DE GRUYTER OPEN

STUDIES IN LOGIC, GRAMMAR AND RHETORIC 41 (54) 2015
DOI: 10.1515/slgr-2015-0019

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INTERACTIVITY AND MENTAL ARITHMETIC: COUPLING MIND AND WORLD TRANSFORMS AND ENHANCES PERFORMANCE

Abstract. Interactivity has been linked to better performance in problem solving, due in part to a more efficient allocation of attentional resources, a better distribution of cognitive load, but perhaps more important by enabling the reasoner to shape and reshape the physical problem presentation to promote the development of the problem solution. Interactivity in solving quotidian arithmetic problems involves gestures, pointing, and the recruitment of artefacts to facilitate computation and augment efficiency. In the experiment reported here, different types of interactivity were examined with a series of mental arithmetic problems. Using a repeated-measures design, participants solved series of five 11-digit sums in four conditions that varied in the type of interactivity: (i) no interactivity (participants solved the problems with their hands on the table top), (ii) pointing (participants could point at the numbers), (iii) pen and paper (participants could note interim totals with a pen), and (iv) tokens (the sums were presented as 11 numbered tokens the arrangement of which participants were free to modify as they proceeded to the solution). Performance in the four conditions was measured in terms of accuracy, calculation error, and efficiency (a ratio composed of the proportion correct over the proportion of time invested in working on the sums). These quantitative analyses were supplemented by a detailed qualitative examination of a participant's actions in the different conditions. The integration of artefacts, such as tokens or a pen, offered reasoners the opportunity to reconfigure the physical presentation of the problem, enacting different arithmetic strategies: the affordance landscape shifts as the problem trajectory is enacted through interactivity, and this generally produced better "mental" arithmetic performance. Participants also felt more positive about and better engaged with the task when they could reconfigure the problem presentation through interactivity. These findings underscore the importance of engineering task environments in the laboratory that offer a window on how problem solving unfolds through a coalition of mental and physical resources.

Keywords: interactivity, mental arithmetic, problem solving, distributed representation, flow

Mathematical problems are embedded in everyday life in a variety of different shapes and forms. When confronted with an arithmetic task, people often rearrange the physical display by interacting with the environment. They might move coins while counting their money, note subtotals with a pen or use their hands to gesture, point, or count (Kirsh, 1995; Neth & Payne, 2001).

Mental arithmetic tasks often entail strategic thinking and deliberate information processing, which require time and effort. Besides basic, well-rehearsed sums, computations are generally said to pose a relatively high load on an individual's internal resources, such as working memory (Ashcraft, 1995; DeStefano & Lefevre, 2004). Numbers are held, added, and manipulated in order to solve the problem, drawing on working memory resources and executive function. The load on working memory varies as a function of the problem complexity and domain specific expertise (and hence the contribution of long term memory). In reducing the load on internal resources, cognitive processes migrate to wherever computations are most easily performed, extending to external resources in a dynamic distributed cognitive system (Kirsh, 2013). The physical actions of an individual within his or her environment are not only integral in distributing working memory load, they also provide a scaffold that can enact new strategies and expand the range of cognition (Kirsh, 2013; Gray & Fu, 2004; Vallée-Tourangeau, 2013).

In this paper, we report an experiment that varied the type of interactivity as participants completed a series of simple mental arithmetic problems. Our analysis profiles performance in quantitative terms—such as the deviations from the correct answer—but also in qualitative terms by describing the different strategies enacted through different forms of interactivity. Increased levels of interactivity have been linked to better performance, possibly due to a stronger focus of attention and better distribution of the load on internal resources (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001; Carlson, Avraamides, Cary & Strasberg, 2007; Vallée-Tourangeau, 2013). Our participants certainly performed better with a greater degree of interactivity, but what we also show is that problem solving efficiency is enhanced by interactivity through the elaboration of qualitatively different strategies. We also sought to capture different levels of engagement and flow promoted in the different interactive environments. We report how participants enjoyed completing the problems as a function of interactivity, but this level of flow was not always a predictor of performance, beyond the benefits conferred by interactivity. The set of quantitative and qualitative analyses offered a more informative window on how interactivity encourages, guides, and constrains the expression of mental arithmetic knowledge and calculation strategy.

