Chapter 4

Black but Not Dark

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Additional information is available at the end of the chapter

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Abstract

Large black holes of millions of solar masses are known to be present in the centre of galaxies. Their mass is negligible compared to the mass of the luminous matter, but their entropy far exceeds the entropy of the latter by 10 orders of magnitude. Strong gravitational fields make them 'black'—but at the same time, they cause them to emit radiation—so they are not 'dark'. What is the meaning of their borders that may only be crossed once and that leads to the information paradox and what are the properties of their interiors? In discussing these and related questions (is it possible that the volume of a black hole might be infinite?), we uncover the unexpected meaning of the term 'strong gravity'.

Keywords: gravity, black holes, horizon, interior, information paradox

1. Introduction

Black holes (BHs) are sources of the strongest gravitational fields in the Universe. On the other hand, they are also the outcomes of these strong gravitational fields. The first time they appeared in science was as a result of speculation. At the end of the XVIIIC, the English geologist (and astronomer) John Michell and the famous French mathematician Pierre-Simon Laplace independently considered the consequences of the presence of a large, compact massive object producing gravitational fields so strong that even light could not escape from them. For obvious reasons, discussions of this kind were limited in their nature at that time.

The next step came at the beginning of 1916, when Karl Schwarzschild, a mathematician and an army officer, found a specific solution for the field equations of Einstein's General Theory of Relativity. He found the solution for the particular case of a static, spherically symmetric spacetime. Schwarzschild sent the results of these studies to Albert Einstein in the form of two chapters. The second of these two chapters contained what was, to Einstein, a controversial result. If the mass of the source of the gravitational field was both big enough and compact enough, then the solution was singular: a particular element of the metric tensor, a tool for describing the geometric properties of the spacetime, became infinite at some distance from the centre. Einstein was concerned by this effect and consequently had been slow to respond; in the meantime, Schwarzschild had died.

Schwarzschild's solution [1] (see subsequent text) reveals a specific form of behaviour and leads to the conclusion that in some circumstances, a so-called horizon (termed an event horizon) is formed around the black hole. Such a horizon acts as a semi-permeable 'membrane' [2]: it may be crossed only once and in one direction only. The radius of the event horizon is called the gravitational radius or the critical or Schwarzschild radius.

The term 'Black Hole', proposed in the 1960s by J.A. Wheeler, represents the reality of a strong gravitational field in which neither massive nor massless objects (i.e. light in the form of photons) could leave its interior. Black holes (BHs) had been regarded as hypothetical objects even as late as the early 1970s; at that time, a famous bet between two prominent physicists, Kip Thorn (Nobel Prize winner in Physics in 2017) and Stephen Hawking, was set. The subject of the bet was the experimental confirmation of the presence of black holes (the annual delivery of a journal from a building sector was the pledge for this bet).

Currently, it is assumed that there is a massive BH with a mass of millions of solar masses $(M_\odot)$ in the centre of each large galaxy [3]. The black hole closest to the Solar System is located at a distance of 1700 light years from us. In the centre of Milky Way, there is a BH of mass $4.3{\cdot}10^6 M_\odot$; one of the largest BHs with a mass of a billion solar masses has been found in the centre of the Sombrero galaxy. This allows us to estimate that the matter confined within black holes is many orders of magnitude smaller than the luminous matter (LM) in each galaxy,

$$\rho_{BH} \leq 10^{-3} \rho_{LM} \tag{1}$$

Hence contributes a negligible fraction of the total energy density. An interesting fact, however, is that the total entropy of black holes, $S_{BH}(tot)$ is 10 s of orders of magnitude higher than the entropy of radiation (CMB), estimated at a value of 10^{90}. Indeed, the entropy of a BH of mass $4.3{\cdot}10^6 M_\odot$ is

$$S_{BH}\left(4.3{\cdot}10^6 M_\odot\right) \cong 10^{90} \tag{2}$$

(see subsequent text), so

$$S_{BH}(tot) \geq 10^{101}, \tag{3}$$

some 20 orders of magnitude smaller than the maximal entropy of our Universe.

The purpose of this exposition is to illuminate the properties of strong gravitational fields. This will be achieved via a discussion of particular processes and phenomena in the vicinity of the event horizon of black holes, on both sides of this horizon.

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2. The Schwarzschild solution and the event horizon

Let us consider the case of mass M as the source of a static and isotropic gravitational field. Then, the geometric properties of the resulting spacetime are determined by the Schwarzschild solution, a metric tensor $g_{\alpha\beta}$. The line element, given in terms of Schwarzschild coordinates, $\{x^\alpha\} = t, r, \theta, \varphi$, is (see [1])

$$ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta = f(r)c^2dt^2 - \frac{1}{f(r)}dr^2 - r^2 d\Omega^2 \tag{4}$$

where $f(r) = 1 - \frac{r_g}{r}$, $r_g = \frac{2GM}{c^2}$ denotes the gravitational radius and $d\Omega^2 = d\theta^2 + \sin^2\theta d\varphi^2$ is a surface element of a unit sphere (we will utilize the system of units such that $c = G = 1$). Solution (2.1) is determined in an empty space outside mass M. Usually, when one deals with a weak gravitational field, the radius R_M of mass M is much larger than its critical radius, $R_M \gg r_g$, then $f(r) \cong 1$. Actually, for the Earth, $r_g(E) \approx 6\ mm$, the strength of the gravitational field is of the order of 10^{-9}; the strength of the solar gravitational field is still very weak, 10^{-6}; but neutron stars yield strong gravitational fields, 10^{-1}. Black holes are the sources of the strongest fields, where an event horizon (defined by $f(r) = 0$) is developed. In such a case, we shall consider that the space outside and inside the horizon is empty—the mass of the black hole is confined at $r = 0$—a singularity. This case will be referred to as an eternal black hole. We shall call the region outside the horizon as the exterior and that inside the horizon as the interior of the black hole.

