbers λ_S, ***such that,***

for all $z(b^k) \in R^N$,

$\square_{(k\in N)}\ z(b^k) = \Sigma_{(S\subset N)}\ \lambda_S \square_{(k\in S)} z(b^k) = \sum_{(K\in N)}\ \sum_{(S:K\in S)}\ \lambda_S z(b^k)$ ***and*** $\Sigma_{(S\subset N)}\ \lambda_S Z(S) >$

$Z(N).$

Thus an efficient structure exists, if and only if, the organization matrix is balanced. If the organization as a whole is not a convex structure, the centre has the role of limiting the links between activities by creating partitions. If interdependence is bounded, and coalitions are expanded as long as the sequence of marginal contributions increases, this creates i. a set of coalitions that are convex, and ii. by excluding negative gains from relatedness, transforms the organization as a whole into a convex structure.

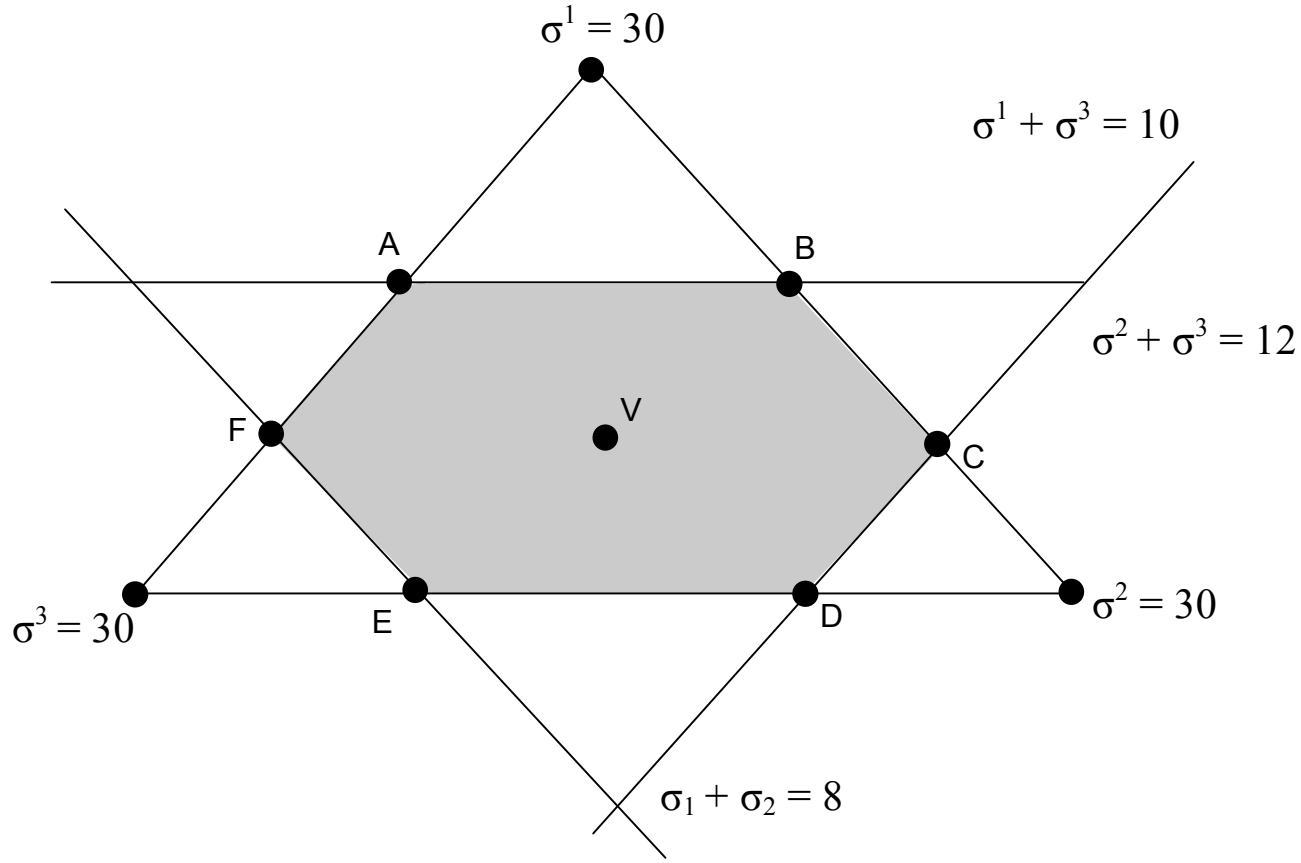
The core of the organization matrix in Figure 1(c) is illustrated in the simplex in Figure 5. The variables in 1(c) have been redefined: imputed rent to any activity must be no less than it is the stand-alone value. So Z(S*) = z(σ^k): only synergies are considered the inequalities must hold: $\sigma^1 \geq 0$, $\sigma^2 \geq 0$, $\sigma^3 \geq 0$.