1. Thinking in the World

The cognitive and physical resources deployed to tackle a problem may be taxed by various features of the task—such as time pressure, level of difficulty, and fatigue. Reasoners naturally recruit artefacts and use the physical space in which they are situated to make thinking easier and more efficient. This interplay between the cognitive and motor system has been associated with improvements in performance, indicated by increased accuracy and speed (Goldin-Meadow et al. 2001). Movement execution such as nodding, pointing, and the manipulation of a problem's spatial arrangement help to surpass the original limitations of working memory capacity thus lowering the expense of internal resources necessary to solve the task and guide attention (Goldin-Meadow, Alibali, & Church, 1993; Vallée-Tourangeau, 2013). Kirsh (1995) describes an organizing activity that recruits external elements to reduce the load on internal resources as a complementary strategy to internal mental processes. Therefore, interacting with the environment and utilizing artefacts can improve performance by distributing the storage and computational demands of the task across resources internal and external to the reasoner. Such distributed cognitive processes shift the cognitive load from the reasoner onto a system in which she is embedded (Vallée-Tourangeau, 2013).

The distribution of the computational cost across resources can enhance performance when solving a problem. However, this distribution of cognition concomitant with the dynamics of interactivity can also enact the application of different problem solving strategies which can improve performance. Yet, it is not only the problem itself or its complexity that impacts how accurately or efficiently an individual performs in a mathematical task. The physical features of the problem presentation can guide behaviours and strategic choices in the path to a solution (Vallée-Tourangeau, Euden, & Hearn, 2011).

The importance of the environment in shaping the content and function of cognitive processes has long been recognized by cognitive psychologists (e.g., Simon, 1996). One research strategy is to explore problem solving performance across superficially different but structurally isomorphic problems. Using isomorphic problems makes it possible to retain the same problem space while at the same time changing the cover story by varying the rules or move operators. It has been suggested that these changes can cause individuals to construct different (internal) problem representations resulting in differences in the solution process (Hayes & Simon, 1977). Zhang and Norman (1994) demonstrated the importance of external representations in an elegant series of experiments on transformation problem solving (using Tower of Hanoi isomorphs). Problems were structurally isomorphic but were presented with different objects. In using different artefacts the external rules and constraints for the problem were changed, which had a substantial impact on problem solving performance. However, in Hayes and Simon (1977) and Zhang and Norman (1994) the role of interactivity in shaping and reshaping the external problem presentation is largely ignored. Embedded in the problem presentation are the varying possibilities for interaction, the nature of these interactions having the potential to direct strategic choices (Neth & Payne, 2001; Kirsh, 2013). Through manipulations of artefacts, a dynamic loop of information and action flows between a person and the outside world: new perspectives are observed leading to new strategies improving the prospect of problem solving (Magnani, 2007). Processing of the problem is shared between the environment, and the body and mind of the agent, configuring a distributed thinking system (Kirsh, 1995).

Mental Arithmetic. Previous research on mental arithmetic has investigated gesturing (Goldin-Meadow et al., 2001), interactivity and additions utilising a computer interface (Neth & Payne, 2001; Neth & Payne, 2011), interactivity and working memory (Vallée-Tourangeau, 2013), and simple coin counting strategies (Kirsh, 1995). Results indicate that interactivity influences performance and the ways by which participants achieve solutions. However, the picture is piecemeal, fragmented by different methodologies, and no study as yet has compared a wide range of different types of interactive behavior using artefacts. Consequently, the current experiment investigated the role of interactivity in adult participants using tangible artefacts with which the problem presentation could be modified through the completion of an arithmetic task in the form of simple additions. Each unique sum consisted of eleven single digit numbers arrayed in a random configuration (see Figure 1). The decision to use simple additions for the exploration of interactivity in mental arithmetic was motivated by three considerations. First, we chose a task requiring only basic arithmetic skills that we anticipated would be unlikely to trigger math anxiety (Ashcraft & Moore, 2009). Second, artefacts, such as tokens and pen-paper that closely resemble commonly used items, were objects that could be introduced in the experimental session with relative ease and efficiency. Finally, the relative length of the sums would make it possible to map the participants' progress in terms of interim totals and hence map the trajectory to solution. Therefore, in the experiment discussed here, the external problem presentation tracks the dynamic interface between the agent's internal representation and the world. Interactivity and the potential to re-shape the problem presentation

