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Developing lean and responsive supply chains: A robust model for alternative risk mitigation strategies in supply chain designs

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ABSTRACT

This paper investigates how organization should design their supply chains (SCs) and use risk mitigation strategies to meet different performance objectives. To do this, we develop two mixed integer nonlinear (MINL) lean and responsive models for a four-tier SC to understand these four strategies: i) holding back-up emergency stocks at the DCs, ii) holding back-up emergency stock for transshipment to all DCs at a strategic DC (for risk pooling in the SC), iii) reserving excess capacity in the facilities, and iv) using other facilities in the SC’s network to back-up the primary facilities. A new method for designing the network is developed which works based on the definition of path to cover all possible disturbances. To solve the two proposed MINL models, a linear regression approximation is suggested to linearize the models; this technique works
based on a piecewise linear transformation. The efficiency of the solution technique is tested for two prevalent distribution functions. We then explore how these models operate using empirical data from an automotive SC. This enables us to develop a more comprehensive risk mitigation framework than previous studies and show how it can be used to determine the optimal SC design and risk mitigation strategies given the uncertainties faced by practitioners and the performance objectives they wish to meet.

Keywords: supply chain management; network design; risk management; robust optimization; responsiveness.

1. INTRODUCTION

In order to compete effectively and manage costs organizations must work out how best to leverage competencies, resources and skills across their supply chains (SCs). The first step is to identify the optimal SC network design (SCND) by deciding which products, processes or services to make or buy, where to source them from, how much capacity to build or reserve in each operation and how to distribute goods or services between them (Xu et al., 2008; Hill and Hill, 2013). SCND is a strategic decision but has direct and indirect impacts on tactical and operational decisions. However, as these SCs become more decentralized, global and dependent on particular suppliers within the chain, they become more vulnerable to supply and demand uncertainties. Managing these uncertainties become increasingly difficult when organizations are also trying to reduce inventory, deliver products with short lifecycles and support customers demanding short lead-times (Norrman and Jansson, 2004; Hill et al., 2011).

Some of the information required to manage a SC will always be uncertain, for example market demand, price and available production, distribution, manpower and energy capacity. This uncertainty can have either a positive or negative impact on the chain (French, 1995; Zimmermann, 2000; Roy, 2005; Stewart, 2005), but risk starts to develop as the probability of a negative impact increases (Klibi et al., 2010). Organizations managing large, global and complex chains such as those for automotive, electronics and consumer appliances need to proactively manage risk as it starts to become more common (Chopra and Sodhi, 2004; Pan and Nagi, 2010; Schmitt, 2011). For example, a recent lightning strike in Albuquerque, New Mexico caused an electrical surge that led to a fire in a Philips plants in the region destroying millions of microchips due to be supplied to Nokia and Ericsson. Nokia’s multi-sourcing strategy enabled it to immediately start buying chips from alternative suppliers, but Ericsson’s single-sourcing agreement with Philips meant it incurred a loss of $400 million as production was delayed for several months (Chopra and Sodhi, 2004).

In order to mitigate risks, SC managers impose appropriate strategies. They must detect the possible risks in their system, predict the possible outcomes and try to use appropriate proactive and reactive strategies. Leading manufacturers such as Toyota, Motorola, Dell, Ferrari and Nokia excel at identifying such risks in their systems and neutralizing their negative effects (Chopra and Sodhi, 2004). Some researchers like Chopra and Sodhi (2004), Tomlin (2006) and Waters (2007) suggest comprehensive classification frameworks for SC risk management strategies. Risk classifications and corresponding risk mitigation strategies in all of these sources, more or less, are the same in nature. Referring to these frameworks, the motivation of this paper can be summarized as follows:
All of SC risk management papers investigate only one or rarely two of the following risk mitigation strategies such as: emergency stock, excess capacity, substitution of facility, etc. This research focuses on several of these strategies together and opens the hand of the designer in selecting the least costly combination of these strategies to neutralize the negative effects of risks.

Another motivation of this research is that all previous research papers have considered supply risk either in the production facilities such as suppliers or transportation links; this paper considers both.

In this paper, two SCND models are considered in the presence of demand and supply-side risks: i) with stochastic market demands and ii) with disruption probability of the supply process. The SC comprises several suppliers, a manufacturer, distribution centers (DCs) and retailers. Each retailer orders its goods; DCs integrate the orders and pass them to the manufacturer. The goods produced by the manufacturer are delivered to retailers through DCs. Disruption in the supply process can occur for two reasons: a) disruption in suppliers or b) disruption in connecting links among the facilities. The made decisions are the number, location and capacity of facilities in the SC's echelons and material/product flow throughout the SC.

The contribution of this research is summarized as follows:

- Modeling SCND problem using path-based formulation,
- Incorporating four different risk mitigation strategies in modeling the problem,
- Including leanness and responsiveness philosophies in modeling the problem,
- Making the model robust by considering several robustness indices,
- Implementing the model on a case example of the automotive industry.

The remainder of the paper is organized as follows: The literature review is presented in section 2. Section 3 describes details of the problem; in section 4 we mathematically model the problem for lean and responsive SCs. Section 5 explains an approximation method to linearize the non-linear parts of the model and in section 6 we apply the proposed model as well as the solution approach to the real-life case. Section 7 discusses the computational results of solving the real-life case. Section 8 applies a robust optimization concept to the model. The paper will be concluded in section 9.

2. LITERATURE REVIEW

2.1. Supply chain risks and mitigation strategies

Every SC has a number of internal (within the chain itself such as impairment of the chain's facilities, disruption of links connecting the facilities, labor strikes, etc.) and external (in the external environment in which the SC operates such as flood, earthquake, supplier bankruptcy, fire, war, terrorist attacks, etc.) vulnerabilities that can potentially delay, disrupt or impair the quality of the material or information flowing through the chain (Chopra and Sodhi, 2004; Neiger et al., 2009). For example:

- **Delay risk** - material or information flow can be delayed by a production failure, system breakdown or a supplier’s inability to respond quickly to a change in demand
- **Disruption risk** - material or information flow can be disrupted by unplanned supply, demand or transportation changes within the chain. For example, a natural disaster (such as an earthquake or flood) or a manmade disaster (such as a fire, war, labor strike, terrorist attack or supplier
bankruptcy).

- **Quality risk** - for example, products might become damaged in production or transportation (material quality) due to factors such as poor supplier knowledge, capability or decision making within the chain. This can lead to incorrect information flowing through the chain and customer requirements not being met.

- **Forecast Risk** - mismatching between the actual demand of the market and a firm’s prediction leads to forecast risk (Chopra and Sodhi, 2004).

Various authors have suggested that organizations can use a number of strategies to mitigate against these risks such as:

- **Emergency stock** - to ensure material flow is not disrupted (Hill and Hill, 2012). For example, Cisco hold emergency stock for low-value, high-demand components manufactured overseas and the USA hold large petroleum reserves to protect the country from potential supplier disruption. As another instance, in 2001, the bankruptcy of UPF-Thompson, the chassis supplier for Land Rover, caused a major problem for the Ford Motor Company. Companies usually hold reserve inventory or redundant suppliers to mitigate the effects of these disruptions (Chopra and Sodhi, 2004).

- **Excess capacity** - holding excess machine, labor or facility capacity within the chain that can be easily initiated or tapped into when a disruption occurs. For example, Toyota employs a special team who can work on all stations (Chopra and Sodhi, 2004).

- **Substitute supplier or facility** - to ensure there are multiple options for particular product or service within the chain (Yang et al., 2009). For example, Motorola use a multi-supplier strategy for procuring high value products (Chopra and Sodhi, 2004).

- **Supplier development** - to increase the process stability of suppliers within the chain and the flow of information across the chain (Hill and Hill, 2012)

- **Profit-sharing** - to increase the financial stability of suppliers within the chain (Babich, 2008; Schmitt et al., 2010)

- **React quickly** - proactive strategies are often very expensive and difficult to implement. Therefore, companies may choose to ‘react quickly’ instead and put in strategies to help them do this such as creating flexible processes and capacities within the chain that can be easily initiated when a disruption occurs (Chopra and Sodhi, 2004).

### 2.2. Modeling supply chain risk

Table 1 summarizes the types of mitigation strategies, disruption and uncertainty that have been modeled within the literature for different numbers of products, planning periods and network flows within a SC. In all of these research works, at least one of the sources of uncertainty is considered in designing the network structure. This table classifies this research in terms of strategies for risk mitigation (columns 2-5), number of products (column 6), number of planning periods (column 7), types of disruption (columns 8-13), types of uncertainty (columns 14-16) and flow direction in their network (columns 17-18). The last row of the table illustrates the contribution of this research, which is considering several risk mitigation strategies and
transport and supply disruptions. According to this table, all possible strategies of risk mitigation in previously published papers are as follows: emergency stock, safety stock, excess capacity, and substitution of facility. Except Romeijn et al. (2007), the other research publications consider only one of these four strategies. Additionally, this table shows all previous research papers consider either supply risk or transport risk.

In Table 2, solution techniques used in SCND and risk literature are categorized. In the table, components of the objective functions (column 2-15), the structure of the studied SC (column 16-25) and the applied solution techniques (column 26) are depicted. In the last row of the table, details of the model and solution technique used in this research are shown.

According to Table 2,

- The number of tiers is usually 2 or 3; like in Longinidis and Georgiadis (2013) and Tabrizi and Razmi (2013). In this paper, we consider a 4-tier SC which is larger, more realistic and easier to be generalized.
- Cost structure in most of the research papers consider some of these cost components: reserve capacity cost, extra inventory cost, shortage cost, purchase/manufacturing/operating/etc. cost, transportation cost, inventory holding cost, and cost related to quality/capacity expansion of facilities. We consider all of them in our problem.

