Undervaluation and Overvaluation of Ambiguous Assets

(work in progress)

by

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Contents:

1. Introduction

- Relevant events:

  - the dot.com bubble

  - the recent financial crisis.

- Basic questions:

  - Can the presence of ambiguity cause the under- and overvaluation of assets?
  - Can it explain the presence of bubbles, crises and recoveries?

- Answer:
  - Yes
  - Yes, if ... .

2. Some Related Literature

- Intuition of ambiguity
  - as in Knight (1921) and Keynes (1937).
- Modelling of ambiguity
  - in the tradition of Schmeidler (1982/1989)
  - Chateauneuf, Eichberger and Grant (2007).
- Dynamics of the ambiguity level
  - dynamic inconsistency in updating, e.g. Gilboa and Schmeidler (1993) and Eichberger and Kelsey (1999)
  - alternatively: following the intuition of ambiguity as in Knight (1921) and Keynes (1937).

Under- and Overvaluation of Ambiguous Assets

- Ambiguity and behavioural finance
  - Herding: Ford, Kelsey, Pang (2005)
  - Momentum Trading: Kelsey, Kozhan and Pang (2010).
- Ambiguity and crises
  - Bank runs: Spanjers (1999/2008a)
  - Currency crises: Spanjers (1999/2008b)
  - The financial crisis: Spanjers (2010).

Keynes (1937) gives a description of what is meant by ambiguity:

"By 'uncertain' knowledge, let me explain, I do not mean merely to distinguish what is known for certain from what is only probable. The game of roulette is not subject, in this sense, to uncertainty [...]. The sense in which I am using the term is that [...] there is no scientific basis on which to form any calculable probability whatever. We simply do not know."

[pp. 113-114]

To Keynes, these implications are not without consequences for economic theory:

“[T]he fact that our knowledge of the future is fluctuating, vague and uncertain, renders wealth a peculiarly unsuitable subject for the methods of the classical economic theory. This theory might work very well in a world in which economic goods are necessarily consumed within a short interval of their being produced. But it requires, I suggest, considerable amendment if it is to be applied to a world in which the accumulation of wealth for an indefinitely postponed future is an important factor; and the greater the proportionate part played by such wealth accumulation the more essential does such amendment become.” [p. 113]

He than continues to discuss its implications:

"Now a practical theory of the future [...] has certain marked characteristics. In particular, being based on so flimsy a foundation, it is subject to sudden and violent changes. The practise of calmness and immobility, of certainty and security, suddenly breaks down. New fears and hopes will, without warning, take charge of human conduct. The forces of disillusion may suddenly impose a new conventional basis of valuation. All these pretty, polite techniques, made for a well-panelled board room and a nicely regulated market are liable to collapse. At all times vague panic fears and equally vague and unreasoned hopes are not really lulled, and lie but a little way below the surface." [pp. 114-115]

3. Ambiguity

- Uncertainty can be distinguished in:
  - (calculable) ***risk*** and
  - (incalculable) ***ambiguity***.
- Risks may fail to be calculable because either
  - one cannot make a reasonable probability estimate for the relevant states of nature, or
  - one does not know the outcome that is obtained for the specific states of nature.

Investors who face ambiguity tend to:

- hope for the best (***optimism***) and/or
- fear the worst (***pessimism***).

Examples for situations with ambiguity are:

- after terrorist attacks of 9/11
  (prob. known, outcomes unknown, pessimism)
- BSE crisis
  (outcomes known, prob. unknown, pessimism)
- Dot.com bubble
  (outcomes known, prob. unknown, optimism).

4. The Basic Model

Investors

- Beliefs are described by:

  - a ***probability estimate*** π over S.

  - a ***level of confidence*** $\gamma \in [0,1]$ in π;

    $\eta = 1 - \gamma$ denotes the ***level of ambiguity***.

- Ambiguity attitude is described by:

  a ***degree of optimism*** $\beta \in [0,1]$ where

  $\beta = 0$ represents full pessimism and

  $\beta = 1$ represents full optimism.