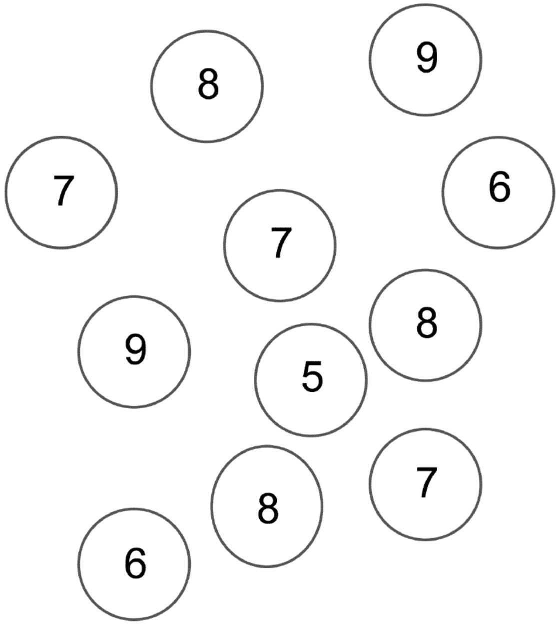

Figure 1. One of the twenty unique sums created. Each sum consisted of 11 single-digit numbers between one and nine. The numbers were categorized as low (1–4) or high (5–9) in order to generate a range of sums in a principled manner. A set of five sums was randomly allocated to each of the four experimental conditions.

were manipulated in terms of four conditions aimed at simulating the tools that might be used by individuals outside the psychologist's laboratory.

Performance was measured in terms of accuracy, calculation error, latency, and efficiency. Not only did we expect accuracy to be influenced by interactivity, but also efficiency should be related to the degree to which participants can modify the problem presentation as they compute the totals. We defined efficiency in terms of the degree of accuracy relative to the resources invested in completing the sum; the latter was operationalized as the time taken to do the sum. We expected to observe different strategies enacted through interactivity, and hence that performance would be best in the conditions that afforded the re-arrangement of the initial problem presentation.

In order to test the effect of altering the physical presentation of a math problem on performance, 60 undergraduate participants were presented with simple addition problems in four different interactivity conditions. In all four modes of interactivity the problems appeared in the same format, as templates consisting of 11 circles (2.2 cm in diameter) covering between 1/4 and 3/4 of a side of A4, presented on a board measuring 39 cm by 34 cm.

In the first condition, participants were asked to add a sequence of single-digit numbers with their hands down and in a second they were allowed to point at the numbers. Thus in these two conditions, the problem presentation could not be modified, but participants could engage in some complementary actions in the latter. In the other two conditions, participants could re-shape the problem presentation. Hence in the third condition, participants were given a pen. Using this pen and the paper on which the digits were presented, they could then recast the sum, as they saw fit. In the fourth, the sums were presented as a set of wooden numbered tokens (2 cm in diameter) that participants were invited to move around the board to arrive at the correct sum. The tokens were initially arranged using templates of tracing paper, created with the same configuration of the constituent numbers as the paper version of the other conditions (to ensure that the perceptual starting point was the same in each condition). Each sum consisted of 11 single digit numbers, 1–9, by varying the combination of high and low digits (see Figure 1 for an example). Participants solved five problems in each of the four conditions, and hence 20 sums were presented to participants; the sums were randomly allocated to each of the four conditions, and the order of the conditions was counterbalanced across participants. Before each set of sums, participants were shown an A4 page with the instructions for the forthcoming task. The wording was similar for all four conditions with changes made to reflect the interactivity opportunities and constraints. Other than in the static condition, where they were instructed not to move their hands, participants were under no obligation to use the pen, the tokens, or to point, whichever was relevant to the current experimental condition.

2. Mental Arithmetic Performance I: Quantitative Results and Discussion

In order to maintain consistency of experience across conditions and participants, each sum was initially obscured from the view of the participant by a screen with instructions from the researcher to start once the screen was removed. Four dependent variables captured different aspects of performance: (i) latency from the time the screen was removed until participants announced an answer; (ii) accuracy in terms of the proportion of correct answers in each condition; (iii) the absolute calculation error, calculated as the absolute difference from the correct solution, for example if the correct solution was 40, an incorrect answer of 42 or 38 would result in the absolute calculation error of 2; and (iv) efficiency operationalized as the ratio of the proportion of correct answers for a set of sums divided by the proportion of time (out of the maximum time) invested in completing the sums. Let's take each of the measures in turn.

Latency. Participants generally took about the same amount of time to complete the task across the four conditions (static $M = 26.79$, $SD = 9.88$; pen-paper $M = 27.26$, $SD = 9.73$; pointing $M = 25.70$, $SD = 10.09$; tokens $M = 26.58$, $SD = 10.41$). The main effect of interactivity in the one-way repeated measures analysis of variance (ANOVA) was not significant, $F < 1$.

Proportion of Correct Answers. The mean number of correct answers was greatest in the tokens ($M = .69$, $SD = .22$) and the pen-paper ($M = .69$, $SD = .23$) conditions. The pointing condition ($M = .66$, $SD = .26$) produced fewer accurate calculations, with the static condition producing the weakest performance ($M = .60$, $SD = .30$). A one-factor repeated measures ANOVA indicated a significant difference between conditions, $F(3, 177) = 3.12$, $p = .027$, $\eta^2 = .050$. Post-hoc tests revealed a significant difference between the static and the pen-paper conditions ($p = .006$) and the static and tokens conditions ($p = .020$), but no significant difference between the pen-paper and tokens conditions.