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Table 1. Type of problems solved in the literature of SCND and risk literature.
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This paper
Considering risk in SCND has drawn researchers’ attention over recent years. Trkman and McCormak (2009) present a new method for identifying and predicting SC risk. In the past, researchers considered fairly simple structures for SCs when solving a SCND problem against risks. Qi et al. (2009) consider a simple chain with predetermined network structure consisting of a retailer and a supplier. The retailer of this chain faces a random internal disruption and a random external disruption from its supplier. The authors formulate the expected inventory holding cost of this chain and numerically solve it to determine the optimal ordering size from the supplier. Schmitt et al. (2010) consider a similar network structure. They investigate the effect of disruption of suppliers and demand uncertainty in setting an optimal inventory system for the SC. Hsu and Li (2011) consider the SCND problem of a manufacturer with several plants in different regions. Their model determines the location and capacity of plants and the production plan through the SC, so as to minimize the average per unit product cost. Xanthopoulos et al. (2012) consider a two echelon SC composed of a wholesaler and two unreliable suppliers. They investigate the trade-off between inventory policies and suppliers’ disruption risk. They optimize the ordering size of the wholesaler before realization of the actual demand so as to maximize the expected total profit.

Modeling demand uncertainty to create an agile SC and using this problem through robust optimization technique is another recent trend in SCND. Pan and Nagi (2010) consider an agile SCND problem in which the market demand is uncertain. They only consider the demand-side uncertainty in their problem. Park et al. (2010) consider the SCND problem in which demand variation is considered as a random variable with normal distribution. They use risk pooling of strategic reserves as the risk mitigation strategy. Mirzapour et al. (2011) propose a robust nonlinear mixed integer model to deal with the aggregate production planning problem of a SC with predetermined network structure. They consider the uncertainty of cost parameters and demand in their modeling.

It can be seen that several works have been done in the field of risk management in SCs by using inventory holding strategy to resist possible disruptions. In these problems, in addition to cyclic stock ordered each period to procure the regular demand and safety stock held to cover probable changes of demand, another kind of inventory is kept which is called emergency reserve or stock. Sheffi (2001) calls this just-in-case inventory, and indicates that firms understand the importance of this inventory after the September 11 terrorist attack. Tomlin (2006) investigates the impact of strategic reserve and a back-up supplier to mitigate the effect of disruption in a SC with a predetermined network. Snyder and Tomlin (2008) develop a threat-advisory system and planning responses to adjust inventory level prior to disruption. Baghalian et al. (2013) consider a SCND problem in a three-tiered SC in markets with random demands and with supply process disruption. They only consider multi-sourcing and having substitutable supply facilities as their risk mitigation strategy. First, they formulate this problem as a stochastic model and then develop a robust model by minimizing the variance of the objective function's stochastic part.

While literature on single-tier chains with supply and demand disruptions is prevalent, few research projects consider multi-tier chains and those few multi-tier works, in which several risk mitigation strategies are considered, assume predetermined network structures for SCs. Obviously, using some of the risk mitigation strategies requires facilities which should be provided in the network designing stage of the SC.
Thus, risk management should be considered in the initial stages of designing a SC. Our research is mainly motivated to fill this gap in the SCND literature. Detailed contribution of this paper compared to literature is explained in the next section.

3. PROBLEM DESCRIPTION

In this paper we consider four different strategies to mitigate against two different risks in a 4-tier SC consisting of suppliers, manufacturers, DCs and retail outlets supplying different markets. The problem setting is for one period, which starts when each retailer places an orders with a DC. The DCs then integrate the orders from the retailers and pass them on to the manufacturers who then order materials from their suppliers. Once the manufacturer has received the materials it makes the products and ships them to the DCs who package and label them before sending them to the retailers. Within the chain, we consider the following four different risk mitigation strategies:

- **Strategy 1** - Emergency stock in the DCs
- **Strategy 2** - Emergency stock in one DC (which all DCs can access) to pool risk within the chain
- **Strategy 3** - Excess capacity in the suppliers
- **Strategy 4** - Substitute suppliers or facilities within the chain.

The SC encounters two different risks as follows:

- **Demand uncertainty** - we consider market demand as known random variable with a defined probability distribution
- **Supply uncertainty** - we define a number of different scenarios to consider various potential supply disruptions that could result from factors within facilities in the chain (such as impairment or overloading) or links between the facilities (such as bad weather, union strike in the ports, closure of entrance borders, custom delays or transport infrastructure repairs).

This enabled us to develop two different models to determine the optimal number, location and capacity of facilities and flow planning in the SC given the performance objectives of the chain:

1. **Lean** - first we develop a lean supply model showing how different risk mitigation strategies can be used to neutralize disruptions and maximize the profitability of the chain using single-objective mathematical model (Section 4.2).

2. **Responsive** - then, we extend the lean model to be responsive by adding another objective function dealing with minimizing the SC’s lead time. These new parts of the model (constraints of the second objective function) include some uncertain time parameters. We use Bertsimas and Sim (2004) method to make the second part robust against the uncertain time parameters. But still the solution of the model is not robust with respect to the first objective function computing the SC’s profit (Section 4.3).

In Section 8.1, we apply a new technique called “revised p-robust” to make the first part of the model (first objective function and it’s including constraints) robust with respect to the possible disruptions considered in the problem.
4. PROBLEM FORMULATION

In this section, we first discuss the alternative paths through the SC we are analyzing before then explaining how we developed the lean and responsive SC models.

4.1. Alternative supply chain paths

Before developing a number of different scenarios within the chain, we first have to establish the alternatives paths that a product can take through the chain from a supplier to a market as shown in Figure 1. Each potential path, $t_{ijm}$, starts from a potential supplier in the first tier (supplier $i$), passes through the manufacturer, passes through a DC candidate in the third tier (DC $j$) and delivers the goods to the retailer of an available market in the fourth tier (retailer $m$). Many different paths can be defined in the potential network structure of the chain; but transportation of products in some of the paths is not feasible because of a lack of necessary infrastructure or the total unit cost of production and distribution through the path ($ct_{ijm}$) being higher than its market price ($p_m$). Using path concept helps us model the availability of the chain in disruptions by defining a single set of scenarios ignoring the cause of the disruption (in the facilities or connecting links).

Although all paths are available in normal conditions, only some of them will be available when a disruption occurs. Each type of disruption is called a scenario whose probability of occurring is independent from the other scenarios. As outlined earlier, four different strategies can be used to optimize the performance of the SC in each scenario by mitigating against the negative effects in each scenario. To enable us to consider strategies 1 and 2 (holding emergency stock in one or all DCs), we need to define new potential paths called ‘emergency paths’ where we consider these stocks as virtual producers that can be used in the case of unavailability of products from either the suppliers or manufacturers. Figure 2 shows ‘emergency paths type 1’ necessary for implementing strategy 1 with virtual producers in the third tier of the chain connected to the retailers; and Figure 3 shows ‘emergency path type 2’ necessary for implementing strategy 2, starting with a virtual producer in a DC, which then has to pass through another DC before reaching a retailer.
4.2. Developing a Lean model: Single Objective Model

We used the following basic notations to formulate this problem.

Sets

$I$: set of all available suppliers can be used by SC; $I=\{i \mid i=1, 2, \ldots, |I|\}$;

$J$: set of all candidate locations for locating DCs; $J=\{j \mid j=1, 2, \ldots, |J|\}$;

$M$: set of candidate locations for locating retailers; $M=\{m \mid m=1, 2, \ldots, |M|\}$;

$S$: set of all possible scenarios;

$T$: Set of all potential paths, $t_{ijm}$, in the potential network structure of SC;

$T^{(i)}$ ($T^{(j)}$ and $T^{(m)}$): Subset of potential paths in the potential network structure of SC, starting from supplier $i$ (passing through DC $j$ and ending at retailer $m$);

$J^{(i)}$: Subset of candidate locations of DCs, which are located along the paths of $T$ started from producer $i$;

$I^{(j)}$: Subset of available suppliers which are located along the paths of $T$, going through candidate DC location $j$. 

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**Figure 2.** Potential emergency paths in the network structure of SC for implementing strategy 1.

**Figure 3.** Potential emergency paths in the network structure of SC for implementing strategy 2.
\( M^{(j)} \): Subset of candidate locations of retailers, which are located along the paths of \( T \) going through candidate DC location \( j \);
\( \bar{I} \): Set of emergency stocks in the DCs' of SC. Each of the DCs can hold products produced from material of its connected suppliers as emergency stocks, \( \bar{I} = \{ i^j, \forall i \in I, \forall j \in J^{(i)} \} \);
\( I^{(j)} \): Subset of emergency stocks, which are held in candidate DC location \( j \);
\( \bar{I}^{(j)} \): Subset of emergency stocks, which are fulfilled by supplier \( i \);
\( \bar{T} \): Set of first type emergency paths usable in disruptions for imposing first risk mitigation strategy;
\( \bar{T}'' \): Set of second type emergency paths usable in disruptions for imposing second risk mitigation strategy;
\( \hat{T}^{(i)}(\hat{T}^{(m)}) \): Subset of \( \bar{T} \) including paths originating from emergency stock \( i^j \) (ending to market \( m \)).
\( T''^{(i)}(T''^{(j)} \text{ and } T''^{(m)}) \): Subset of \( T'' \) including paths originating from emergency stock \( i^j \) (passing through DC \( j \) and ending to market \( m \)).