- For state-contingent consumption the leads to in:

$$U(x, \eta; \beta) := (1 - \eta)\, \mathbf{E}\{u(x))\} + \eta\,(1 - \beta) \min_{s \epsilon S} u(x(s)) + \eta\, \beta \max_{s \epsilon S} u(x(s)).$$

- Investors are characterized by their ambiguity attitude β.
- Investors are uniformly distributed over [0,1].
- Each investor has one unit of wealth to invest.

Assets

- Unambiguous asset with expected payout 1.
- Ambiguous asset:
  - level of ambiguity η
  - expected payout $\mathbf{E}\{x\} = E(\eta)$
  - institutionally determined ambiguity for the (targeted) marginal investor β is denoted by $\eta^i(\beta)$.

Investment in the ambiguous asset

- The critical level of ambiguity for investor β to invest in the ambiguous asset is denoted by $\eta^p(\beta)$.

Equilibrium

- A combination (η^*, β^*) of ambiguity level and marginal investor in the asset is an equilibrium if:

  i) $\eta^* = \eta^p(\beta^*)$

  ii) $\eta^* = \eta^i(\beta^*)$.

5. An Example of Undervaluation

- Expected payout of the ambiguous asset

  $\mathbf{E}\{x\}= E(\eta) := 1 + \eta.$

- Possible payouts: $x \in [0, 2]$.
- This leads to

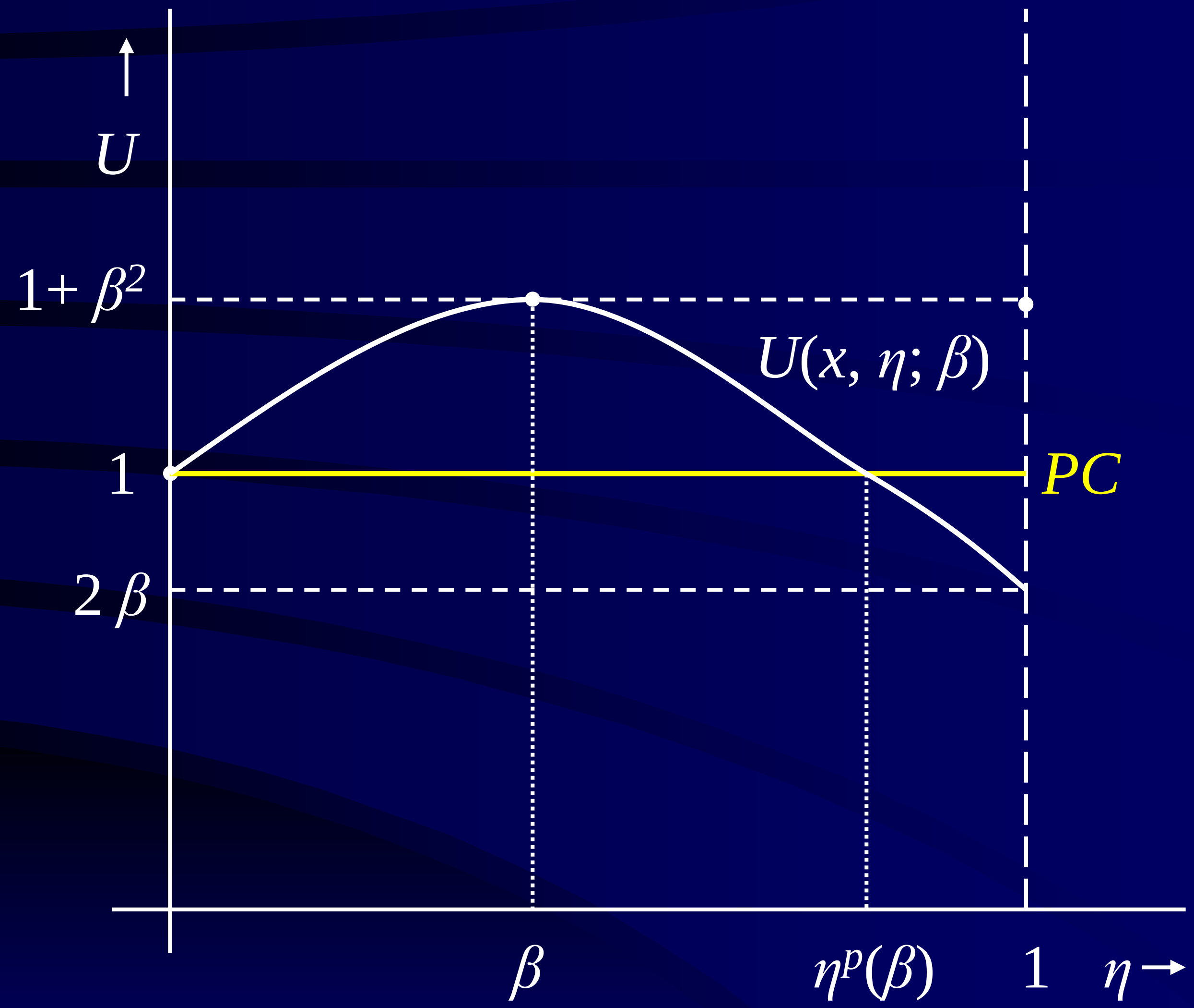
$$U(x, \eta; \beta) := (1\text{-}n)\ E(\eta)$$