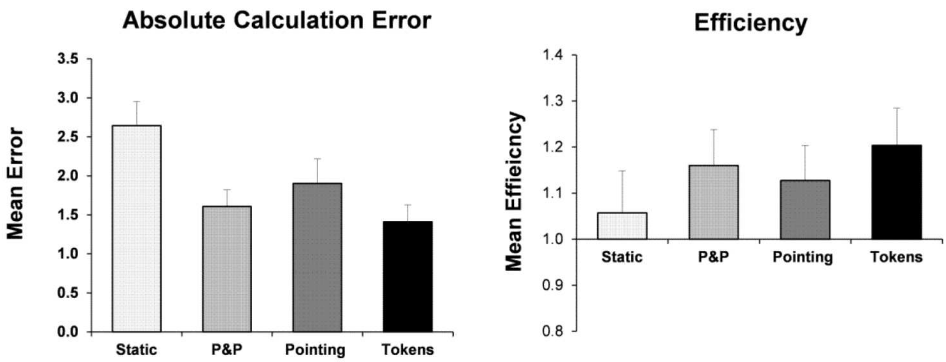

Figure 2. Mean absolute calculation error (left panel) and mean calculation efficiency (right panel) in the four experimental conditions. Error bars are standard errors of the mean.

Absolute Calculation Error. As illustrated in the left panel of Figure 2, the more interesting trends in performance were evident in the deviation from the correct answers: the best results being observed in the token condition and the worst in the hands down static condition. Deviation from the correct answer was lowest when using the tokens ($M = 1.41$, $SD = 1.69$); the pen-paper ($M = 1.61$, $SD = 1.65$) and the pointing ($M = 1.90$, $SD = 2.43$) conditions produced higher deviations, with the static condition ($M = 2.64$, $SD = 2.39$) eliciting the poorest results with the highest mean absolute calculation error. The one-factor repeated measures ANOVA revealed a significant difference between interactivity conditions, $F(3, 177) = 6.34$, $p < .001$, $\eta^2 = .097$, with post-hoc tests indicating no significant difference between pen-paper and tokens but again there was a significant difference between the pen-paper and the tokens conditions when compared to the static condition ($p = .005$, $p < .001$ respectively).

Efficiency. As these efficiency ratios were defined, higher ratios mean relatively better performance as a function of the resources invested in completing the problem. As the right panel of Figure 2 shows, calculations were most efficient in the tokens ($M = 1.20$, $SD = .62$) and the pen-paper conditions ($M = 1.15$, $SD = .60$) with the static ($M = 1.05$, $SD = .71$) and the pointing ($M = 1.12$, $SD = .59$) conditions being least efficient. While the static condition produced the lowest level of efficiency, the main effect of interactivity was not significant however, $F(3, 177) = 1.39$, $p = .247$.

Discussion. The performance differences across the conditions revealed that even with these very simple sums, interactivity with artefacts helped to transform the execution of the calculations. While the participants used around the same amount of time to complete the sums regardless of the interactivity condition, they made more errors and the magnitude of the errors was larger when no artefacts were in use. We also note the marginally significant difference in the efficiency of performance between the hands-down static condition and the high interactivity tokens condition ($p = .060$). Next, we turn to more a more qualitative characterization of performance that offers an interesting window onto how artefacts alter performance.

3. Mental Arithmetic Performance II: Qualitative Observations

In the present study, the performance of a random selection of participants ($n = 26$) was captured on video in order to analyze the pathways to solution as they unfolded for these simple additions. Each participant consented to the video recording, which took place across the entire experimental session in a purpose-built laboratory with unobtrusive and noise-free cameras. The sample was determined by the availability of the audio-video observation lab and participants' consent to being filmed. We based our analysis of the participants' problem-solving trajectory on two simple measures, namely the nature of the groupings of numbers, and whether these groupings were "good provisional sums" (GPS; defined as

Σ 5 MOD = 0; Vallée-Tourangeau, 2013) offering congenial stepping-stones to promote more efficient problem solving. For example, grouping a "7" and an "8" to form an interim total of "15", may encourage participants to group a "6", a "4" and a "5" to create "30" as the next interim total on the way to the final solution.