**Variables**

\( y_i \): 1 if available supplier \( i \) is selected to be used by SC; 0 otherwise \( (\forall i \in I) \);
\( y_i' \): Production capacity of supplier \( i \) assigned to SC \( (\forall i \in I) \);
\( z_j \): 1 if candidate location \( j \) is selected to locate a DC; 0 otherwise \( (\forall j \in J) \);
\( z_j' \): Capacity of DC located in candidate location \( j (\forall j \in J) \);
\( w_m \): 1 if candidate location \( m \) is selected to locate a retailer; 0 otherwise \( (\forall m \in M) \);
\( x_{jt} \): Flow of product through path \( t \) in scenario \( s \) \( (\forall t \in T \cup T' \cup T'', \forall s \in S) \);
\( v_{ij} \): Amount of product kept in emergency stock \( i^j (\forall i \in I) \);
\( P_{ts} \): Profit of each market in scenario \( s \);

**Parameters**

\( a_i \): Fixed cost of selecting supplier \( i \) as a component of SC \( (\forall i \in I) \);
\( a_i' \): Cost of unit capacity of supplier \( i \) assigned to SC \( (\forall i \in I) \);
\( b_j \): Fixed cost of locating a DC in candidate location \( j (\forall j \in J) \);
\( b_j' \): Cost of unit capacity of DC \( (\forall j \in J) \);
\( h_j \): Holding cost of unit emergency stock in DC \( j \) along the considered planning period \( (\forall j \in J) \);
\( l_j \): Percentage of emergency stock value in DC \( j \) considered as cost of inventory lost income \( (\forall j \in J) \);
\( c_m \): Fixed cost of locating a retailer in candidate location \( m (\forall m \in M) \);
\( SV_m \): Salvage value of unit excess product at the end of period in retailer \( m (\forall m \in M) \). Negative value of this parameter can be interpreted as the unit holding cost of extra inventory at the end of the planning period for durable (not perishable) products.
\( LS_m \): Lost sale cost of unit shortage at the end of period in retailer \( m (\forall m \in M) \);
\( e_t \): 1 if path \( t \) is usable in scenario \( s \); 0 otherwise \( (\forall s \in S, \forall t \in T \cup T' \cup T'') \);
\( D_m \): Demand of product in market \( m (\forall m \in M) \);
\( E(D_m) \): Expected demand of product in market \( m (\forall m \in M) \);
$F(D_m)$: Cumulative distribution function of variable $D_m$ ($\forall m \in M$);
$P_m$: Price of one unit of product in market $m$ ($\forall m \in M$);
$Pr_s$: Probability of occurrence of scenario $s$ ($\forall s \in S$);
$ct$: Cost of producing and distributing unit along the path $t$. If $t \in T$, then $ct_{jm} = a''_i$ (normal procuring cost of unit in supplier $i$ and converting it to a product in manufacturer) + $d_{ij}$ (transportation cost of unit from supplier $i$ to DC $j$) + $b''_j$ (handling cost of unit in DC $j$) + $d_{jm}$ (transportation cost of unit from DC $j$ to retailer $m$) + $c''_m$ (handling cost of unit in retailer $m$). If $t \in T'$, then $ct'_{jm} = a''_i + d_{ij} + b''_j + d_{jm} + c''_m + a''''_i$ (extra procuring cost received by supplier $i$ for components produced out of order for fulfilling emergency stocks). If $t \in T''$, then $ct''_{jm} = a''_i + d_{ij} + b''_j + d_{jm} + c''_m + a''''_i + d_{ij}$ (transportation cost of unit from DC $j$ to DC $j$). Since production cost of the unit in the manufacturer is same and common for products of all paths, we ignore it in our calculations. Also we assume that $\gamma^+ = \max\{\gamma, 0\} \text{ and } \gamma \land \eta = \min\{\gamma, \eta\}$.

In the traditional flow planning in a SC problem, instead of paths, flow variables are defined between the facilities of different echelons. In other words, the traditional approach is link-based, which means we need to define two kinds of variables such as $x_{ij}$ representing flow quantity between supplier $i$ ($\forall i \in I$) and DC $j$ ($\forall j \in J$) and $x_{jm}$ representing flow quantity between DC $j$ ($\forall j \in J$) and retailer $m$ ($\forall m \in M$). In this traditional modeling, again, the number of variables grows exponentially by the number of facilities in the SC's echelons. However, the path-based approach introduced in this paper enables us to compute the production and distribution cost per unit through each path and if it is more than ended market's price, we can remove them from possible path sets ($T$, $T'$ and $T''$) and reduce the number of possible options. So in this way we can reduce the number of variables. Therefore, it is the expected that complexity of the path-based approach should be less than the traditional link-based approach.

**Mathematical Model**

In order to develop the mathematical formulation of the problem, first we calculate the profit of each market in scenario $s$.

$$ Pt_s = P_m \left( \sum_{t \in T(m) \cup T'(m) \cup T''(m)} x^s_t \land D_m \right) + SV_m \left( \sum_{t \in T(m) \cup T'(m) \cup T''(m)} x^s_t - D_m \right) + LS_m \left( (D_m - \sum_{t \in T(m) \cup T'(m) \cup T''(m)} x^s_t)^+ - \sum_{t \in T(m) \cup T'(m) \cup T''(m)} ct \cdot x^s_t \right) $$

The first term of Eq. (1) represents the SC income in market $m$. The second term is the salvage value (for perishable products with positive $SV_a$) or holding cost (for long-lasting products with negative $SV_a$) of unsold products. The third term is the shortage cost of the lost sales. The last term represents products' production and shipment cost along the paths. Each retailer predicts its demand for the period and orders it before the beginning of that period. At the end of the planning period, unsold products can be sold for salvage value or kept for the next period in charge of some holding cost and lost sales leave shortage costs.

Considering demand uncertainty, the following function is used to calculate costs:

$$ (\sum_{t \in T(m) \cup T'(m) \cup T''(m)} x^s_t \land D_m) = \sum_{t \in T(m) \cup T'(m) \cup T''(m)} x^s_t - \left( \sum_{t \in T(m) \cup T'(m) \cup T''(m)} x^s_t - D_m \right)^+ $$

(2)
where \( (\sum_{t \in T(m)I_{T'(m)}} x^t_x^i - D_m)^+ = \int_0^{\sum_{t \in T(m)I_{T'(m)}} x^t_x^i} F(D_m), dD_m \) and
\[
(\sum_{t \in T(m)I_{T'(m)}} x^t_x^i - D_m)^+ - \left( D_m - \sum_{t \in T(m)I_{T'(m)}} x^t_x^i \right)^+ = \sum_{t \in T(m)I_{T'(m)}} x^t_x^i - E(D_m).
\]

By substituting the equations, the profit function for each market can be shown as follows:
\[
P_t = (P_m + LS_m) \cdot \sum_{t \in T(m)I_{T'(m)}} x^t_x^i - (P_m + LS_m - SV_m) \cdot \int_0^{\sum_{t \in T(m)I_{T'(m)}} x^t_x^i} F(D_m), dD_m - \sum_{t \in T(m)I_{T'(m)}} c_t x^t_x^i
\]
\[
(3)
\]

We use bi-level stochastic programming method to formulate this problem to show that we deal with two different kinds of decisions with different planning horizons. Strategic and long-term decisions of designing the network of SC (number, location and capacity of SC’s facilities) are made in the upper level. Operational and short-term decisions of flow planning throughout the SC’s network are made in the lower level. The upper model is as follows (Santoso et al., 2005):

\[
\text{Max } Z_1 = -\sum_i (a_i \cdot y_i + \hat{a}_i \cdot \hat{y}_i) - \sum_j (b_j \cdot z_j + \hat{b}_j \cdot \hat{z}_j) - \sum_m c_m \cdot w_m + E(z''(x^s_x, v_{ij}))
\]
\[
(4)
\]

\[
\text{S.T.}
\]
\[
\hat{y}_i \leq N \cdot y_i \quad (\forall i \in I)
\]
\[
\hat{z}_j \leq N \cdot z_j \quad (\forall j \in J)
\]
\[
y_i, z_j, w_m \in \{0, 1\} \quad (\forall i \in I, \forall j \in J, \forall m \in M)
\]
\[
\hat{y}_i, \hat{z}_j \geq 0 \quad (\forall i \in I, \forall j \in J)
\]
\[
(5, 6, 7, 8)
\]

This is the upper level model dealing with network design decisions and \( z''(x^s_x, v_{ij}) \) is the optimal solution of the following problem (lower model) which demonstrates the operational flow planning throughout the network of SC and computes its obtainable income in scenario \( s \):

\[
\text{Max } Z''(x^s_x, v_{ij}) =
\]
\[
\sum_{t \in T(m)I_{T'(m)}} x^t_x^i - \sum_{t \in T(m)I_{T'(m)}} (P_m + LS_m - SV_m) \cdot \int_0^{\sum_{t \in T(m)I_{T'(m)}} x^t_x^i} F(D_m), dD_m
\]
\[
- \sum_{t \in T(m)I_{T'(m)}} LS_m \cdot E(D_m), w_m - \sum_{t \in T(m)I_{T'(m)}} \sum_{t' \in T(m)I_{T'(m)}} c_t x^t_x^i
\]
\[
- \sum_i (v_{ij} - \sum_{t \in T(m)I_{T'(m)}} x^t_x^i \cdot (h_j + I_j \cdot (a''_{ij} + d_{ij} + a'''_{ij})))
\]
\[
(9)
\]

\[
\text{S.T.}
\]
\[
\sum_{t' \in T(m)I_{T'(m)}} x^t_x^i \leq N \cdot y_i \quad (\forall i \in I)
\]
\[
\sum_{t' \in T(m)I_{T'(m)}} x^t_x^i \leq N \cdot z_j \quad (\forall j \in J)
\]
\[
\sum_{t \in T(m)I_{T'(m)}} x^t_x^i \leq N \cdot w_m \quad (\forall m \in M)
\]
\[
v_{ij} \leq N \cdot y_i \quad (\forall i, j \in I)
\]
\[
v_{ij} \leq N \cdot z_j \quad (\forall i, j \in I)
\]
\[
x^t_x^i \leq N \cdot e^t_x^i \quad (\forall t \in T \cup T' \cup T'')
\]
\[
(10, 11, 12, 13, 14, 15, 16, 17, 18, 19)
\]
The first term of objective function (9) (in bracket) represents the profit of the SC in its markets; the term inside the bracket is the same as equation (5). The last term describes the holding cost of emergency stocks in the DCs of the chain. Based on Constraints (10), (11), and (12), facilities along the active paths should be located (\( N \) is a large constant). According to constraints (13) and (14), only the products produced from raw materials of selected suppliers of the SC can be kept as emergency stocks in the located DCs. Constraint (15) imposes that in scenario \( s \) the shipments are done only through the paths which are available in that scenario. Constraint (16) forces that in each scenario the amount of flow from each supplier should be lower than the reserved capacity of that supplier. According to constraint (17), the sum of passing flow and reserved emergency stock in each DC should be lower than the capacity of that DC. Constraint (18) implies that during disruption, the amount of output from each emergency stock cannot be higher than the amount of that reserve.