$$+ \eta\ \beta\ \max_{s \in S} x(s) + \eta\ (1\text{-}\beta)\ \min_{s \in S} x(s)$$

$$= (1\text{-}\eta)\ (1+\eta) + 2\ \eta\ \beta + 0 =$$

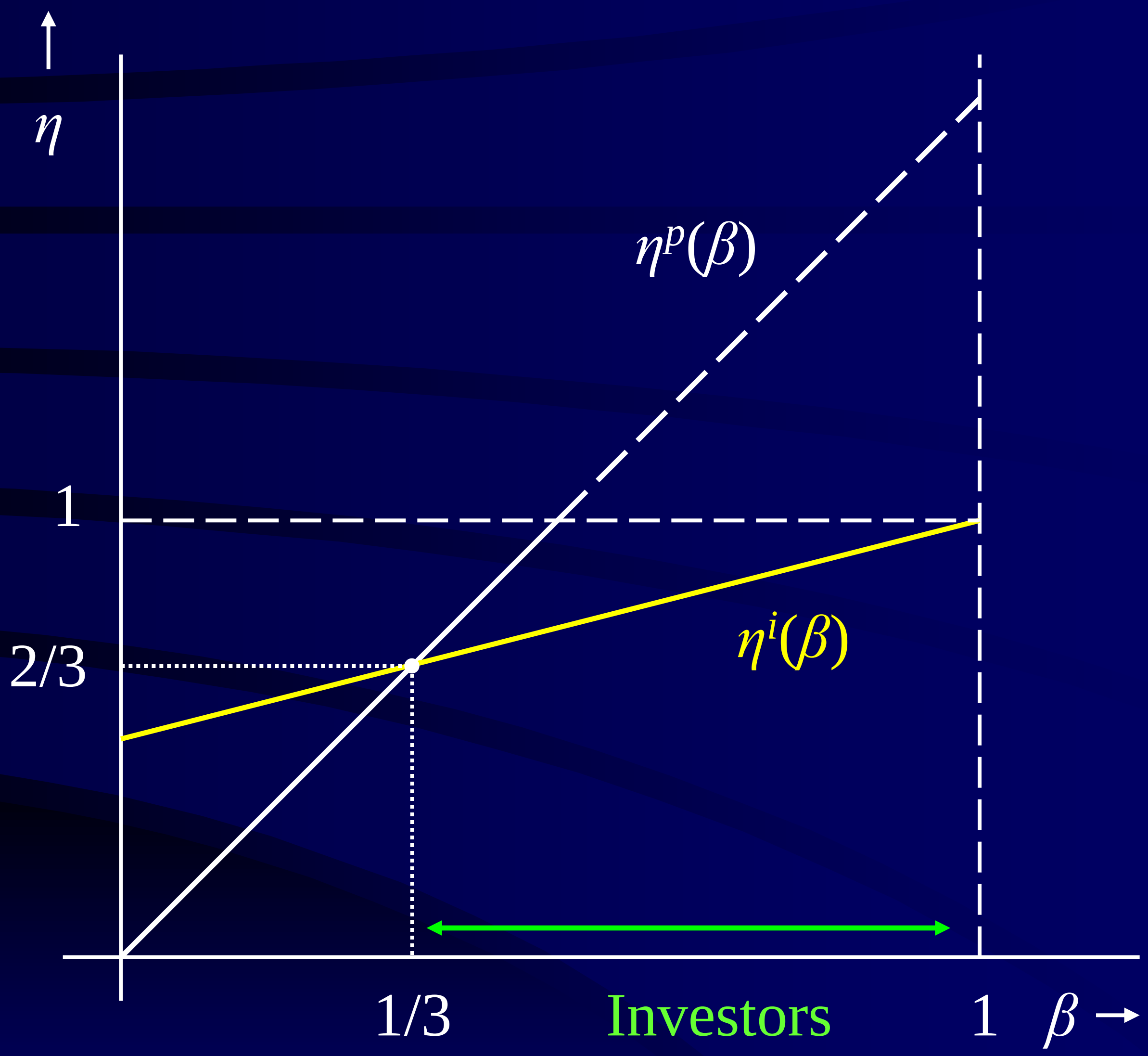
$$-\eta^2 + 2\ \eta\ \beta + 1.$$

Under- and Overvaluation of Ambiguous Assets



- The participation constraint: $U(x, \eta; \beta) \geq 1$ yields
  $\eta^p(\beta) := 2\,\beta.$
- The level of ambiguity is determined by majority voting of the investors in the asset.
- For marginal voter β the median voter is
  $\beta^m(\beta) := 1/2 + 1/2\,\beta.$
- The optimal level of ambiguity for investor β is
  $\eta^*(\beta) := \beta$
  and
  $\eta^i(\beta) = 1/2 + 1/2\,\beta.$

Under- and Overvaluation of Ambiguous Assets



Under- and Overvaluation of Ambiguous Assets

- If time passes, investors become more familiar with the ambiuous asset and the maximal level of ambiguity falls.
- For $t \in [0,1]$ the attainable levels of ambiguity $\eta(t)$ are restricted to be in $[0, \psi(t)]$ with $\psi(t) := 1 - t$.
- It follows that