Here we will explore the actions of one participant in detail, concentrating on a selection of the sums completed (see Figure 1 for an example of how the digits were arranged), for three of the interactive conditions. (The static condition is excluded from analysis as the video revealed no insight into the calculations.) A number of video recordings were discarded from analysis due to technical reasons such as the hands or head obscured the camera's view of the participant's actions. From the remaining sample, we selected for detailed discussion a 21 year old, right-handed male undergraduate psychology student. His performance was considered typical of other participants as he utilized the pen and paper, pointed, and grouped the tokens in a similar manner to those participants analyzed frame by frame. In addition, as these behaviours were identifiable across all conditions for this individual, there is the opportunity to build a picture of how the presentation of the sums subtly alters his arithmetic strategy. The groupings will be labeled in keeping with the protocol established by Vallée-Tourangeau (2013) as good provisional sums (GPS). In addition, those groupings made up of the same digits, for example grouping all the nines together, will be called same digit groupings (SDG).

The Token Condition. The sums were presented as a random array of wooden tokens. All the token movements in this condition made by the participant were of a sliding nature. He generally moved the tokens into distinct groupings on different parts of the board before adding them together.

With the first presentation of the initial five sums, the participant initially arranged some of the tokens into GPS: 2 and 8, then a separate group of 7 and 3, then 4 and 6. These three groups were moved together to make one large group equaling 30. Two further groups were created: 5 and 6, 8 and 3. The 5 and 6 were moved to the ungrouped token 9 to make 20. The group of 8 and 3 was then combined with the 5, 6 and 9 to total 31. Thus 30 from the first large group added to 31 from the second large group summed to the correct answer of 61. The final order of groupings (with interim totals in brackets) was 2, 8 (10) + 7, 3 (20) + 4, 6 (30) and 5, 6, 9 (50) and 8, 3 (61). Thus, moving the tokens into the three groups of ten then the forming another of 5 and 6 unveiled the opportunity to add to this 9, bringing together the groups in a total of 50, which eased the way to add the final 11.

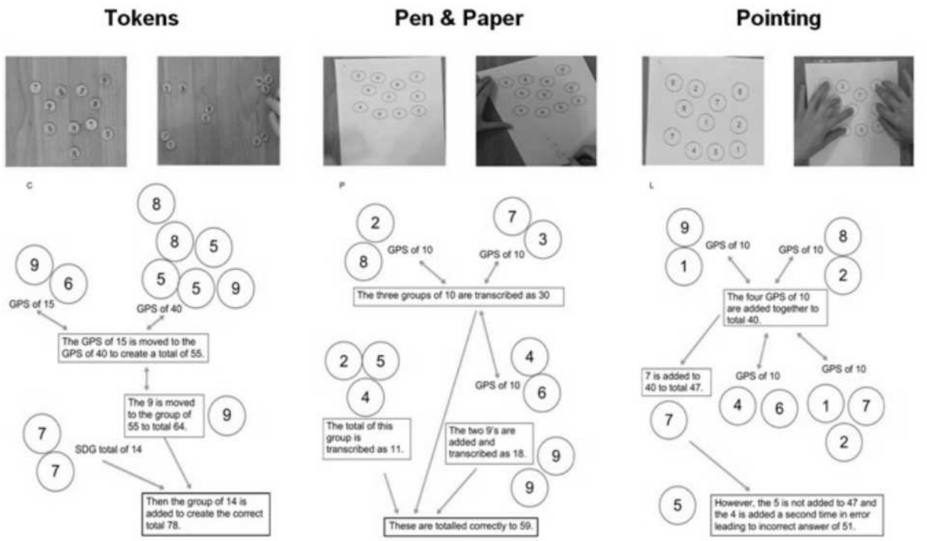

Figure 3. The three panels show the stages of progress as the participant solves simple math problems in varying conditions of interactivity. The first screen shot in each of the three panels shows the initial problem presentation. The second screen shot captures the participant part way through the path to solution. The bottom image is a reconstruction of the final stages of the problem-solving process as revealed in frame-by-frame analysis of the participant's actions.

This pattern of using GPS was clearly evident in the next two problems and the fifth one. In the latter, the answer given was wrong by 10. Yet by that time, this participant was an efficient mover of tokens, reconfiguring the problem space into congenial subtotals of 10, and he generated an answer, albeit an incorrect one, in only 11 seconds. A careful analysis of the participant's movements shows that he moved his hand across the groupings created; probably then, tallying the GPSs, he missed a grouping that added to 10; hence the calculation error.