Different possible disruptions in the SC are defined by scenarios. In each scenario, some of the potential paths of the chain are inactive due to unavailability of their including facilities or links. For instance, considering \( n \) different impairments of facilities and \( m \) different disruptions of connecting links, in the worst case \( 2^n m \) different scenarios can be defined for the problem. For each scenario, we solve the above model and calculate the amount of \( z'' \) for that scenario. So the expected profit can be computed as:

\[
E(z''(x^s_j, v_i)) = \\
\sum_s \sum_{\text{paths}} \left[ \sum_M (P_m + LS_m) \sum_{t \in T(m) \cup T''(m)} x^s_t - \sum_M (P_m + LS_m - SV_m) \int_0^{D(m)} F(D_m) \cdot dD_m - \sum M LS_m \cdot E(D_m) \cdot w_m - \sum_M \sum_{t \in T(m) \cup T''(m)} \sum_{i \in M} (v_i - \sum_{t \in T(i) \cup T''(i)} x^s_t) \cdot (h_j + l_j + (a''_i + d_{ij} + a''''_i)) \right] 
\]

Then the bi-level problem is reformulated into a single-level one and can be solved by commercial solvers. Based on this model, when a path \( t \in T(m) \) servicing market \( m \) in a normal condition (without any disruption) is out of use in a scenario, other active paths of subset \( T(m) \) can be used to serve that market (risk mitigation strategy 4 – having back-up facilities). In this case extra capacities are needed in the included facilities of this new path to be able to fulfill the production request of this new path (risk mitigation strategy 3 – having extra capacity). Or the demand of market \( m \) can be fulfilled by the emergency stock of path \( t \) stored in its corresponding DC (risk mitigation strategy 1 – keeping emergency stock) or emergency stock of other DCs (risk mitigation strategy 2 – risk pooling in emergency stock).

The modeling mechanism proposed in this paper is not only restricted to the specific SC investigated in this paper (a SC with four echelon including several suppliers, one manufacturer, several DCs and several retailers). For instance consider another SC with a different network structure including several first tier suppliers, several second tier suppliers, several manufacturers, a DC and several retailers. We can easily use the model proposed in this paper for this new SC only by modifying the concept of path. In the new SC, each path starts from a first tier supplier in the first echelon, passes through a second tier supplier and a
manufacturer in the second and third echelons respectively, and after going through the single existing DC ends at a retailer in the last echelon. By this new definition of path, we can apply the same model for this new SC. The only assumption about the network structure of SC is that “the network should be an acyclic digraph”.

4.3. Responsive model: Bi-Objective Model

A responsive SC should be able to respond quickly to unpredictable market changes, which is normally achieved by reducing its lead time and cost (Gunasekara et al., 2008). Therefore, we must add the objective function of minimizing the SC’s lead-time to the mathematical model of the problem by adding the following notations.

**Variables**

$r^s_t$: 1 if path $t$ is used by SC in scenario $s$ (when $x^s_t > 0$); 0 otherwise (when $x^s_t = 0$) ($\forall s \in S, \forall t \in T \cup T' \cup T''$).

**Parameters**

$D^\text{max}_m$: Maximum potential demand of market $m$ ($\forall m \in M$);
$\hat{t}_i$: Unit production time in supplier $i$; $\hat{t}_i \in [\tilde{t}_i - \hat{\tilde{t}}_i, \tilde{\tilde{t}}_i + \hat{\tilde{\tilde{t}}}_i]$;
$\tilde{t}_{ij}$: Transportation time from supplier $i$ to DC $j$. We assume this time is independent of product amount;
$\tilde{t}'_{ij}$: Unit transportation time from supplier $i$ to DC $j$; $\tilde{t}_{ij} \in [\tilde{t}_{ij} - \hat{\tilde{t}}_{ij}, \tilde{\tilde{t}}_{ij} + \hat{\tilde{\tilde{t}}}_{ij}]$;
$t_j$: Unit handling time in DC $j$; $t_j \in [\tilde{t}_j - \hat{\tilde{t}}_j, \tilde{\tilde{t}}_j + \hat{\tilde{\tilde{t}}}_j]$;
$\tilde{t}_{jm}$: Transportation time from DC $j$ to retailer $m$. We assume this time is independent of product amount;
$\tilde{t}'_{jm}$: Unit transportation time from DC $j$ to retailer $m$; $\tilde{t}_{jm} \in [\tilde{t}_{jm} - \hat{\tilde{t}}_{jm}, \tilde{\tilde{t}}_{jm} + \hat{\tilde{\tilde{t}}}_{jm}]$;
$t_{ij'}$: Transportation time from DC $j$ to DC $j'$ ($j' \neq j$). We assume this time in independent of product amount;
$\tilde{t}_{ij'}$: Unit transportation time from DC $j$ to DC $j'$ ($j' \neq j$); $\tilde{t}_{ij} \in [\tilde{t}_{ij} - \hat{\tilde{t}}_{ij}, \tilde{\tilde{t}}_{ij} + \hat{\tilde{\tilde{t}}}_{ij}]$;
$tm^s_t$: Time of producing and distributing $x^s_t$ units of product through path $t$ in scenario $s$.

\begin{align*}
\text{If } t & \in T \text{ then } \quad tm^s_t(x^s_t) = t_i \cdot x^s_t + t_{ij} \cdot x^s_t + t_{ij} \cdot x^s_t + t_{jm} \cdot \tilde{t}^s_{ij} + t_{jm} \cdot \tilde{\tilde{t}}^s_{ij} + t_{jm} \cdot \tilde{\tilde{\tilde{t}}}^s_{ij}; \\
\text{If } t & \in T' \text{ then } \quad tm^s_t(x^s_t) = t_j \cdot x^s_t + t_{jm} \cdot \tilde{t}^s_{jm} + t_{jm} \cdot \tilde{\tilde{t}}^s_{jm}; \\
\text{If } t & \in T'' \text{ then } \quad tm^s_t(x^s_t) = t_j \cdot x^s_t + t_{jj'} \cdot \tilde{t}^s_{jj'} + t_{jj'} \cdot \tilde{\tilde{t}}^s_{jj'} + t_{jj'} \cdot \tilde{\tilde{\tilde{t}}}^s_{jj'}.
\end{align*}

$\lambda_1$: Importance of reducing lead time in increasing the responsiveness of the chain;
$\lambda_2$: Importance of responding to possible demand of the markets in increasing the responsiveness of the chain.

We can ignore the production time of the manufacturer because this time is the same for all possible paths. In this problem, for minimizing the process lead time of the SC, we try to minimize the maximum time required to deliver the products to the retailers. Since the longest lead time of the SC and the largest non-responded demand of the chain are numbers with different dimensions, therefore we normalized them. For normalizing these two terms we define $D_{\text{max}} = \max_{m}(D^\text{max}_m)$ and
\[ l_{\text{max}} = \max_M(\max_{T'(m) \cup T''(m)}(tm^2_i(D^\text{max}_m))). \]

In the bi-objective mathematical formulation of the response SCN, the following objective function and constraints should be added to the aforementioned base model:

\[
\text{Min } z_2 = \lambda_1 \cdot \frac{l}{l_{\text{max}}} + \lambda_2 \cdot \frac{g}{d_{\text{max}}} \quad (24)
\]

\textbf{S.T.}

\[
l^s_t \geq t^s_m \quad (\forall t \in T'(m) \cup T''(m), \forall m \in M, \forall s \in S) \quad (25)
\]

\[
l_m \geq l^s_m \quad (\forall m \in M) \quad (26)
\]

\[
l \geq l_m \quad (\forall m \in M) \quad (27)
\]

\[
g^s_m \geq (D^\text{max}_m - \sum_{T'(m) \cup T''(m)} x^2_t) \quad (\forall s \in S, \forall m \in M) \quad (28)
\]

\[
g_m \geq g^s_m \quad (\forall m \in M) \quad (29)
\]

\[
g \geq g_m \quad (\forall m \in M) \quad (30)
\]

\[
r^s_i \in \{0, 1\} \quad (\forall t \in T' \cup T'', \forall s \in S) \quad (31)
\]

\[
l^s_m, l_m, l, g^s_m, g_m, g \geq 0. \quad (32)
\]

The first part of objective function (24) minimizes this longest lead time of the SC and the second part minimizes this largest non-responded demand of the chain.