3. Exterior of the Schwarzschild BH

The meaning of a strong gravitational field is revealed via investigation of the properties of the exterior and then the interior of a BH. It is natural to start from the former region. Let us underline the first, nearly trivial fact that the (relativistic) definition of the gravitational radius as the singularity of the metric (2.1) coincides with a purely classical physics definition of a critical radius such that the escape velocity becomes equal to the speed of light in a vacuum, c (see [4]). The generalization of this observation [4, 5] leads to the conclusion that the speed of a freely falling test particle tends to c, independently of the initial conditions. This and the other properties of the exterior of the event horizon may be described by means of geodesics of both kinds, that is, for massive and massless particles (light rays). The geodesic equations may be derived from the following Lagrangian (see Eq. (4)):

$$\mathcal{L} = f(r)\dot{t}^2 - \frac{1}{f(r)}\dot{r}^2 - r^2\dot{\theta}^2 - r^2\sin^2\theta\dot{\varphi}^2 \tag{5}$$

in a standard manner leading to the Euler–Lagrange equations; $\dot{x}^\mu \equiv \frac{dx^\mu}{d\sigma}$ and σ is an auxiliary parameter. There are two conserved quantities resulting from the symmetry conditions: energy, e (due to time independence of the Lagrangian), and angular momentum, l (due to the invariance of the Lagrangian with φ). The latter condition results in the planar character of geodesic motions, so one may, without loss of generality, choose an equatorial plane, $\theta = \frac{\pi}{2}$ and express these conservation laws as follows:

$$f(r)\dot{t} = e, \tag{6}$$

$$r^2\dot{\varphi} = l. \tag{7}$$

One can determine then arbitrary geodesics from the normalization condition

$$g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = \eta \tag{8}$$

where $\eta = 1$ or 0 for time-like (massive object) geodesics or for light-like (massless object) geodesics, respectively. Indeed, the radial component of the velocity vector, u ($\eta = 1$), or the wave vector, k ($\eta = 0$), takes the form:

$$\dot{r} = \pm\sqrt{e^2 - f(r)\left(\frac{l^2}{r^2} + \eta\right)} \tag{9}$$

Using Eqs. (6)–(9), one can characterize both types of geodesics and illustrate in this way selected features of gravitational fields outside the BH horizon.

Apart from geodesic motions, we will also be employing systems of static observers, SO, whose spatial coordinates are fixed. They are characterized by velocity four-vector,

$$u_{SO} = \left(\frac{1}{\sqrt{f(r)}}, 0, 0, 0\right) \tag{10}$$

3.1. Travel time towards BH horizon

Let us consider the situation of observer A (Alice) whose frame of reference is in a radial free fall, $l = 0$ towards the BH horizon (4). A's frame (or "spaceship") initially was at rest at a Mother Station, MS, located at r_0. The coordinate time to cover the radial coordinate range (r_0, r) in this case is found from Eqs. (6)–(9)

$$t = -\int_{r_0}^{r} \frac{erdr}{\sqrt{\left(r - r_g\right)\left[r_g - r(e^2 - 1)\right]}} \tag{11}$$

It diverges, $t \to \infty$ as A's spaceship approaches the horizon, $r \to r_g$. The proper time, which is the time measured by Alice herself,

$$\tau = -\int_{r_0}^{r} \frac{e\sqrt{r}dr}{\sqrt{\left[r_g - r(e^2 - 1)\right]}} < \infty \tag{12}$$

turns out to be finite. This illustrates a manifestation of the most dramatic time delay: for distant observers (but actually for all observers exterior to the horizon), Alice's frame of reference would need an infinite time to reach the event horizon, while a finite time elapses for the co-moving observer, Alice herself. Another aspect of this outcome has already been mentioned. The speed, V, of the freely falling test particle as measured by a static observer, SO, follows from the expression (see also [5]),

$$u_{SO}U = f\dot{t}\frac{1}{\sqrt{f(r)}} = \frac{1}{\sqrt{1-V^2}} \tag{13}$$

One finds then a general outcome: the speed of a test particle radially freely falling

$$V^2 = \frac{e^2 - f(r)}{e^2} \xrightarrow[f\to 0]{} 1 \tag{14}$$

approaching the event horizon tends to the value of the speed of light in the vacuum. And this result is independent of the initial conditions. One may ask: how would that speed be changing inside the horizon? We discuss this question subsequently.

3.2. Generalized Doppler shift: how to fix the instant of crossing of the Schwarzschild BH horizon

It is a well-known fact that due to the equivalence principle, an observer confined within a frame freely falling towards the horizon cannot identify the instant at which he/she crosses the horizon and if a black hole is large enough, such an observer would harmlessly cross the horizon without even noticing [6]. On the other hand, one can quite precisely determine that instant. How is this seeming contradiction possible?