Vertices denote the sum of the off diagonal elements, the total synergy available to the grand coalition. Rent attributed in the grand coalition, $Z^*(N) = \sigma^1 + \sigma^2 + \sigma^3 = 30$, must be at least as great as that to any proper coalition ($S \subset N$):

$\sigma^1 + \sigma^2 \geq 8$, $\sigma^1 + \sigma^3 \geq 10$, $\sigma^2 + \sigma^3 \geq 12$,

The shaded area ABCDEF illustrates the core. The Shapely value is located at V. Note that though the Shapely value exists, the core of the matrix in 4(a) is empty because it does not satisfy a similar set of inequalities: $Z^*(N) = 2$ but the coalition structure {1, 2} and {3} can get 34 ($\sigma^1 + \sigma^2 \geq 22$ and $\sigma^3 = 12$).

If the core is empty, there is no reason why binding agreements should not be made between coalitions, which create an efficient structure. If the core is not empty, a set of arrangements might exist within an organization that inhibits efficient structures. One role of the centre in the case of an empty core is to create agreements and determine the limits of interdependence that permit efficient structures to be created.



Firms and markets

Viewing organizations as co-operative games emphasises the parallel between firms and markets. The core equivalence theorem (Aumann, 1964) shows that for sufficiently large market games, the core is equivalent to the set of competitive equilibria, so a price system underlies a convex organizational structure. The attractions of internal markets in practice are highlighted if corporate structure is seen in terms of a linear programme. The minimisation problem is that of minimising the rent that no coalition can improve payoffs,

$$\textbf{minimise}_{(bk)} \quad \sum_{(k \in N)} \mathbf{z(b^k)} \textbf{ subject to } \sum_{(K \in S} \mathbf{z(b^k) \geq Z(S).}$$

$$\text{all } S \subseteq N \qquad (15).$$

and the dual problem is that of finding the balanced weights on coalitions that maximise the rent of the organization,

$$\text{maximise}_{\lambda S} \quad \sum_{(S \in N} \lambda_S Z(S) \text{ subject to } \Sigma_S \lambda_S = 1. \qquad (16).$$

Payoffs to coalitions have a natural interpretation as shadow prices, but as demonstrated, the problem has a solution, $\sum_{(K \in N)} z(b^k) = \lambda_S Z(S)$, only if the organization matrix is balanced. If it is not balanced (convex), no underlying set of shadow prices exists.

Even if the organization matrix is convex, and the core is not empty, a role exists for the centre and for hierarchical control. The relatively small numbers of activities that are involved even within large organizations, means that the relationship between the core and an underlying set competitive prices is approximate. Bargaining power of individual agents may be sufficient to block the attainment of an appropriate set of competitive prices. Thus in the case of convex structures, the role of an organizations exist because they are able to offer participants higher rents than they would otherwise achieve through decentralised bargaining. Somewhat paradoxically, one interpretation of this result is that firms exist because the alternative market arrangement is monopolistic and potentially offers lower payoffs.

The rationale for the existence of firms, both in the case of convex and non convex structures is expressed as follows:

> *Proposition 9: firms exist because no underlying set of market prices exists that are capable of achieving efficient structures.*

This explanation of the existence of firms is complementary to the transactions cost approach. Holding benefits constant, whether exchange occurs in a market or within a firm, depends on the respective transactions costs (Coase, 1937; Williamson, 1975): a frequent criticism of Williamson's work is that he does not say enough about ways in which costs in the hierarchical framework of the firm differ from those incurred on markets (Kreps, 1996). The point made here is that benefits are not constant since a function of organizations is to internalise potential gains from coalitional structures: these gains are not achievable through markets. The rationale for organizations is reinforced by the observation that in circumstances which require team effort, that is when off diagonal elements of the organization matrix are substantial, high powered market like incentives are quite destructive of co-operation and learning (Teece and Pisano, 1994).