Constraint (25) calculates the maximum time required to deliver the products to retailer \( m \) in scenario \( s \) as \( l^s_m = \max_{t \in T'(m) \cup T''(m)}(tm^2_i) \). Constraint (26) calculates the maximum time required to deliver the products to retailer \( m \) in all the possible scenarios as \( l_m = \max_S(l^s_m) \). The amount of \( l_m \) should be calculated for all the retailers. The longest lead time of the SC, \( l = \max_M(l_m) \), is calculated in constraint (27). To increase the ability of the SC in response to possible demands of the markets, constraint (28) calculates the difference between the maximum potential demand of market \( m \) and its procured product in scenario \( s \) as \( g^s_m = (D^\text{max}_m - \sum_{T'(m) \cup T''(m)} x^2_t) \). Constraint (29) calculates the maximum unmet demand of retailer \( m \) in all the possible scenarios, \( g_m = \max_S(g^s_m) \). The value of \( g_m \) should be calculated for all the retailers. So the largest non-responded demand of the SC, \( g = \max_M(g_m) \), is calculated in constraint (30). The second part of the objective function (24) minimizes this largest unfulfilled demand of the SC.

Calculating the exact amounts of \( t_i, t_j, t'_{ij}, t'_{jm} \) and \( t''_{jj} \) parameters is not easy and contains a kind of uncertainty in reality. So we consider their uncertainty as interval parameters defined by mean values, \( \bar{t}_i, \bar{t}_j, \bar{t}'_{ij}, \bar{t}'_{jm} \) and \( \bar{t}''_{jj} \), and domain variations \( \bar{t}_i, \bar{t}_j, \bar{t}'_{ij}, \bar{t}'_{jm} \) and \( \bar{t}''_{jj} \), respectively. This problem belongs to a group of stochastic programming problems in which the distribution functions of uncertain parameters are unknown. Several works have been performed in this field (Bertsimas and Sim, 2004; Ben-Tal and Nemirovski, 2000; Soyster, 1973). We use an approach, which is chiefly based on Bertsimas and Sim (2004) to make the model robust against the uncertainty of parameters. For more details about this approach refer to Bertsimas and Sim (2004). To use this approach new notations should be defined:

\textbf{Sets}

\( L_t \): Set of uncertain coefficients in the constraint (25) of the model written for path \( t (l_t \in L_t, \ t \in T' \cup T' \cup T'') \).

\textbf{Parameters}
\( \Gamma_t \): Number of uncertain coefficients in set \( L \) that can deviate from their nominal values simultaneously. This parameter is called *Budget of Uncertainty* by Bertsimas and Sim (2004) and is used to adjust the robustness against the level of conservatism. It is obvious that \( 0 \leq \Gamma_t \leq |L_t| (\forall t \in T \cup T' \cup T'') \).

### Variables

\( \alpha_t \): Dual variable which should be defined in Bertsimas and Sim (2004) method for constraint (25) of the model written for path \( t \in T \cup T' \cup T'' \).

\( \beta_{tl} \): Dual variable which should be defined in Bertsimas and Sim (2004) method for uncertain coefficient \( l_t \) of constraint (25) of the model written for path \( t (l_t \in L_t, t \in T \cup T' \cup T'') \). We generally show the mean value of this coefficient by \( \tilde{t}_{tl} \) and its tolerance by \( \bar{t}_{tl} \).

### Mathematical Model

The mathematical formulation of the problem is as follows:

\[
\text{Min} \quad z_2 = \lambda_1 \frac{1}{\bar{l}_{max}} + \lambda_2 \frac{\bar{g}}{\bar{d}_{max}} \quad (33)
\]

\[
\text{S.T.} \]

\[
\begin{align*}
\sum_{m} l_m \tilde{t}_i \cdot x_i^S - t_{ij} \tilde{r}_j^S - \tilde{\bar{e}}_j \cdot x_j^S - t_{jm} \tilde{r}_m^S - \tilde{\bar{e}}_j \cdot x_j^S - \tilde{\bar{e}}_j \cdot x_j^S - \Gamma_t \alpha_t - \sum_{l=1}^{L_t} \beta_{tl} & \geq 0 \\
(\forall t \in T^{(m)}, \forall m \in M) & \quad (34)
\end{align*}
\]

\[
\begin{align*}
\alpha_t + \beta_{tl} & \geq \tilde{t}_{tl} \cdot y_t \\
\forall t \in T^{(m)}, \forall m \in M, \forall l_t \in L_t & \quad (35)
\end{align*}
\]

\[
\begin{align*}
- \gamma_t & \leq \alpha_t \leq \gamma_t \\
\forall t \in T^{(m)}, \forall m \in M & \quad (36)
\end{align*}
\]

\[
\begin{align*}
\sum_{m} l_m \tilde{t}_i \cdot x_i^S - t_{ij} \tilde{r}_j^S - t_{jm} \tilde{r}_m^S - \tilde{\bar{e}}_j \cdot x_j^S - \tilde{\bar{e}}_j \cdot x_j^S - \tilde{\bar{e}}_j \cdot x_j^S - \Gamma_t \alpha_t - \sum_{l=1}^{L_t} \beta_{tl} & \geq 0 \\
(\forall t \in T^{(m)}, \forall m \in M) & \quad (37)
\end{align*}
\]

\[
\begin{align*}
\alpha_t + \beta_{tl} & \geq \tilde{t}_{tl} \cdot y_t \\
(\forall t \in T^{(m)}, \forall m \in M, \forall l_t \in L_t) & \quad (38)
\end{align*}
\]

\[
\begin{align*}
\gamma_t & \leq \alpha_t \leq \gamma_t \\
\forall t \in T^{(m)}, \forall m \in M & \quad (39)
\end{align*}
\]

\[
\begin{align*}
\sum_{m} l_m \tilde{t}_i \cdot x_i^S + t_{ij} \tilde{r}_j^S - t_{jm} \tilde{r}_m^S - \tilde{\bar{e}}_j \cdot x_j^S - \tilde{\bar{e}}_j \cdot x_j^S - \tilde{\bar{e}}_j \cdot x_j^S - \Gamma_t \alpha_t - \sum_{l=1}^{L_t} \beta_{tl} & \geq 0 \\
(\forall t \in T^{(m)}, \forall m \in M) & \quad (40)
\end{align*}
\]

\[
\begin{align*}
\alpha_t + \beta_{tl} & \geq \tilde{t}_{tl} \cdot y_t \\
(\forall t \in T^{(m)}, \forall m \in M, \forall l_t \in L_t) & \quad (41)
\end{align*}
\]

\[
\begin{align*}
\gamma_t & \leq \alpha_t \leq \gamma_t \\
(\forall t \in T^{(m)}, \forall m \in M) & \quad (42)
\end{align*}
\]

\[
\begin{align*}
l_m & \geq l_m^S \\
(\forall m \in M, \forall s \in S) & \quad (43)
\end{align*}
\]

\[
\begin{align*}
l^* & \geq l^* \\
(\forall m \in M) & \quad (44)
\end{align*}
\]

\[
\begin{align*}
g_m & \geq \bar{g}_m \\
(\forall m \in M, \forall s \in S) & \quad (45)
\end{align*}
\]

\[
\begin{align*}
g & \geq \bar{g} \\
(\forall m \in M) & \quad (46)
\end{align*}
\]

\[
\begin{align*}
r_x^S & \in [0,1] \\
(\forall t \in T \cup T' \cup T'', \forall s \in S) & \quad (47)
\end{align*}
\]

\[
\begin{align*}
l_m^S, l_m, I_m, g_m, g, \alpha_t, \beta_{tl}, \gamma_t & \geq 0 \\
(\forall m \in M, \forall s \in S, \forall t \in T \cup T' \cup T'', \forall l_t \in L_t) & \quad (49)
\end{align*}
\]

Adding this second objective function and its corresponding constraints (33-49) to the first objective function and its corresponding constraints (6-19) leads to a comprehensive model for designing lean and responsive network structure for the SC which is a Mixed Integer Non-linear (MINL) programming. In the next section a method is proposed to solve this model.

### 5. SOLUTION METHOD

The proposed model in the previous section is a MINL model, which is nonlinear due to the integral cumulative distribution function in its first objective function. According to the type of distribution function
of the market's demand, the form of this term is different and for some cases such as normal distribution calculating this term is not straightforward and does not have a closed form equation. Since these functions are always upward and convex, we use linear regression approximation to linearize them. We use a piecewise linear transformation to break the range of nonlinear term into several intervals and substitute the convex function of each interval with a straight line (with a unique constant and coefficient). For the uniform and normal distribution functions which are more common in the demands of markets, you can see the approximations in Figures 4 and 5. In Figures 4 and 5, the ranges of functions have been broken to five and four intervals respectively. By increasing the number of intervals, the bias of approximations will decrease. This approximation converts the nonlinear model to a linear one; but we need to add new variables and constraints to the model.

![Figure 4. Approximation of accumulative standard normal distribution function.](image1)

![Figure 5. Approximation of accumulative uniform distribution function.](image2)

After linearizing the cumulative distribution functions of markets' demands, we have several approximation intervals for each market and need to define some new variables for appropriate interval selection.

**Set**

$N^m$: Set of approximation intervals for market $m$ ($n^m \in N^m$).

**Parameters**

$coeff_{n^m}$: Coefficient for interval $n^m$ of market $m$ ($\forall m \in M, n^m \in N^m$).

$const_{n^m}$: Constant for interval $n^m$ of market $m$ ($\forall m \in M, n^m \in N^m$).

$lower_{n^m}$: Lower bound for interval $n^m$ of market $m$ ($\forall m \in M, n^m \in N^m$).

$upper_{n^m}$: Upper bound for interval $n^m$ of market $m$ ($\forall m \in M, n^m \in N^m$).