Before resolving this, let us recollect a well-known result, that of the gravitational frequency shift. In order to do this, one considers radial signals of a fixed frequency, $\overline{\omega}$ emitted at r_0 (the location of the Mother Station) and recorded by a static observer at $r > r_g$. The wave vector k of those radial light rays, $k = \left(k^t, , k^r, 0, 0\right)$ is (see Eqs. (6), (7)):

$$k^t = \frac{\omega}{f}, \tag{15}$$

$$k^r = \pm\omega, \tag{16}$$

where $\pm$ corresponds to out- and ingoing rays, respectively. If MS emits such a signal with frequency

$$\omega^e_{MS} = \frac{\omega}{\sqrt{f(r_0)}} \equiv \overline{\omega}, \tag{17}$$

SO records it at r and measures its frequency as

$$\omega_{SO} = u_{SO}k = \frac{\omega}{\sqrt{f(r)}} \tag{18}$$

so

$$\frac{\omega_{SO}^{r}}{\omega_{MS}^{e}} = \sqrt{\frac{f(r_0)}{f(r)}} \xrightarrow[f \to 0]{} \infty. \tag{19}$$

The frequency recorded by SO is indefinitely blueshifted: when r tends to r_g, $f \to 0$.

When such radial signals are recorded by Alice, $\omega_A = u_A k$, at her instantaneous position at r, then she finds (see Eqs. (6)–(9) and (15, 16)

$$\omega_A^r = \frac{\frac{\omega}{\sqrt{f(r_0)}}}{1+V} \tag{20}$$

where V is the speed of her spaceship as measured by SO (placed at r) (see Eq. (14)).

Exchanging such signals, one can observe a (generalized) Doppler shift of the following form [7]:

$$\frac{\omega_A^r}{\omega_{MS}^e} = \frac{1}{1+\sqrt{\frac{e^2-f(r)}{e^2}}} = \frac{1}{1+V} \tag{21}$$

and

$$\frac{\omega_{MS}^r}{\omega_A^e} = 1 - V \tag{22}$$

The meaning of result (22) is as follows: signals coming from a frame infalling towards the black hole horizon are indefinitely redshifted (and ultimately disappear from the screens/ sensors)—such a journey seems to take infinitely long for external observers. This confirms our former conclusion. The result (21) on the other hand means that the Doppler shift of signals coming from MS allows Alice to identify the horizon quite precisely—the Doppler shift reaches a value of ½ on the horizon.

3.3. Image collision or the 'touching ghosts' anomaly

With the speed of free fall tending to the speed of light in a vacuum, the generalized Doppler shift as characterized by Eqs. (21) and (22) and the dramatic form of the time delay in this case, this leads to yet another anomaly—*image collision* [8] or *touching ghosts* [9]. Signals emitted by Alice located within the infalling frame appear to get "frozen" in the proximity of the horizon (see, however, [10, 11]).

Let us consider another observer, B (Bob), whose spaceship also starts from MS, following Alice's spaceship. Alice and Bob exchange electromagnetic signals; how (when) does Bob perceive the instant of Alice's crossing of the horizon? The answer has been referred to as 'image collision' or 'touching ghosts' and it is as follows [8, 9]. Alice sends an encoded message: a signal that means 'I am crossing the horizon' (at the instant when her Doppler shift is half); Bob receives that message at the instant when he himself crosses the horizon.

An interesting fact is that this effect, originally illustrated by means of Kruskal-Szekeres coordinates, may be interpreted in a general manner, without reference to any specific system of coordinates. Indeed, if Bob received such a message before crossing the horizon, that information would be transmitted to our part of the universe; this would contradict the fact that the horizon crossing can never be observed.

3.4. Photon sphere

In the case of null geodesics in the equatorial plane, the wave vector components are as follows:

$$k^t = \frac{\omega}{f}, \; k^\varphi = \frac{l}{r^2} \tag{23}$$

$$k^r = \pm\sqrt{\omega^2 - f\frac{l^2}{r^2}} \equiv \pm l\sqrt{\frac{1}{b^2} - V_{eff}(r)} \tag{24}$$

where b is a so-called impact parameter. The function $V_{eff}(r) = f\frac{r_g^2}{r^2}$ is regarded as an 'effective potential' for null geodesics (see **Figure 1**). The shape of null geodesics depends on the value of b. The deflection angle

$$\int d\varphi = \pm\int \frac{dr}{r^2\sqrt{\frac{1}{b^2} - V_{eff}(r)}}$$

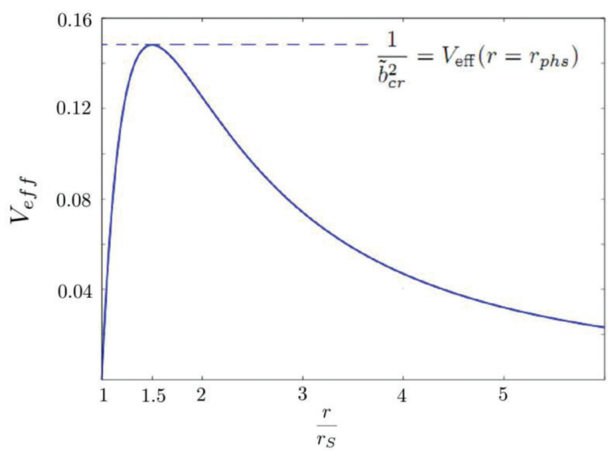

Figure 1. Effective potential $V_{eff}(r) = f\frac{r_g^2}{r^2}$ in the case of Schwarzschild spacetime (horizontal axis—r expressed in units, $r_S \equiv r_g = 2M$).

is small for large values of b—light rays are only slightly deflected. It grows indefinitely as the impact parameter value tends to its critical value, $b_{cr}^{-1} = V_{eff}\left(r_{phs}\right)$. The impact parameter b_{cr} corresponds to the so-called 'photon sphere' composed of circular trajectories, $r = r_{phs}$,

$$r_{phs} = \frac{3}{2} r_g \equiv 3M \tag{25}$$

which are (unstable) null geodesics:

$$\left(k^t, 0, 0, , k^\varphi\right) = \left(\frac{\omega}{f\left(r_{phs}\right)}, 0, 0, \frac{l}{r_{phs}^2}\right) \tag{26}$$

3.5. The shape of light cones

It should be noted that in approaching an event horizon, the shape of a light cone evolves in a characteristic manner. Indeed, observing radial in- and outgoing signals

$$ds^2 = f dt^2 - \frac{1}{f} dr^2 = 0 \tag{27}$$

one finds,

$$\frac{dr}{dt} = \pm f \xrightarrow[r \to r_g]{} 0 \tag{28}$$

which may be illustrated as a sequence of vanishingly narrow cones.