When summing the digits it may seem obvious to create good provisional sums from the layout of the tokens; however, the important observation is that the secondary rearrangement of tokens into the two large groups emerged from cues provided by moving the tokens into the original smaller groupings. This is a key observation that highlights the importance of interactivity: *the affordance landscape shifts as the problem trajectory is enacted through interactivity.* This was also discernible when the participant completed the fourth set of sums (see the leftmost panels of Figure 3). In this case, the GPS were not as obvious and the participant had no choice but to employ a different approach. Initially, the participant slid tokens into the

SDG of 5, 5 then 9, 9, the third 5 was moved but not grouped; 7, 7 followed by 8, 8 then 6 and 9. He then proceeded to re-arrange the tokens into these altered groupings: 5, 5, 5 + 9 + 8, 8 were merged to form a group totalling 40, then 9 was added and 6 to equal 55, then 9 to make 64, which made for an easy addition of 14 from 7 and 7. In changing the configuration, initial efforts of grouping cued and prompted a next level of grouping that simply could not occur without artefacts and interaction. In other words, he could only create the final groups after re-configuring the tokens from the first attempt.

The Pen and Paper Condition. The participant employed a strategy of crossing through the digits with the pen provided and writing numbers and additions at the bottom of the page using the paper on which the digits were printed as a worksheet. This system resulted in the largest mean latency across the four conditions. There was evidence of attempted grouping in provisional sums in groups of 10's; however, he used his fingers and the pen to point to numbers in order to keep track of his additions.

The middle panels of Figure 3 illustrate a typical effort in this condition. The participant began by crossing off 2 and 8 to make 10, then 3 and 7 for another interim 10, then 6 and 4. He then pointed to 2, 5 and 4 and wrote 11. Next he added the 9 and 9, writing 18. He dragged his pen across the six digits crossed through earlier that made the three groups of 10 and wrote 30. He added the transcribed numbers of 11, 18 and 30 to announce the correct total of 59. Note the additional tracking and mapping necessitated by the re-transcription of the random number configuration into a more canonical columnar arrangement (something that was neither required nor observed in the tokens condition).

Another sum in this condition provides a window into how the formation of good groupings was not as obvious; the task of mentally rearranging digits was more challenging from a fixed arrangement unlike the manipulable token presentation. The participant began by crossing off 2 and 8, then 4 and 6, writing the number 20 below the printed digits. He then dragged his pen across the 4, 5 and 9 and wrote 18 on the worksheet. Again moving his pen across 7, 8 he added the figures 9, 9, 6 to the worksheet, then 18 again and crossed off the hand written numbers 9, 9 and 6, then wrote 6 above the 20. He scanned his pen over the written numbers 18 and then added 40, he crossed out 40, writing 36 on the right, then below the 20 he penned a 6 and announced the answer of 62. Ultimately, this convoluted series of crossing outs and adding on paper resulted in a deviation of four from the correct answer. Thus, in the pen and paper condition, the participant sometimes re-transcribed the additions, sought judicious pairings, but the mapping process was slow and the participant had to be particularly vigilant as he systematically converted struck-off digits into provisional sums.

Unlike the highly manipulable token condition, here the participant appeared to enact a form of iterative consulting by switching between his own reconstructed representation of figures or crossing-offs on the worksheet, and the random array of static digits, potentially increasing the chance of transcription error. The marking of the digits into GPS was more hesitant in the pen-paper condition than in the token condition, where the tokens were grouped together swiftly. It may be the case that the crossing off slowed him down; therefore it appears he may have rapidly switched to relying on internal memory rather than maximizing the use of the external resources. It appears that the transcription of digits into calculations on the worksheet became more convoluted and time consuming. This indicated a possible cost-benefit trade-off occurring during the distribution of cognitive resources across the continuum between internal and external memory, even at the risk of making an error (cf. Fu & Gray, 2000).

The Pointing Condition. The participant used both hands and most fingers to point and to anchor counting points. At times throughout the task there was hovering with the fingers above numbers, pauses, counting and re-counting of digits. The strategy is mixed with some grouping at the beginning of each task, with more grouping of like numbers together than in the token condition.

In one set, the SDG of 5, 5 then 7, 7 then 8, 8 were created, then 9, 7 followed by 6. Therefore, the totals were 10, 24, 36, 52, 68 and 74. In the final set, there are very few like numbers to group together, therefore another strategy was required. In this case, the participant used congenial totals of 8, 2, then 1, 9, then 6, 4 followed by 2, 7, 1. However, in order to achieve this he pressed his fingers on top of these 9 digits on the page; in doing so his hands obscured the remaining two numbers 5 and 7 (see the rightmost panels in Figure 3). The upshot is that, upon taking his hands away, he added the 4 that was already accounted for and did not touch or point to the 5 that remained uncounted. Therefore he announced an incorrect answer.