**Variables**

$IS_{n^m}^s$: Binary variable equal to 1 if interval $n^m$ is selected in scenario $s$ in market $m$ and 0 otherwise ($\forall s \in S, \forall m \in M, \forall n^m \in N^m$).

$AI_{n^m,t}^s$: Amount of product flows in scenario $s$ through path $t$ in interval $n^m$ ($\forall s \in S, \forall t \in T \cup T' \cup T'', \forall m \in M, \forall n^m \in N^m$).
After defining the above notations, the nonlinear part of the first objective function of the proposed model is linearized as follows:

\[
\text{Max } z''(A_{n,m,t}^S, IS_{n,m}^S, v_{ij}) = (\sum_{M}(P_n + LS_m), \sum_{t \in T^{(m)}} A_{n,m,t}^S + \text{const}_m. IS_{n,m}^S) \sum_{n^m \in N^m} A_{n,m,t}^S
\]

\[
- \sum_{M}(P_n + LS_m - SV_m), \sum_{n^m \in N^m} (\text{coeff}_{n^m} \sum_{t \in T^{(m)} \cup T''^{(m)}} ct \cdot \sum_{n^m \in N^m} A_{n,m,t}^S)
\]

\[
- \sum_{M} t \in T^{(m)} \cup T''^{(m)} (n^m \in N^m) m \text{ is destination market of path } t \quad AI_{n,m,t}^S \leq h_j + j. (a''_i + d_{ij} + a'''_i)
\]

\[
\text{S.T.}
\]

\[
\sum_{n^m \in N^m} A_{n,m,t}^S \leq N. y_i \quad (\forall i \in I)
\]

\[
\sum_{n^m \in N^m} A_{n,m,t}^S \leq N. z_j \quad (\forall j \in J)
\]

\[
\sum_{n^m \in N^m} A_{n,m,t}^S \leq N. w_m \quad (\forall m \in M)
\]

\[
IS_{n,m}^S = 1 \quad (\forall m \in M)
\]

\[
AI_{n,m,t}^S \leq \text{upper}_m \cdot IS_{n,m}^S \quad (\forall n^m \in N^m, \forall m \in M)
\]

\[
AI_{n,m,t}^S \geq \text{lower}_m \cdot IS_{n,m}^S \quad (\forall n^m \in N^m, \forall m \in M)
\]

\[
AI_{n,m,t}^S \leq N. e_t^S \quad (\forall t \in T \cup T' \cup T'')
\]

\[
\sum_{n^m \in N^m} A_{n,m,t}^S \leq y_i \quad (\forall i \in I)
\]

\[
\sum_{n^m \in N^m} A_{n,m,t}^S + \sum_{i \in I} v_{ij} + \sum_{n^m \in N^m} A_{n,m,t}^S \leq z_j \quad (\forall j \in J)
\]

\[
\sum_{n^m \in N^m} A_{n,m,t}^S \leq v_{ij} \quad (\forall i \in I)
\]

\[
AI_{n,m,t}^S, v_{ij} \geq 0 \quad (\forall n^m \in N^m, \forall m \in M, \forall t \in T \cup T' \cup T'', \forall i \in I)
\]

\[
IS_{n,m}^S \in \{0,1\} \quad (\forall n^m \in N^m, \forall m \in M)
\]

\[
\text{Constraints (13–14)}
\]

Eq. (54) expresses that for each market only one interval can be chosen. Eq. (55) and (56) determine bounds for the defined intervals. The other relations were described in the previous sections.
By this linearization method, the proposed model transforms to a mixed integer linear (MIL) model, which can be solved easily by existing commercial solvers such as CPLEX, GAMS or LINGO.

6. EXPLORING THE MODELS USING A REAL-LIFE CASE STUDY

We have explored how our models operate using empirical data from one of the largest automotive SC in the Middle East as it faces demand and supply uncertainty. IKC (Market 1), SAC (Market 2) and KKC (Market 3) are the largest automotive manufacturers in this region and all have multi-tier supplier networks that provide them with different components. NMC (Market 4) assembles some kinds of engines and supplies to all three automotive manufacturers using components from many national and international suppliers. NMC in turn purchases gear pins from SMAC (Manufacturer). SMAC produces a fifth gear pin for NMC. SMAC can procure its raw material, CK45 steel, from either a national supplier YIIC (Supplier 2) or an overseas supplier (Supplier 1). YIIC (Supplier 2) also supplies steel to a number of other industries in the country and the high productivity of its factory means SMAC’s (Manufacturer) orders are sometimes delayed. Equally, orders with its overseas steel suppliers (Supplier 2) can also be delayed in customs. Once the steel is receive by SMAC (Manufacturer), it is stretched, cut, ground, rasped and milled into a gear pin before being sent to a DC where it is then brushed, inspected and labeled before being sent to a number of different customers including NMC (Market 1). However, recent delays and disruptions in steel supply from YIIC (Supplier 2) and its overseas suppliers (Supplier 1) mean that SMAC (Manufacturer) has been supplying orders late to its customers and, as a result, demand for gear pins has started to fall. To improve its performance, SMAC decided to redesign its network structure. Its potential structure and potential usable paths are shown in Figures 6, 7, and 8.

The cost and time components of each of these paths shown in Tables 3 and 4 model the real challenges, issues and objectives of the companies within the SC. The fixed costs of contracting with first and second suppliers; locating first and second DCs and opening a retailer in first, second, third, and fourth markets are 100, 100, 1000, 1000, 500, 500, 550, and 500 respectively. The costs of reserving a capacity unit in suppliers and building a capacity unit in DCs are considered as 0.2 and 0.03 respectively in this problem. The cost of holding one unit of inventory of product for one period is 0.5, and 1.5 percent of the monetary value of the holding inventory is considered inventory sleep cost. We assume $p_m = 10, LS_m = 2$ and $SV_m = 0 \ (\forall m \in M)$.

<table>
<thead>
<tr>
<th>Path</th>
<th>Cost of producing and distributing through path</th>
<th>Path</th>
<th>Cost of producing and distributing through path</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{1111}$</td>
<td>8.1</td>
<td>$t_{4023}$</td>
<td>8.4</td>
</tr>
<tr>
<td>$t_{1112}$</td>
<td>8.1</td>
<td>$t_{4004}$</td>
<td>8.4</td>
</tr>
<tr>
<td>$t_{2122}$</td>
<td>8.1</td>
<td>$t_{1022}$</td>
<td>8.9</td>
</tr>
<tr>
<td>$t_{1123}$</td>
<td>7.9</td>
<td>$t_{1023}$</td>
<td>8.9</td>
</tr>
<tr>
<td>$t_{2123}$</td>
<td>7.9</td>
<td>$t_{1024}$</td>
<td>8.9</td>
</tr>
<tr>
<td>$t_{2124}$</td>
<td>7.9</td>
<td>$t_{1011}$</td>
<td>8.7</td>
</tr>
<tr>
<td>$t_{2111}$</td>
<td>7.9</td>
<td>$t_{1012}$</td>
<td>8.7</td>
</tr>
<tr>
<td>$t_{101}$</td>
<td>8.6</td>
<td>$t_{2011}$</td>
<td>8.7</td>
</tr>
<tr>
<td>$t_{102}$</td>
<td>8.6</td>
<td>$t_{2012}$</td>
<td>8.9</td>
</tr>
<tr>
<td>$t_{103}$</td>
<td>8.4</td>
<td>$t_{2022}$</td>
<td>8.9</td>
</tr>
<tr>
<td>$t_{201}$</td>
<td>8.6</td>
<td>$t_{2023}$</td>
<td>8.7</td>
</tr>
<tr>
<td>$t_{202}$</td>
<td>8.4</td>
<td>$t_{2024}$</td>
<td>8.7</td>
</tr>
</tbody>
</table>
Table 4. Time parameters of SAMAC problem.

<table>
<thead>
<tr>
<th>Time components</th>
<th>Volume-independent time component (t')</th>
<th>Volume-dependent time component (t'')</th>
<th>Mean (t')</th>
<th>Tolerance (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{i=1}$</td>
<td>-</td>
<td></td>
<td>0.025</td>
<td>0.01</td>
</tr>
<tr>
<td>$t_{i=2}$</td>
<td>-</td>
<td></td>
<td>0.04</td>
<td>0.01</td>
</tr>
<tr>
<td>$t_{j=1}$</td>
<td>-</td>
<td></td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>$t_{j=2}$</td>
<td>-</td>
<td></td>
<td>0.002</td>
<td>0.001</td>
</tr>
<tr>
<td>$t_{ij} (i=1,j=1)$</td>
<td>0.2</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$t_{ij} (i=1,j=2)$</td>
<td>0.2</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$t_{ij} (i=2,j=1)$</td>
<td>0.4</td>
<td></td>
<td>0.1</td>
<td>0.04</td>
</tr>
<tr>
<td>$t_{ij} (i=2,j=2)$</td>
<td>0.4</td>
<td></td>
<td>0.1</td>
<td>0.04</td>
</tr>
<tr>
<td>$t_{ij'} (j=1,i'=2)$</td>
<td>0.6</td>
<td></td>
<td>0.15</td>
<td>0.05</td>
</tr>
<tr>
<td>$t_{jm} (j=1,m=1)$</td>
<td>0.2</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$t_{jm} (j=1,m=2)$</td>
<td>0.2</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$t_{jm} (j=2,m=2)$</td>
<td>0.3</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$t_{jm} (j=2,m=3)$</td>
<td>0.3</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
<tr>
<td>$t_{jm} (j=2,m=4)$</td>
<td>0.3</td>
<td></td>
<td>0.05</td>
<td>0.02</td>
</tr>
</tbody>
</table>