4. Interior of Schwarzschild BH

In order to describe the interior of the horizon of the Schwarzschild spacetime, one can follow an approach proposed by Doran et al. [12]. These authors showed that discussing the problem of an empty, but dynamically changing spacetime, one finds, using specific boundary conditions, the metric (4) of the interior of the Schwarzschild spacetime, that is,

$$ds^2 = \left(1 - \frac{2M}{r}\right) dt^2 - \frac{1}{\left(1 - \frac{2M}{r}\right)} dr^2 - r^2 d\Omega^2 \tag{29}$$

for $r < r_g = 2M$ (see also [13]). This means that formally one can use Schwarzschild coordinates also for the interior of the horizon, but then one must remember about the exchange of the roles of the t and r coordinates. Inside the horizon, r plays the role of a temporal coordinate: it changes from r_g to 0 and $dr < 0$; t plays the role of a spatial coordinate, changing between $-\infty$ and $+\infty$ with dt taking both positive and negative values. The important consequence is a

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change of the symmetry of the system: instead of a static, spherically symmetric spacetime, one encounters a homogeneous, spherically symmetric and dynamically changing spacetime; energy is no longer conserved but (due to the homogeneity along the t-axis), appropriately, the t-momentum component is conserved.

Therefore, one can consider spacetime (29) as representing the interior of a Schwarzschild black hole. Accordingly, analogues of the phenomena described above outside the horizon will be analyzed.

First, one introduces a class of resting observers, RO, that is, those, whose spatial coordinates, t, θ, φ are fixed. Then, the velocity u_{RO} four vector's only nonvanishing component is a temporal one,

$$u_{RO} = -\sqrt{-f}\,\partial_r. \tag{30}$$

The class of infalling test particles located in Alice's frame of reference is described in the same way as given outside the horizon (Eqs. (6)–(9))—in this case, however, $r < r_g = 2M$, so $f < 0$. In this region, ingoing ($-$) and outgoing (+) null geodesics (that are planar) described as

$$f\frac{dt}{d\sigma} = \pm\omega \qquad r^2\frac{d\varphi}{d\sigma} = l \qquad \frac{dr}{d\sigma} = -\sqrt{\omega^2 - f\left(\frac{l^2}{r^2} + \eta\right)} \tag{31}$$

differ from their counterparts outside the horizon by a small but important feature—the $\pm$ sign is designated to a spatial coordinate, namely the r-coordinate outside the horizon and the t-coordinate inside the horizon. Having said this, one may now discuss specific effects (see [14, 15]).

4.1. The speed of an infalling test particle

A test particle located in A's framework (Eqs. (6)–(9)), $l = 0$, is freely falling FF. Then, a resting observer (30) measures its (squared) speed $\tilde{V}^2$ as follows:

$$u_{RO}U_{FF} = -\frac{1}{f}\sqrt{-f}\sqrt{e^2 - f} = \frac{1}{1-\tilde{V}^2}. \tag{32}$$

One finds then that (c.f. Eq. (14))

$$\tilde{V}^2 = \frac{e^2}{e^2 - f}. \tag{33}$$

This is, at first sight, a rather unexpected outcome: the speed is given by a formula inverse to the one obtained outside the horizon, Eq. (14). Another aspect of this result is revealed when one illustrates the speed outside and inside the horizon as measured by observers that are static, SO, and resting, RO (30), respectively (see **Figure 2**).

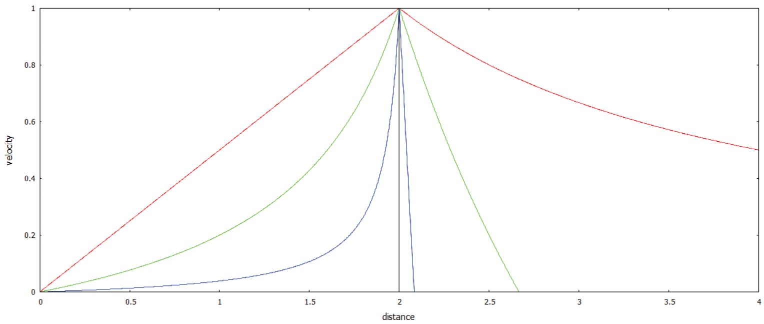

Figure 2. Values of 'velocity' V^2 measured by SO (outside horizon) and $\tilde{V}^2$ by RO (inside horizon) of different test particles in the Schwarzschild spacetime. The red curve corresponds to $e = 1$, $\tilde{V}^2 = \frac{r}{r_g}$, the green curve to, $e = 0.5$ and the blue one to $e = 0.2$. The vertical line represents the horizon located at $r_g = 2$ (horizontal axis—r expressed in units M).

4.2. The Doppler shift

Let us consider an analogy of the generalized Doppler effect inside the horizon.