$$\beta = 1/2 - 1/2\, t$$

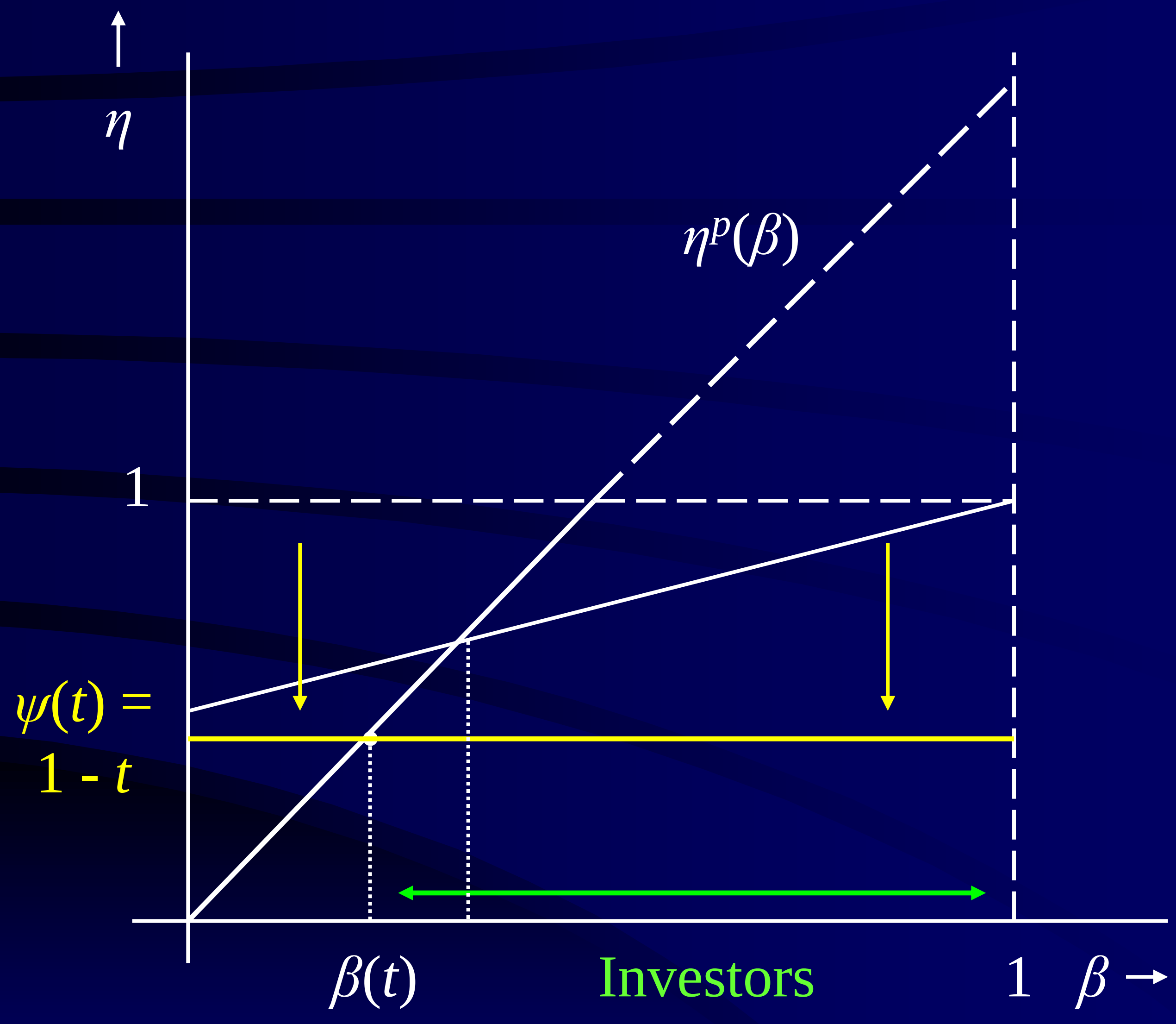
so

$$\beta^m(\beta) := 1/2 + 1/2\, \beta = 3/4 - 1/4\, t$$

and

$$\eta^p(\beta) := (1-t) + (3/4\, t - 1/4) = 3/4 - 1/4\, t > 1 - t.$$

Under- and Overvaluation of Ambiguous Assets



Under- and Overvaluation of Ambiguous Assets

- If time passes, investors become more familiar with the ambiuous asset and the maximal level of ambiguity falls.
- The ambiguous asset becomes attractive for more cautious investors.
- If it is not possible to futher increase investment in the ambiguous asset, its price increase.
- When the level ambiguity of the asset becomes sufficiently low, the price of the asset starts falling, reaching parity with the unambiguous asset for the ambiguity level of zero.