Discussion. In scrutinizing the movements of the participant, frame-by-frame patterns emerge from what appear to be often random, unconnected strategies. However, what is particularly interesting is how the emergent pathway to solution differs between conditions of interactivity. The different types of interactivity transform the initial problem presentation with the agent behaving differently as a function of the artefacts offered, and hence the distributed system reconfigures itself on the path to solution. In using the wooden tokens to add the numbers, shifting them into different groupings disclosed a number of ways to ease calculation. At times actions resulted in good provisional sums, other times same digit groupings were made or tokens were separated from the other tokens to ease thinking. This re-configuration of the problem space expands the range of cognition revealing new ways to sum the numbers; ways that would not have been achievable without the possibility for arrangement and re-arrangement of the artefacts offered by the level of interactivity. The pen and paper condition presented the opportunity to use a more traditional method of summing numbers. In every sum the participant crossed off some or all of the numbers and re-transcribed figures onto the page. The examples scrutinized here showed that it was more effortful to keep track of groupings and that new strategies were not as easily enacted as with the tokens. Similar observations were made in the pointing condition: while the GPS and the SDG appeared to be easily identified by the participant, it was not an easy task to maintain the totals using his fingers. There was no evidence that this condition opened new pathways to alternative strategies for adding these digits. The frame-by-frame scrutiny of movements in problem solving afforded by different artefacts has emerged as an important tool in exposing strategies used by this participant. It may well be that, in future studies, verbal protocols or the measurement of eye movements in conjunction with filming could uncover different strategies employed in a low interactivity static condition.

4. Engagement and Flow

Student engagement in performing academic tasks may be an influential factor in learning and achievement, with the activity by which learning is experienced providing a stimulus for deep engagement and attaining the feeling of being in the "flow" (Shernoff, Csikszentmihalyi, Schneider, & Shernoff, 2003). It is also possible that a task that offers a student a sense of connection to the real world is more likely to maximize student engagement (Newmann, Wehlage, & Lamborn, 1992). Furthermore, Schiefele and Csikszentmihaly (1995) discuss the importance of the affective experience on performance, while engaging in mathematics in the classroom. Positive emotions elicited by the task experience may contribute to increased problem-solving capacities (Shernoff et al., 2003). With these observations on task engagement in mind, we developed an eight-item scale to assess participants' attitude towards the familiar and novel tasks presented in this experiment. The objective was to ascertain whether any performance differences were attributable to the use of artefacts. The scale was found to be highly reliable in all four conditions (Static, Cronbach's $\alpha = .80$; Pen-paper, Cronbach's $\alpha = .77$; Pointing, Cronbach's $\alpha = .78$; Tokens, Cronbach's $\alpha = .77$). The attitude of participants was more positive toward the pen-paper ($M = 37.98$, $SD = 8.38$) and the tokens ($M = 37.78$, $SD = 8.94$) conditions, than the pointing condition ($M = 34.12$, $SD = 8.76$) and least favourable for the static condition ($M = 31.63$, $SD = 9.13$). In a one-way repeated measures ANOVA, the main effect of interactivity was significant, $F(3, 117) = 17.07$, $p < .001$, $\eta^2 = .231$. Post-hoc tests further identified significant differences between the static and the tokens conditions and the static and pen-paper conditions ($p < .001$ for both comparisons). The static and pointing conditions were also significantly different ($p = .025$). Feelings toward the pointing condition differed significantly from those in the pen-paper ($p < .001$) and tokens conditions ($p = .008$).

It is noteworthy that the participants' attitude towards completing the sums in the different conditions paralleled the impact of interactivity on performance. Conditions involving external resources, pens or tokens, seemed to elicit a more positive, engaged attitude towards the simple arithmetic problems than the restricted, static condition. Of course, participants were also more accurate in the interactive conditions. But the more positive attitudes towards the problems cannot be attributed to task success since participants were not given feedback about their performance; that is, after announcing each sum, the participants were not informed by the experimenter whether their answer was right or wrong. However, results also showed that as task engagement scores increased, efficiency increased, with significant correlations in the tokens ($r(58) = .25$, $p = .056$) and pen and paper ($r(58) = .26$, $p = .045$, see Table 1), the token and pen-paper condition being the two conditions in which participants exerted some control over problem configuration. This suggests that engagement with the task tended to encourage more efficient performance. These findings are in keeping with the notion that higher levels of personal involvement positively affect performance (Shernoff et al., 2003). Also, changing the visual display may ease the task and thereby lighten the cognitive load, which increases effectiveness and alters attitudes (Vallée-Tourangeau, Sirota, & Villejoubert, 2013). We speculate that this preference for the use of artefacts elicits a more positive attitude by participants, as utilizing pen and paper is a familiar method of computation and the use of manipulatives is often considered a "fun" learning technique (Moyer, 2001). Therefore, one may argue that improvement in performance in the higher interactivity conditions may be explained by affect, since employing artefacts seemed to be preferred.