4.2.1. Frequency shift of signals coming from MS

4.2.1.1. Resting observers

One can start from an analogy of the gravitational frequency shift: a resting observer (30) records radially incoming signals coming from the Mother Station. Then, according to Eq. (18), the frequency shift is

$$\frac{\omega^r_{RO}}{\omega^s_{MS}} = \sqrt{\frac{f(r_0)}{-f(r)}} \rightarrow \begin{cases} \infty & r \rightarrow r_g \\ 0 & r \rightarrow 0 \end{cases} \tag{34}$$

One finds then that the gravitational frequency shift of the signals coming from MS and recorded by static, SO, and resting, RO, observers, outside and inside the horizon, respectively, as having a symmetric form with respect to the horizon itself (see Eqs. (19) and (34)).

4.2.1.2. Freely falling observers

The frequency shift of signals coming from MS and recorded by Alice, who is radially freely falling, is

$$\frac{\omega^r_{A}}{\omega^s_{MS}} = \frac{1}{1 + \sqrt{\frac{e^2 - f}{e^2}}} \rightarrow \begin{cases} \frac{1}{2} & r \rightarrow r_g \\ 0 & r \rightarrow 0 \end{cases}. \tag{35}$$

Expression (35) is the same as its counterpart outside the horizon (21): it turns out that the frequency shift is a continuous and decreasing function from 1 to 0 during the trip through the horizon; as emphasized earlier, with the factor $\frac{1}{2}$ marking the horizon (see **Figure 3**)

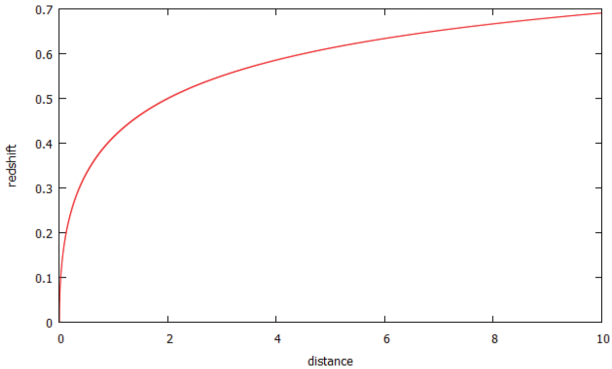

Figure 3. Monotonic and continuous change of the frequency ratio $\frac{\omega_A^r}{\omega_{MS}^s}$ (redshift—Vertical axis) outside and inside the horizon (horizontal axis—r expressed in units M, $r_g = 2$).

4.2.2. *Frequency shift of signals inside the horizon of BH*

One can consider the exchange of signals by observers at rest inside the horizon. One can distinguish two types of signals: going along the direction of homogeneity, that is, the t-axis, and signals propagating perpendicularly to this axis.

4.2.2.1. *Signals propagating along the t-axis*

The frequency shift of signals exchanged by two observers at rest at t_1 and t_2 depends on the emission instant, r_1 (recording instant r_2 is fixed by the distant t_1, t_2):

$$\frac{\omega^r(t_2)}{\omega^e(t_1)} = \frac{\sqrt{-f(r_1)}}{\sqrt{-f(r_2)}} \xrightarrow[r_2 \to 0]{} 0. \tag{36}$$

One finds that in this case, the frequency redshift tends to zero at the ultimate singularity (see **Figure 4**).

4.2.2.2. *Signals propagating perpendicularly to the t-axis*

The wave vector of signals propagating perpendicularly to the t-axis has two non-vanishing components $k = k^r \partial_r + k^\varphi \partial_\varphi$ (because of the planar character of the trajectory, one can choose $\theta = \frac{\pi}{2}$, i.e. the equatorial plane). Then, the frequency shift, for two static observers placed within this plane perpendicular to the t-axis, is given by

$$\frac{\omega^r(\varphi_2)}{\omega^e(\varphi_1)} = \frac{r_1}{r_2} \xrightarrow[r_2 \to 0]{} \infty. \tag{37}$$

One finds an indefinite blueshift at the ultimate singularity (see **Figure 5**).

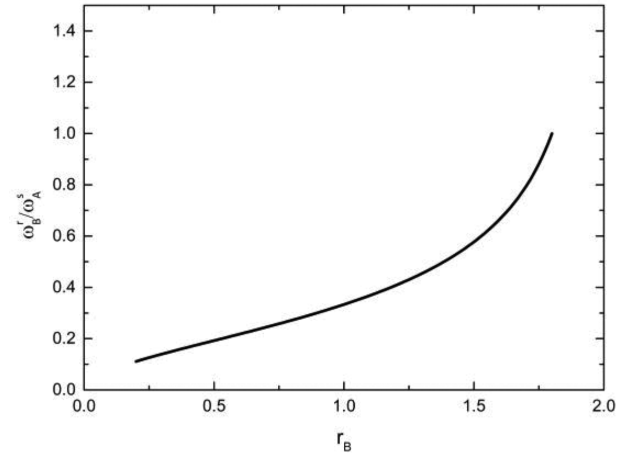

Figure 4. Frequency redshift (vertical axis) for the signal propagating along homogeneity direction between instants, $r_B = 0.9r_g = 1.8$ and $r_A = 0.2r_g = 0.4$ as a function of r (horizontal axis—In M units, $r_g = 2$).