6. An Example of Overvaluation

- Expected payout of the ambiguous asset

  $\mathbf{E}\{x\} = E(\eta) := 1\text{-}\eta.$

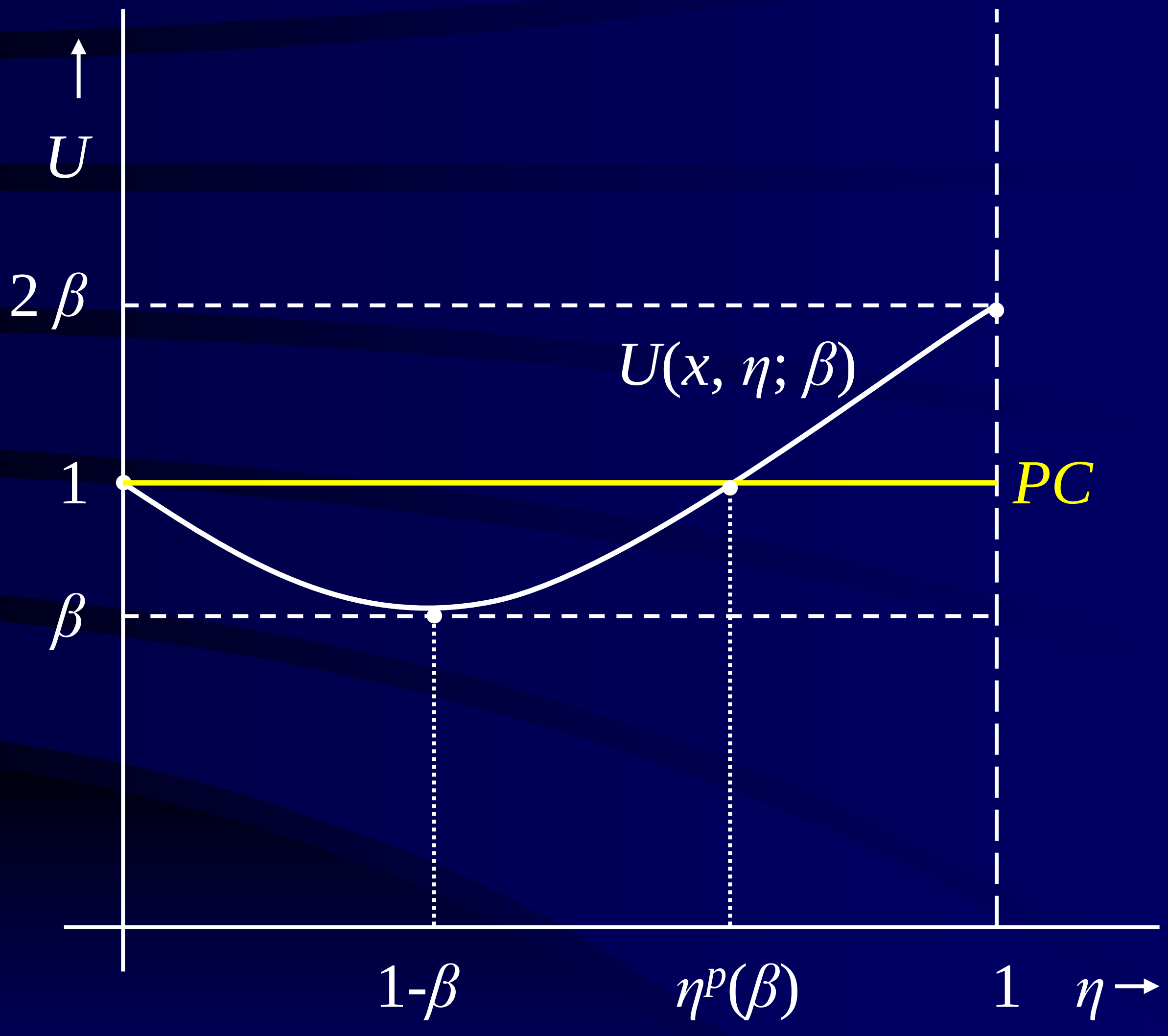
- Possible payouts: $x \in [0, 2]$.
- This leads to

$$U(x, \eta; \beta) := (1\text{-}\eta)\, E(\eta) + \eta\, \beta \max_{s \in S} x(s) + \eta\, (1\text{-}\beta) \min_{s \in S} x(s)$$