Table 1
Correlations between task engagement scores and the performance measures in all four experimental conditions

	Static	P&P	Point	Tokens
Accuracy	.171	.315*	.062	.226
Calc. Error	–.215	–.386**	–.156	–.137
Latency	–.175	–.156	–.264*	–.270*
Efficiency	.197	.260*	.174	.248

Note. Calc. Error = Absolute Calculation Error, P&P = Pen and paper, Point = Pointing.
* $p < .05$, ** $p < .01$

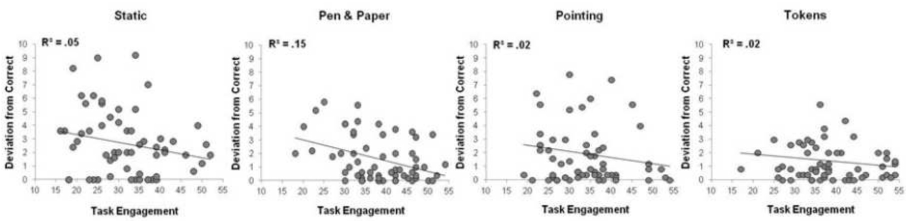

Figure 4. Scatterplots of task engagement scores and mean absolute deviation errors in the four experimental conditions. The slope of the linear trend in all four conditions was negative, but only significantly so in the pen and paper condition, $r(58) = -.386$, $p = .002$.

However, there is good evidence in the correlational data that this cannot be the explanation; that is, beyond task enjoyment, interactivity simply confers a distinct performance advantage. This is revealed in the pattern of correlations between engagement and calculation errors in the four conditions (as plotted in Figure 4). A negative correlation trend between task engagement and absolute calculation error was observed in all four conditions (as engagements increased, errors decreased) but the correlation between task engagement scores and deviation was only significant in the pen and paper condition (see Figure 4). The absence of correlations in the token condition when compared to the correlations for these measures in the pen-paper condition implies that the familiarity of using pen-paper may be a large component of performance, whereas it is the manipulability of the tokens themselves rather than affect alone that accounts for the improvement in performance.

5. Conclusion

In calculating simple arithmetic sums, individuals usually create opportunities to deploy a range of complementary strategies as a function of the level of interactivity that binds mental and physical resources. Studying systems rather than individuals poses theoretical and methodological challenges. Theoretically, the nature of the problem representation and the trajectory of the solution as it evolves from an embryonic to a fully formed answer, should perhaps be understood as being distributed and configured in terms of a transaction between the participants' internal resources and the shape and nature of the resources in the external environment. Attempting to segment and independently specify the components of a cognitive system, namely the thinking agent and his or her immediate environment, is not as productive as seeking to characterise the system as a whole (see Baber, Parekh, & Cengiz, 2014). The methodological implications of this transactional perspective are important. Of course, systems can be more complex, and composed of a much wider range of functional elements, which challenges the traditional toolkit of experimental cognitive psychologists designed to deal with a cognitively sequestered individual in a laboratory environment that generally prevents interactivity. The findings and methods reported here suggest that a more qualitative idiographic cognitive science supported by an observational toolkit that can code at a much smaller time scale the evolution of a problem representation and its solution would make a substantial contribution to problem solving research.

In considering the impact of interactivity on problem solving it is not only the performance and task engagement that is altered with a changing problem presentation but also the affordances offered by the artefacts to hand. The qualitative analysis here clearly illustrates how the agent behaved differently as a function of the artefacts offered and hence how the system reconfigures itself on its path to solution. "An artifact is not a piece of inert matter that you *act upon*, but something active with which you *engage* and '*interact*'" (Malafouris, 2013, p. 149, emphasis in the original). Interactivity encourages the reconfiguration of the problem space, opening windows to new strategies, improving efficiency and enjoyment.

Finally, adapting the cognitive psychologist's laboratory to permit the physical manipulation of a problem presentation offers a more representative window onto thinking outside the laboratory. To be sure, people can simulate and think in their head without physically interacting with the outside world (although this internal cogitation may well reflect the internalization of much interactivity); but they often "go to extraordinary lengths to *avoid* having to resort to . . . fully environmentally detached reflection(s)" (Clark, 2010, p. 24, emphasis in the original). The data presented here reveals the importance of engineering task environments in the lab that support distributed problem representations to better understand the engagement of individuals as they explore and manipulate the external world to solve problems.

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