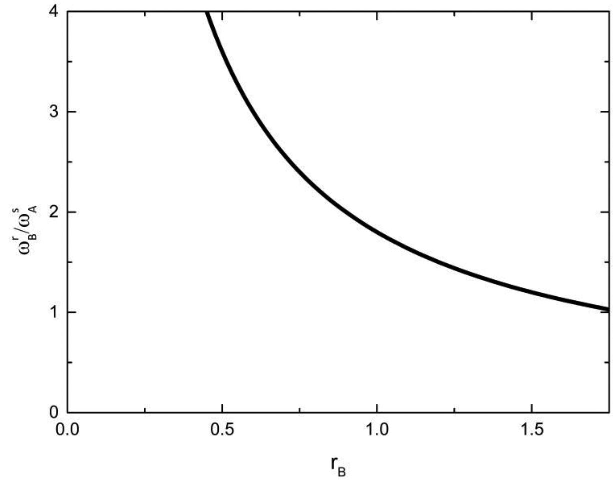

Figure 5. Frequency blueshift (vertical axis) for the signal propagating perpendicularly to the homogeneity direction, between instants, $r_B = 0.9r_g = 1.8$ and $r_A = 0.3r_g = 0.6$ as a function of r (horizontal axis—In units M, $r_g = 2$).

4.3. Photon sphere analogue

Null geodesics propagating perpendicularly to the t-axis resemble trajectories belonging to the photon sphere. Indeed, they are determined by the wave vector,

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$$(0, k^r, 0, , k^\varphi) \equiv \left(0, -\sqrt{\frac{-1}{f}\frac{l^2}{r^2}}, 0, \frac{l}{r^2}\right) \tag{38}$$

having only one spatial component, the angular component, k^φ corresponding to a circular-like motion. There is one significant feature distinguishing these null geodesics from the circular trajectories of radius $r = r_{phs}$ outside the horizon: they circulate over a sphere of an ever-decreasing radius. One can see from the null condition:

$$-\frac{1}{f}dr^2 - r^2 d\varphi^2 = 0 \tag{39}$$

that the rate of change of the radius of such a sphere is proportional to r, which is a temporal-like (decreasing) coordinate. Therefore, one finds inside a black hole an interesting phenomenon: a photon sphere analogue. Outside the horizon, a light ray belonging to the photon sphere can (in principle, as it is a circular trajectory of unstable equilibrium) unwind infinitely many times. One can ask then: inside a black hole, how many times can a light ray orbit along a photon sphere analogue before reaching the ultimate singularity?

The answer to this question is quite unexpected: it is exactly *a single half rotation*.

Indeed, by using Eq. (39), one obtains

$$\Delta\varphi = \int \frac{dr}{r\sqrt{\frac{-f}{r^2}}} = \int_0^{2M} \frac{dr}{\sqrt{r(2M - r)}} = \pi \tag{40}$$

This means that the angle traversed by a light ray is equal in this case to π. A general property is that the deflection angle for a light ray within the BH horizon cannot exceed π.

5. The horizon of a Schwarzschild BH

Among various interesting properties of the Schwarzschild BHs horizon, there are at least two that are relevant to our discussion.

The first relates to the speed of an object crossing the horizon. As described earlier, the value of the speed of Alice's spaceship tends to the value of the speed of light c as it approaches the horizon. Does that speed take the value c on the horizon? There are no observers residing on the horizon, but other observers, crossing the horizon, would in principle be able to perform such a measurement. Performing this kind of thought experiment, one obtains the following: the speed of Alice's spaceship crossing the BH horizon is less than the speed of light. The value of that speed depends on the initial conditions.

The second is linked to any outgoing light ray trapped at the horizon. It may be a signal emitted by Alice at the instant she was crossing the horizon with the encoded message: 'I am crossing the horizon now'. If it was a signal of some specific frequency, what would be its frequency as recorded by Bob, when he crossed the horizon? It turns out that such a signal 'ages': it is redshifted and the value of the redshift becomes greater as the original distance between Alice and Bob increases [15].

6. The meaning of a strong gravitational field

Let us underline the rather unexpected and counterintuitive observations that accompany the presence of the event horizon of a Schwarzschild BH. The strange intimate symmetry of the outer versus inner region: static observers outside the horizon and observers at rest inside the horizon measuring the Doppler shift of signals incoming from MS would record basically the same outcomes. The speed of a test particle falling towards the BH appears to be impeded after crossing the horizon. As described elsewhere, the speed of a test particle uniformly accelerated inside the horizon after reaching its maximal value starts to diminish. A null geodesic follows exactly half a circular orbit within the horizon. Signals exchanged within the horizon seem to mimic the cosmological model expanding along one specific direction and contracting perpendicularly to this direction. All of these are manifestations of the presence of such a strong gravitational field that the event horizon of the BH is developed.

Inside the horizon of a Schwarzschild BH, one comes across a unique phenomenon: an interchange of the roles of the r and t coordinates. Outside the horizon $r > r_S$, the radial coordinate is an ordinary spatial coordinate, which may change from r_S to ∞ in both directions, $dr = \pm|dr|$ and the time coordinate t is a temporal one, that is, such that, $dt > 0$. Inside the horizon $r < r_S$, and coordinate r becomes a temporal one: r changes from r_S to 0 and $dr < 0$; coordinate t then plays the role of a spatial coordinate: $-\infty < t < \infty$ and $dt = \pm|dt|$.

Such an interchange results in a dramatic difference of the symmetry properties of the spacetime. As mentioned earlier, the Schwarzschild spacetime outside the horizon is static, independent of time and isotropic; this results in the conservation of energy and angular momentum, respectively. Inside the horizon, the spacetime is still independent of t but this is now a spatial coordinate in that spacetime, leading to t-component momentum conservation; it is no longer static but instead dynamically changing, being r-dependent. Inside the horizon of the Schwarzschild BH, spacetime is cylindrical-like, homogeneous along the t-axis and spherical-like, of radius r perpendicularly to this axis (see also [2, 16]).