In this problem, supply is disrupted when material is delivered late. Data collected from SMAC (Gear pin) showed that 25% of YIIC's (Supplier 2) orders and 5% of the overseas supplier's (Supplier 1) orders are delivered late. However, the cost of buying steel from the overseas supplier (Supplier 1) is higher than YIIC (Supplier 2). We therefore have four scenarios to explore in this problem. In the first scenario, only the overseas supplier (Supplier 1) delivers late, which has a probability of 5%. In the second scenario, only YIIC (Supplier 2) delivers late, which has a probability of 25%. SMAC (Manufacturer) can use all four risk mitigation strategies (outlined earlier in our model) in both scenarios. In the third scenario, disruption occurs simultaneously in both suppliers with a probability of 1.25%, and the only possible risk mitigation strategy to overcome this is to hold emergency stock in the DCs. In the fourth scenario, there is no disruption in the system, which has a probability of 68.75%.
As can be seen, the new problem formulation method of this paper using the concept of potential paths is so flexible that it can be easily adjusted to more complicated SCs. Solving the mathematical model of this problem (linearized bi-objective model including Equations (50-62) and Equations (33-49)) resulted in the SC network structure and product flows shown in Figures 9 to 12 (using a weighting approach to deal with bi-objective model considering the same weights for both objective functions).
Our results suggest that SMAC (Manufacturer) should select YIIC (Supplier 2) as its main steel supplier and supply products only to Markets 1, 2 and 3 through its second DC (DC 2). It also suggests SMAC (Manufacturer) should use the following risk mitigation strategies:

- Substitute supplier and facility – SMAC (Manufacturer) should have a back-up DC and steel supplier. In this case, its overseas steel supplier (Supplier 1) and second DC (DC 2) are the back-up facilities in its chain. Although these are not its main facilities, it is important to maintain them to manage against supply risk within its chain.

- Emergency stock - should be put in place at its first DC (DC 1). This stock will be partially used to supply market demand if YIIC (Supplier 2) is disrupted and completely used if both YIIC (Supplier 2) and the overseas supplier (Supplier 1) are disrupted.

7. DISCUSSION OF THE MODEL

In this section, we will make several changes to the condition of the SMAC case problem in order to investigate the reaction of the model with respect to these changes. This analysis can approve the correctness of the proposed model.

First, we solve the proposed model only by considering its first objective function, which maximizes the SC's expected profit (Equations 50-62). In this problem, the probability of scenarios 4, 3, 2 and 1 are assumed as 0.6875, 0.0125, 0.05 and 0.25 respectively. The unit capacity reserving costs of the first and second suppliers are the same and equal to 0.01 (sample problem 1). Thus supplier 2 not only has lower
procurement cost, but also its impairment probability is 0.05, which is lower in respect to that of the first supplier's. The results of the model, network structure of the SC and the product flows in different scenarios, are depicted in Figure 13 (for abbreviation, we delete the third tier of the SC that includes the SMAC manufacturer because the number, capacity and location of the facilities in this tier of the SC are completely predetermined).

![Network structure of the SC and the product flows in sample problem 1.](image)

The profit of the SC in this model is 30136.22. The longest time of supplying goods and the largest demand lost are 2506.84 and 7120 respectively. As we expect, the model tried to use paths originated from supplier 2 because the production cost of this supplier is lower and its reliability is higher. In this case, we suggest the following risk mitigation strategies for the SC:

- Substitute supplier and facility (first supplier, oversea supplier, is the back-up facility for this chain).
  - This supplier will be substituted by the main supplier in the second scenario when second supplier is disrupted to supply 7120 material units.
- In the third scenario which has a very low occurrence probability, "do nothing" is suggested.

In the next step, we increase the unit capacity reserving cost of supplier 1 from 0.01 to 0.2 (sample problem 2). The results of this model are depicted in Figure 14.

![Network structure of the SC and the product flows in sample problem 2.](image)

By increasing the cost of capacity, the model discards supplier 1 from its network structure. Since the probability of impairment in supplier 2 is very low, the new chain prefers not to pay for reserving capacity or keeping emergency reserve for these occasions which occur so rarely. By inactivating path $t_{123}$, the profit of the SC reduces to 28920.8. In this case, we suggest the following risk mitigation strategy for the SC:

- When second supplier is disrupted (second and third scenarios), select "do nothing" strategy. This is the most profitable strategy.
Now, we change the probability of scenarios 4, 3, 2 and 1 to 0.6875, 0.0125, 0.25 and 0.05 respectively (sample problem 3). So supplier 1, with high production cost, has higher reliability in the timely delivery of ordered goods, which is more logical than the assumption of the previous problems. In this case, the optimal network structure of the SC and its product flows in different scenarios are depicted in Figure 15.

Since the probability of impairment in supplier 2 is increased from 0.0625 to 0.2625, the model decides to activate some of the paths originated from supplier 1 and keep some emergency stocks for consumption in the occasion of disruption in supplier 1. In this case, the following risk mitigation strategies are suggested for the SC:

- Substitute supplier and facility (first DC and first supplier are the back-up facilities). First supplier will be used instead of the main supplier in the second scenario when second supplier is disrupted. In this scenario both main and back-up DCs will be used to distribute products to the markets.
- An emergency stock of products provided by the second supplier should be stored in the first DC. This stock will be used partially in the second scenario (Supplier 2 is disrupted) and completely in the third scenario (both suppliers are disrupted).

The profit of the SC in this model is 23965.55. The longest time of supplying goods and the largest demand lost are 1873.16 and 7120 respectively. High reliability of supplier 1 increases its attractiveness.

In the next step, we add the second objective function to the model of the problem (Equations 33-49), which tries to increase the responsiveness of the SC by increasing the speed of supplying products to the markets through the SC and augmenting its ability to respond to all the demands of the markets (sample problem 4). If we consider 1 as weights of both first and second objective functions in the model, the network and product flows of this chain in different scenarios will be as in Figure 16.

In this case, by considering the second objective function, the model tries to decrease maximum demand
loss, which in previous sample problem occurs in market 3, from 7120 to 5090. For this purpose, the model has to activate path \( t_{2123} \) for market 3 in scenario 3. The longest path of this network structure for supplying products to the markets is 1873.16. The same risk mitigation strategies as sample problem 3 are suggested here.

Now, for checking the behavior of the model with respect to objective function 2, we increase its weight in the objective function from 1 to 5 (sample problem 5). By increasing the importance of responsiveness, the model activates even less profitable paths such as \( t_{1112} \) and \( t_{1123} \) to decrease maximum demand loss of the SC in the markets and consequently to amplify its responsiveness (Figure 17). Since paths originated from the first supplier are usually shorter, the number of active paths of this kind increases in the network structure of the SC in this sample problem. The longest path of this network structure for supplying products to the markets reduces to 1339.37.

![Network structure of the SC and the product flows in sample problem 5.](image)

In this case, first and second suppliers are the main suppliers in the SC's network structure which are used both in the normal condition. We suggest the following risk mitigation strategy for this SC:

- An emergency stock should be established in the first DC by the products of the second supplier. This stock will be used partially in the second scenario (Supplier 2 is disrupted) to service market 4 and completely in the third scenario (both suppliers are disrupted) to service the second and third markets.

Now we increase the weight of the second objective function from 5 to 20 (sample problem 6). In this case, responsiveness of the SC is much more important than economic considerations. As you can see in Figure 18, retailer 4 is discarded from the SC's structure. Only one path, originated from supplier 2, \( t_{2124} \), supplies product to retailer 4. Since paths of supplier 2 are long, the only way to reduce the longest time of supplying products to the markets in the SC is to ignore market 4. By discarding this retailer, the longest lead time of the SC reduces to 756.35 from 1339.37. We suggest the following risk mitigation strategy for this SC:

- An emergency stock should be established in the first DC by the products of the second supplier. This stock will be used completely in the second and third scenarios to service the second and third markets.
Specifications of these six problems are summarized in Table 5.

Table 5. Specifications of six test problems.

<table>
<thead>
<tr>
<th>Sample problem</th>
<th>Probability of scenarios</th>
<th>Weights of objective functions</th>
<th>Capacity cost</th>
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<tr>
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<td>0.0125</td>
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<tr>
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</tr>
<tr>
<td>6</td>
<td>0.6875</td>
<td>0.0125</td>
<td>0.25</td>
</tr>
</tbody>
</table>

8. ROBUST SOLUTION AND COMPUTATIONAL RESULTS

8.1. Robust solution

In the proposed model, the profit of the SC changes in different scenarios. The objective of this proposed model is to maximize the expected profit of the SC. But as is known, in the realization of each scenario, the resulting profit of the SC can be different from this expected amount. Some of the researches that have been done in the field of risk management in SC network design problems ignore these deviations and only optimize the expected value of objective functions (Mirhassani Al-e-Hashem et al., 2011; Tsiakis et al., 2001; Santoso et al., 2005; Qi et al., 2009; Xanthopoulos et al., 2012; Georgiadis et al., 2011; Cardona-Valdés et al., 2010; Schütz et al., 2009; Goh et al., 2007). They used stochastic programming techniques for modeling their problems. For controlling the deviation of the objective function in each scenario from its expected value, different methods, which are called robust optimization methods, have been proposed in the literature. Robust optimization techniques lead to solutions that are less sensitive to the materialization of the data in a scenario set and remain close to optimum in response to changing input data. Mulvey et al. (1995) and Mulvey and Ruszczyński (1995) are the first works to have been done in the field of robust optimization. In addition to maximizing the expected value of the objective function, they minimize the variance / standard deviation of objective functions in different scenarios. Yu and Li (2000) present a novel robust optimization model to minimize variance in this kind of problem. This model is transformed into a linear program by adding less deviation variables compared with Mulvey et al. (1995) and Mulvey and Ruszczyński (1995). The other way for robust optimization is that proposed by Gulpinar and Rustem (2007) which ensures that performance of the model is optimal in the worst case. This method is called “worst case” robust optimization or “min-max” optimization.