The question arises as to why establishing an internal market is so important in current corporate strategy, both in the public and private sector, when it is based upon such frail foundations. Perhaps the answer lies in organizational politics. Moving towards more efficient structures increases total rent, but setting up internal markets may be a more effective way of increasing the leverage of senior management to divert a greater share in absolute terms to themselves and their immediate constituents, the shareholders.

Concluding Remarks

Decisions about corporate structure have been treated in the framework of a co-operative game (N,z) with transferable payoffs in the form of economic rents. Efficiency conditions have been shown to differ in an integrated structure, in which the grand coalition of all activities is convex, and a partitioned organization, in which the core is empty. The path of an organization matrix oscillates between a configuration of positive elements signalling rent or competitive advantage and the effects of competition and changes in the internal and external environment erode a zero value as the position. The role of the centre, by adaptation and learning is continually to recreate competitive advantage: reforming the coalitional structure is an important strategic variable for so doing.

Why do firms exist and what determines their size? These are two key questions in the theory of the firm. The paper suggests a clue to both. Firms exist because an underlying price system that would support the markets required for efficient structures do not exist. Two roles of the management are to enhance activities and create relatedness. Both are expressed in the marginal value of activities: if this is positive, the search for competitive advantage implies an increase in the size and scope of enterprises, and negativity signifies the reverse. The current fashion for outsourcing, unbundling, disintegration, indicates the feeling, in corporations, over the last fifteen years or so, those markets (even quasi markets) may be more parsimonious over costs, than organizational structures. given the theory outlined in the paper, this indicates an over confidence in markets. A number of consultancy companies anticipate a demand in the near future for their services in reassembling organizations that have been dismantled over the last five years or so. In one sense the trend towards alliances mirrors this; creating coalitions of activities between organization by more open agreements than is possible within them.

The paper demonstrates that focus on core businesses is not a sufficient condition for structural efficiency. In a dynamic environment, where a critical task is that of identifying new businesses, it is probably not even necessary. So why the focus on them? The existence of internal markets is

often questionable, so why have they become such an important component of strategy? One answer is in terms of bounded rationality. It is the best firms can do in the circumstances. Another is in terms of power. At the centre, organizations have always been as much concerned with distributing the surplus among stakeholders, as with its growth through enhancing, creating relatedness, and restructuring. The literature of management and economics recognises this, but tends to concentrate on the interdependence between the distribution and creation of surplus; emphasising incentives and the relationship between profits, investment, and the growth of firms.

Some restructuring in organizations is prompted by competitive pressures, creating zeros in organization matrix, but the extent to which it is adopted by high rent firms, and in the public, and even the charitable sector, signifies an increasingly ruthless economy. Current preoccupations, like focusing on core businesses, internal markets, and reengineering, affirm the power of top management to dispose of assets, including human resources, a metonymy for human beings, at will, and these strategies augment the power of top managers, to redistribute the surplus generated by organizations to themselves, their stockholders and financial arbitrageurs.

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[1] An organization may contain non core businesses and yet be justified by the condition.
Let $Z(B') = \Sigma\Sigma\,\zeta(b_{ij})$ so $Z(B) = Z(B\square) + \zeta(b^s) \geq 0$ all $s \in N$.
If $\left|Z(B')\right| > \left|\zeta(B^S)\right|$ and $\zeta(b^s) < 0$, Z(B) is justified even though it contains a non core businesses.
ii. An organization can be made up entirely of core businesses and still not be justified.

Suppose $\zeta(b^s) > 0$ all s $\in$ N: so $\sum \zeta(b^s) = \Sigma\Sigma\,\zeta(b_{ij}) + 2\sum\sum \zeta(b_{ij}) > 0$. Given the definition, $Z(B) = \Sigma\Sigma\,\zeta(b_{ij}) + \Sigma\Sigma\,\zeta(b_{ij})$, if $\sum \Sigma\,\zeta(b_{ij}) < 0$ and

$2\left|\sum\sum\zeta(b_{ij})\right| > \left|\sum\sum\zeta(b_{ij})\right| > \left|\sum\sum\zeta(b_{ij})\right|$, Z(B) is not justified even though it is made up entirely of core businesses.

iii. If $\zeta(b_{ss}) > 0$, all s $\in$ N, then the organization is justified.

Satisfying the core condition means that $\Sigma\,\Sigma\zeta(b_{ij}) + 2\,\Sigma\,\Sigma\,\zeta(b_{ij}) > 0$, hence if the first term is positive, it must outweigh any negative synergy effects.