All of this presents the above-seemingly unexpected or counterintuitive phenomena in a new perspective. The speed of the infalling test particle is measured as 'distance'/'time' so the interchange of the roles of 'distance' and 'time' leads to the inverse expressions to those exterior to the horizon, V and interior, $\tilde{V}$; hence, the speed turns out to decrease inside the horizon. The cylindrical-shape BH interior is a dynamically changing spacetime: expanding along the t-axis and contracting perpendicularly to this axis. This results in both red- and blueshifts, respectively [12, 17]. Hence, it actually *is* a realization of a specific *cosmology*. The fact that light rays propagating perpendicularly to the cylinder axis occupy a semicircular photon sphere analogue is found to have a deeper significance [18]. The same value π is found

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for other kinds of black holes, and this appears to be a fundamental discovery; it may be a symmetry property linked to a 'new physics' of black holes [19]. Also, other observations may need deeper analysis but, whatever the interpretation, they are caused by the strong gravitational fields that form the BH horizon.

Let us emphasize that the common sense property of the BH, namely. 'nothing, not even light can leave their interior' takes on a new sense now: crossing the event horizon, a test object can never reach it again as this would mean travelling backwards in time.

There is a more formal interpretation of the interchange of the role of radial and temporal coordinates in the theory of relativity. The Killing vector representing time independence symmetry, being time-like outside the horizon becomes space-like inside the horizon—this actually means that the time-like component of the momentum four vector is converted into a space-like momentum component, respectively. This opens the door for radiation emitted by black holes—Hawking radiation.

7. Astrophysical black holes

Generations of thermonuclear reactions support stars against gravitational collapse [3, 20]. The first stage is a process of hydrogen burning to make helium. When a substantial amount of hydrogen is exhausted, gravitational contraction raises the temperature until helium burning, the so-called triple alpha process, can start. This evolution eventually leads, for massive stars, to the last stage where an element with the largest binding energy per nucleon, ${}^{56}_{26}Fe$, is produced. What happens then?

One can consider the state of a star of mass M and radius R, which exhausted its thermonuclear fuel, $T = 0$. It is supported by a nonthermal pressure, due to the fermionic nature of electrons, protons and neutrons. There are two competing contributions to the energy of such an object. A negative one arises from a gravitational origin

$$E_g \propto -\frac{M^2}{R} \tag{41}$$

and a positive one, the kinetic energy of the electronic gas:

$$E_k \propto nR^3 \langle E \rangle \tag{42}$$

where n denotes the density of electrons and $\langle E \rangle$ is the electronic mean energy. Taking the following relation between the characteristic electron momentum, p_F, and the corresponding wavelength, $\lambda \propto n^{-1/3}$,

$$p_F \propto \lambda^{-1} \propto n^{1/3} \tag{43}$$

one obtains for a nonrelativistic range of energies, $\langle E \rangle \propto p_F^2$

$$E_k \propto \frac{M^{5/3}}{R^2}. \tag{44}$$

It appears that the kinetic energy term dominates in the range of decreasing values of R, preventing further contraction. However, for more massive stars, higher energies are available and the electrons would be regarded as relativistic, $\langle E \rangle \propto p_F$ and then,

$$E_k \propto \frac{M^{4/3}}{R} \tag{45}$$

In such a case for a mass M larger than the Chandrasekhar limit, $M_{WD} \approx 1.4 M_\odot$ (for white dwarfs), the pressure of the electron gas could not support a star against its gravitational contraction.

For even more massive stars, one comes across inverse beta decay leading to the formation of a neutron star core. In such a case, the Pauli exclusion principle, this time for neutrons, prevents gravitational collapse, up to some specific limit, $M_{cr} \propto 2-3\, M_\odot$. For masses larger than this limiting case, nothing can stop the ongoing gravitational collapse eventually leading to a singular state of matter—a black hole.

8. Entropy and Hawking radiation

In early 1970s, it was indicated by Bekenstein [21] and Hawking [22] that BH entropy is proportional to their surface area:

$$S = k_B \frac{4\pi r_S^2}{4 l_P^2} = k_B \frac{4\pi M^2}{l_P^2} \tag{46}$$

where l_P denotes the Planck length and k_B Boltzmann's constant. It was also recognized that BHs may be regarded as simple thermodynamic systems (the black hole 'no hair' theorem) characterized by three parameters, mass M, angular momentum J and charge Q. Accordingly, one can identify four different kinds of black hole: Schwarzschild (nonrotating and uncharged, characterized by their mass M), Reissner-Nordstrom (charged but nonrotating, characterized by M and Q), Kerr (rotating, characterized by M and J) and Kerr-Newman (rotating and charged, characterized by M, J and Q). In the case of Schwarzschild spacetime, one can apply a simple thermodynamic formula [21, 22].

$$dU = TdS \tag{47}$$

and identifying $U = M$ to determine the BH temperature T_{BH} as being proportional to its inverse mass,

$$T_{BH} = \frac{\hbar c^3}{8\pi M k_B G} \tag{48}$$

where we use standard notation. It was Hawking's idea that black holes should lead to a new kind of uncertainty [23], other than the one having a quantum mechanical origin. When matter or radiation falling in towards a black hole crosses its horizon, the information it carries is inevitably lost. This led to two controversies. Firstly, information itself is lost. Secondly, one can consider black hole formation due to the gravitational collapse of matter (or radiation) as the unitary evolution of a pure quantum state. After the formation of the horizon, further evolution has to be regarded in terms of mixed states due to the loss of information. This means the breakdown of quantum mechanical predictability. Both elements of such an information problem, loss of information and breakdown of unitary quantum evolution, were objected to from the very beginning.