These methods have been used by the other researchers in various problems, sometimes with some
variations to make them more efficient. Pan and Nagi (2010), Mirzapour et al. (2011), Leung et al. (2007), Gulpinar et al. (2003) and Zanjani et al. (2010) use a different method for minimizing the variance of the objective function is scenarios in their SC network design problems. Sawik (2011) and Peng et al. (2011) try to optimize the performance of the problem in the worst case. Peng et al. (2011) consider the problem of designing a reliable network structure for a SC that performs as well as possible under normal conditions, while also demonstrating an acceptable performance in disruptions. The objective function of their model optimizes the performance of the SC in normal conditions and some new constraints are added to the model, which guarantee that the performance of the SC in each scenario will not deviate by more than \( p \) percent (%) from its optimal amount. They called this method \( p \)-robust technique. This method only tries to improve the worst performance of the SC and does not consider the probability of the scenarios. So by using \( p \)-robust method, the performance of the SC in different scenarios can be very dispersed with a high standard deviation and weak expected value. This problem of the method will increase by reducing the probability of normal condition. To solve this problem, we propose a new robust optimization model for our problem called revised \( p \)-robust technique. This method not only improves their worst case by imposing a low bound for the performance of the system in each scenario, but also improves their expected amount by minimizing the standard deviation of the performances. Using this technique in our problem resulted in the following model:

\[
\begin{align*}
\text{Min} & \quad Z'''(x^s_i, v^i_j) = \sum_{s} Pr_s (Z^*_s - Z''(x^s_i, v^i_j)) \\
\text{S.T.} & \quad Z''(x^s_i, v^i_j) = \sum_i (a_i y^i_j + \hat{a}_i \hat{y}^i_j) - \sum_j (b_j z^j_j + \hat{b}_j \hat{z}^j_j) - \sum_M c_m w_m \geq p \times Z^*_s \quad (\forall s \in S) \quad (77) \\
& \quad \sum_{r(i)} u_{j(r(i))} (u_{j(r(i))}^r (d^i_j)) x^s_i \leq N i y_i \quad (\forall i \in I, \forall s \in S) \quad (65) \\
& \quad \sum_{r(i)} u_{j(r(i))} (u_{j(r(i))}^r (d^i_j)) x^s_i \leq N j z_j \quad (\forall j \in J, \forall s \in S) \quad (66) \\
& \quad \sum_{r(m)} u_{j(r(m))} (u_{j(r(m))}^r (d^i_j)) \sum_{r(i)} x^s_i \leq N w_m \quad (\forall m \in M, \forall s \in S) \quad (67) \\
& \quad v^i_j \leq y_i \quad (\forall i \in I) \quad (68) \\
& \quad v^i_j \leq z_j \quad (\forall i \in I) \quad (69) \\
& \quad x^s_i \leq N e^s_i \quad (\forall t \in T \cup T' \cup T'', \forall s \in S) \quad (70) \\
& \quad \sum_{r(i)} x^s_i \leq y_i \quad (\forall i \in I, \forall s \in S) \quad (71) \\
& \quad \sum_{r(i)} x^s_i + \sum_{j \in J} v^i_j + \sum_{r(i)} x^s_i \leq z_j \quad (\forall j \in J, \forall s \in S) \quad (72) \\
& \quad \sum_{r(i)} x^s_i + \sum_{r(j)} x^s_j \leq v^j_j \quad (\forall i \in I, \forall s \in S) \quad (73) \\
& \quad x^s_i, v^i_j \geq 0 \quad (\forall i \in I, \forall s \in S, \forall t \in T \cup T' \cup T'') \quad (74) \\
& \quad y_i, z_j, w_m \in \{0, 1\} \quad (\forall i \in I, \forall j \in J, \forall m \in M) \quad (75) \\
& \quad y_i, z_j \geq 0 \quad (\forall i \in I, \forall j \in J) \quad (76)
\end{align*}
\]

The amount of \( Z^*_s \) for each scenario is separately calculated by solving the following mathematical model:

\[
\begin{align*}
\text{Max} & \quad Z^*_s = Z''(x^s_i, v^i_j) - \sum_i (a_i y^i_j + \hat{a}_i \hat{y}^i_j) - \sum_j (b_j z^j_j + \hat{b}_j \hat{z}^j_j) - \sum_M c_m w_m \\
\text{S.T.} & \quad \text{Constraints (7, 8, 10-19)}
\end{align*}
\]
sum of differences between the profit of the SC in each scenario and the best obtainable profit of the SC in that scenario, $Z^*$ (this objective function, by minimising the standard deviation of the performances, improves their expected amount). Constraint (64) guarantees that the profit of the SC in each scenario should be at least more than $p$ percent (%) of its best obtainable profit (this constraint, by imposing a low bound for the performance of the system in each scenario, improves the worst case of the problem).

8.2. Computational results
In this section, we investigate the performance of $p$-robust method of Peng et al. (2011) with our revised $p$-robust technique for several random sample problems generated based on the case study problem. When the probability of first, second, third and fourth scenarios are 0.1, 0.3, 0.03 and 0.57, respectively, the results of these methods for different values of $p$ are shown in Table 6 (First group problems).

<table>
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<th>p</th>
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<th>Second group problems</th>
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<td>Expected value</td>
<td>Worst case</td>
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<tr>
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<td></td>
<td>25371.15</td>
<td>24980.63</td>
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It is obvious that the two methods have similar performance with respect to the worst case index. The performances, with respect to mean and standard deviation of the SC's profit, are shown in Figures 19 and 20 respectively.
As can be seen, our method always results in higher expected profits and lower standard deviations. By decreasing the amount of $p$ and consequently increasing the feasible region of the mathematical model, the profit mean of our method starts to improve and inversely the profit mean of the $p$-robust technique gradually decreases. By decreasing $p$, our method performs much better than $p$-robust with respect to expected value of profit. As can be seen in Figure 20, in all of these problems our method always has lower standard deviation. By decreasing $p$, the deviations of our method remain roughly constant but those of $p$-robust start to increase considerably. Our method performs for about 1411 and 2530 units better than $p$-robust in mean and deviation indexes respectively.

When we change the probability of first, second, third and fourth scenarios to 0.01, 0.25, 0.0025 and 0.7375 respectively, the results of these two methods for different amounts of $p$ are shown in Table 3 (Second group problems).

In these problems, the behavior of the model is similar to the first ones (Figures 21 and 22). Our method performs for about 15137 and 26673 units better than $p$-robust in mean and deviation indexes respectively. Again these two methods have similar performance with respect to the worst case index.

**8.3. Run time**

SC network design problem includes strategic and long-term decisions and such a problem is only solved once over a long period of time such as once per 10 or 15 years. On the other hand, network structure of a SC is rarely designed from scratch and mainly the existing network is improved by adding new entities. Also note that for solving such strategic problems such as SC network design, even one month running time is
acceptable whereas to solve operational and tactical problems (like inventory, sequencing and scheduling) even 10 hours can be unacceptable. In this section to make a sense about the solvable size of the problems, we used this modeling and problem solving approach for some randomly generated numerical problems. The size of the problems and their solving time are summarized in Table 7. For the problem of last row in Table 7, we could not gain a solution even after 72 hours. The models of problems are solved by a computer with the following feature: Intel(R) Core(TM) 4 Duo CPU, 3.6 GHz, with 12276 MB RAM using the default settings.

<table>
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<th>Number of manufacturers</th>
<th>Number of DCs</th>
<th>Number of retailers</th>
<th>Number of paths</th>
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9. CONCLUSION

In this paper, we develop models for a four-tier SC that can be used by organizations to identify their optimal SC design and risk mitigation strategies given their performance objectives (lean or responsive). We then use empirical data from a large automotive SC to explore how this model works in practice. In doing so, we make a number of contributions to the existing academic literature:

- Considering several risk mitigation strategies opens the hand of the designer to select the least costly combination of these strategies to neutralize the negative effects of the risks. The importance of considering several risk mitigation strategies increases when the costs of imposing these strategies are different in the SC’s facilities which are mainly right in the global SCs in which facilities are located in different countries with different labor, space, facility, etc. costs.
- A path-based model formulation for the problem is introduced. We show that this approach can be used to simultaneously model disruptions in the SC’s both facilities and connecting links by defining a single scenario set. We demonstrate how this path-based formulation can be extended to handle different risk mitigation strategies such as back-up facilities, reserved capacity and emergency stock.
- Several measures are introduced in the literature to for robustness such as variance or standard
deviation of performances, worst-case scenario, lower bound for the performance in each scenario, etc. Using each of these measures to make the model robust (from solution perspective) improves the performance of the chain from one aspect and makes it worse from the other aspects. This situation gets much worse when there is a huge difference between the probabilities of scenarios which is very prevalent in considering disruptions and extreme events. In these cases making an appropriate tradeoff between these measures to get a good solution is necessary.

Future research can now build on this by using our methodology to develop models for other risk mitigation strategies (such as supplier development or profit-sharing), SC designs (with various network structures) or performance objectives (such as social or environmental performance). Equally, it would be useful to understand how SCs should be designed to mitigate against vulnerabilities in their external environment (such as competition and industry factors), when organizations should decide to bring processes, products or services back in house (a trend that seems to be emerging) and how they should best manage ‘reverse’ flows of information and products through the chain (such as how to recycle products after customers have finished using them). All of these are complicated problems faced by practitioners where modelling could help them assess different options and understand the expected impact of the decisions they make.

REFERENCES
Babich V. (2008) Independence of capacity ordering and financial subsidies to risky suppliers. Working paper, Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor, MI.