Hawking himself [24] formulated the idea of black hole decay. Due to the existence of an event horizon and the conversion of one of the Killing vectors from a temporal to a spatial one, a pair of entangled particles, one of positive and one of negative energy, would be created in the proximity of the horizon. Two scenarios are then possible when one (the one with negative energy) or both of the particles fall behind the horizon. The point is that the particle with negative energy could not 'survive' in our part of the Universe for fundamental reasons, but it could exist within the horizon. This is so because the energy, the t-component of the particle momentum vector within the horizon, takes on a spatial character, so it might then be either positive or negative. Hence, one of the particles, the one with positive energy, departs to infinity, being recorded as Hawking radiation and the other member of the pair, with negative energy, falls behind, 'tunnelling through' [25] the horizon and reducing the BH mass. This is the meaning of BH evaporation. Hawking evaporation is the radiation of a black body of temperature, T_{BH} (8.2).

Therefore, BHs turn out to be evaporating nonequilibrium systems with a decay time

$$t_{BH} \cong 10^{74} \left(\frac{M}{M_{\odot}} \right)^3 \tag{49}$$

fifty seven orders of magnitude larger than the age of the Universe for moderate BH masses M. According to the generalized second law of thermodynamics, the entropy during evaporation is an increasing function of time. Indeed, during evaporation, the BH entropy decreases,

$$dS_{BH} = -\frac{dU}{T_{BH}} \tag{50}$$

yet the entropy of the respective BH radiation is larger by one-third [26].

$$dS_r = \frac{4}{3} \frac{dU}{T_{BH}}. \tag{51}$$

One may suspect that information lost due to the presence of the horizon may be retrieved due to evaporation, thus restoring this fundamental aspect of quantum mechanical unitary evolution [27–29]. A closer scrutiny shows that this is not so obvious: at the initial stages of the BH decay, both BH and radiation are close to their maximally mixed states, thus no information is retrieved. Although the process of releasing information might be of a non-perturbative character, the information problem (referred to as the information paradox) still remains unsolved. It was indicated that smooth quantum mechanical unitary evolution should lead to the breakdown of the smoothness of the proximity of the event horizon, leading to a 'firewall' [30]. This concept was objected to in more recent papers [31–33]; nevertheless, the information paradox is still far from being removed. It may currently be formulated in many different ways and one of those ways can be expressed as follows:

Hawking radiation consists of particles born as entangled pairs; those recorded far away are then entangled with a diminishing BH. Finally, the BH disappears. What, then, are those particles recorded at distant locations still entangled with [34]?

9. Final remarks

The purpose of this presentation was to illustrate selected features of strong gravitational fields. Black holes are the sources of the strongest gravitational field in the sense that an event horizon has developed. Let us briefly consider the point '*black but not dark'.* The presence of black holes may be recognized primarily due to gravitational interactions: the dynamics of their environment. In this sense they may be regarded as a component of a dark matter sector. Accepting such an oversimplified or naïve point of view for a while, one may ask about the character of this component. Partly, the answer is obvious: this is baryonic matter, as massive stars collapsing into black holes are composed of baryonic matter. But due to instability, there are no extremely massive stars, so BHs of millions of solar masses have a different origin (eternal black holes), so they might not necessarily be composed of baryonic matter. In principle, as they evaporate, they emit radiation; also, they could be charged so they could therefore affect their environment not only gravitationally. Hence, although they are black they are hardly 'dark'. As mentioned at the beginning of this exposition, BHs constitute a small fraction of the density of baryonic matter, so they are interesting objects in the Universe rather for the local properties imposed by their gravitational field, than for other reasons (at least so it seems to us at the moment).

The outcome of the presence of the horizon of the BH is a dramatic difference in the symmetry properties of the exterior and interior of the BH. Energy conservation related to the time-like Killing vector is changed into a corresponding momentum component conservation as the Killing vector is converted into a space-like one. That is a consequence of the static spacetime outside the horizon being transformed into a homogeneous one, along the t-direction, but it also becomes a dynamically changing spacetime inside the horizon: expanding along the homogeneity direction and contracting perpendicularly to that direction. On the one hand, this leads to the information paradox. On the other hand, the presence of the BH's event horizon may be interpreted as an interchange of the roles of the time and radial coordinates. And this leads to unexpected scenarios, with some surprising processes and phenomena taking place outside the horizon yet with even more striking properties of the interior of the horizon. It should be underlined that the discussion presented here has dealt mostly with eternal BHs, which have not been created due to gravitational collapse but rather have existed forever (since the Big Bang). However different these may seem, they have a lot in common. They both decay due to Hawking radiation [2]; as suggested by various authors [16], the interior of gravitationally collapsing black holes is also of a cylindrical shape, and both eternal and collapsing BHs share one more common but bizarre property, their volume is *infinite* [16, 35]. Hence, though it is not guaranteed that the interiors are the same their properties might turn out to be quite similar. But there is a still a deeper problem of a much more fundamental character: could the interior of black holes be described by the approach presented here? Or more specifically, could a very strong gravitational field, inside the BH horizon, be described in terms of the theory of relativity? Or is a new physical approach necessary, as emphasized by G. t'Hooft [19] (see also [36]) involving quantum mechanical aspects also? As usual, the answer will come in time, but even if the answer is satisfactory, in this case, it will probably never be the final word.

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