$$= (1\text{-}\eta)(1\text{-}\eta) + 2\, \eta\, \beta + 0 =$$

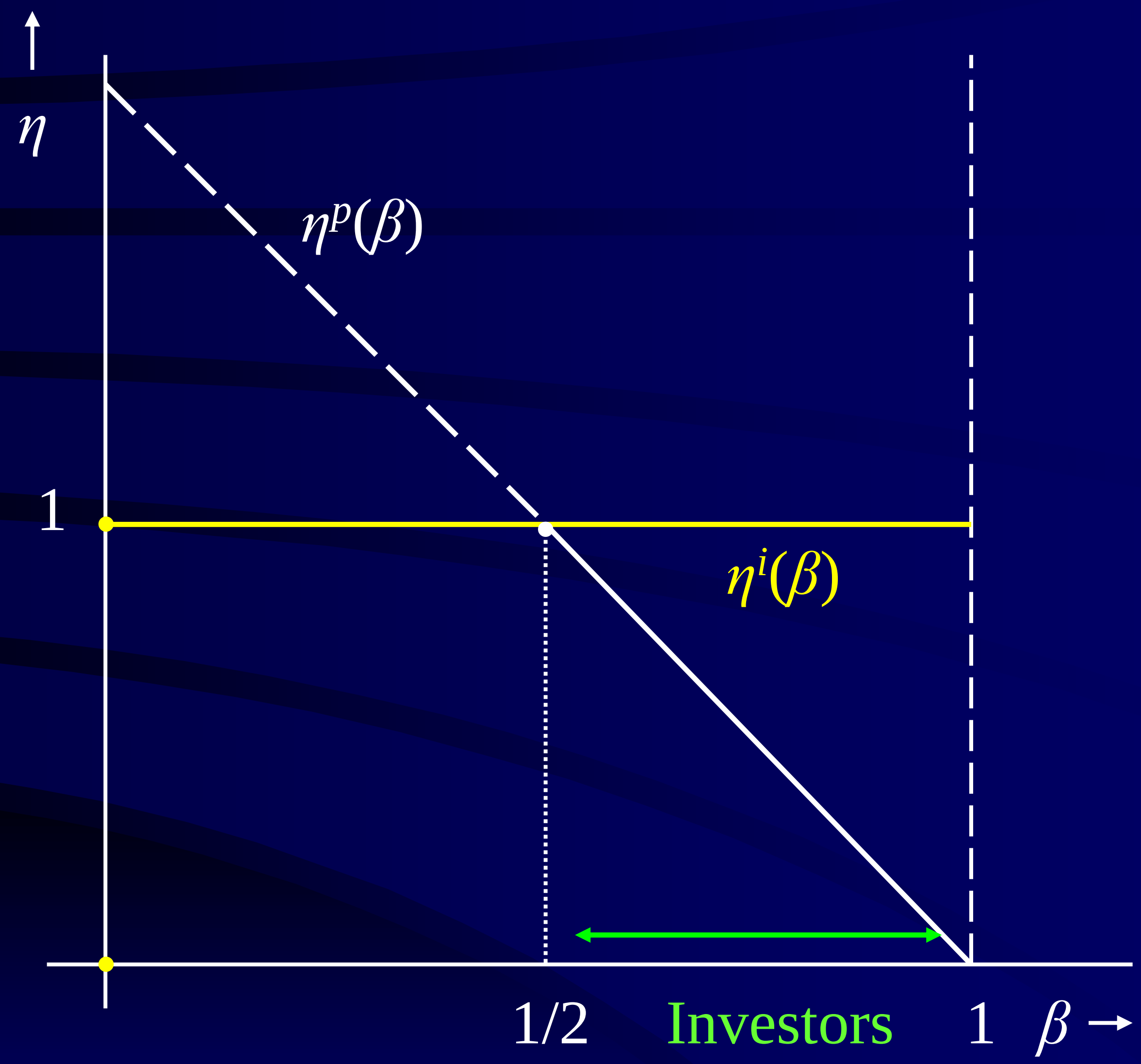
$$\eta^2 - 2\, \eta\, (1\text{-}\beta) + 1.$$

Under- and Overvaluation of Ambiguous Assets



- The participation constraint $U(x, \eta; \beta) \geq 1$ yields
  $\eta^p(\beta) := 2 - 2\,\beta$.
- The ambiguity $\eta^i(\beta)$ is determined by majority voting of the investors in the asset.
- For marginal voter β the median voter is
  $\beta^m(\beta) := 1/2 + 1/2\,\beta$.
- For $\eta = 0$: $U(x, \eta; \beta) = 1$
  $\eta = 1$: $U(x, \eta; \beta) = 2\,\beta$.
- The endogenous level of ambiguity is

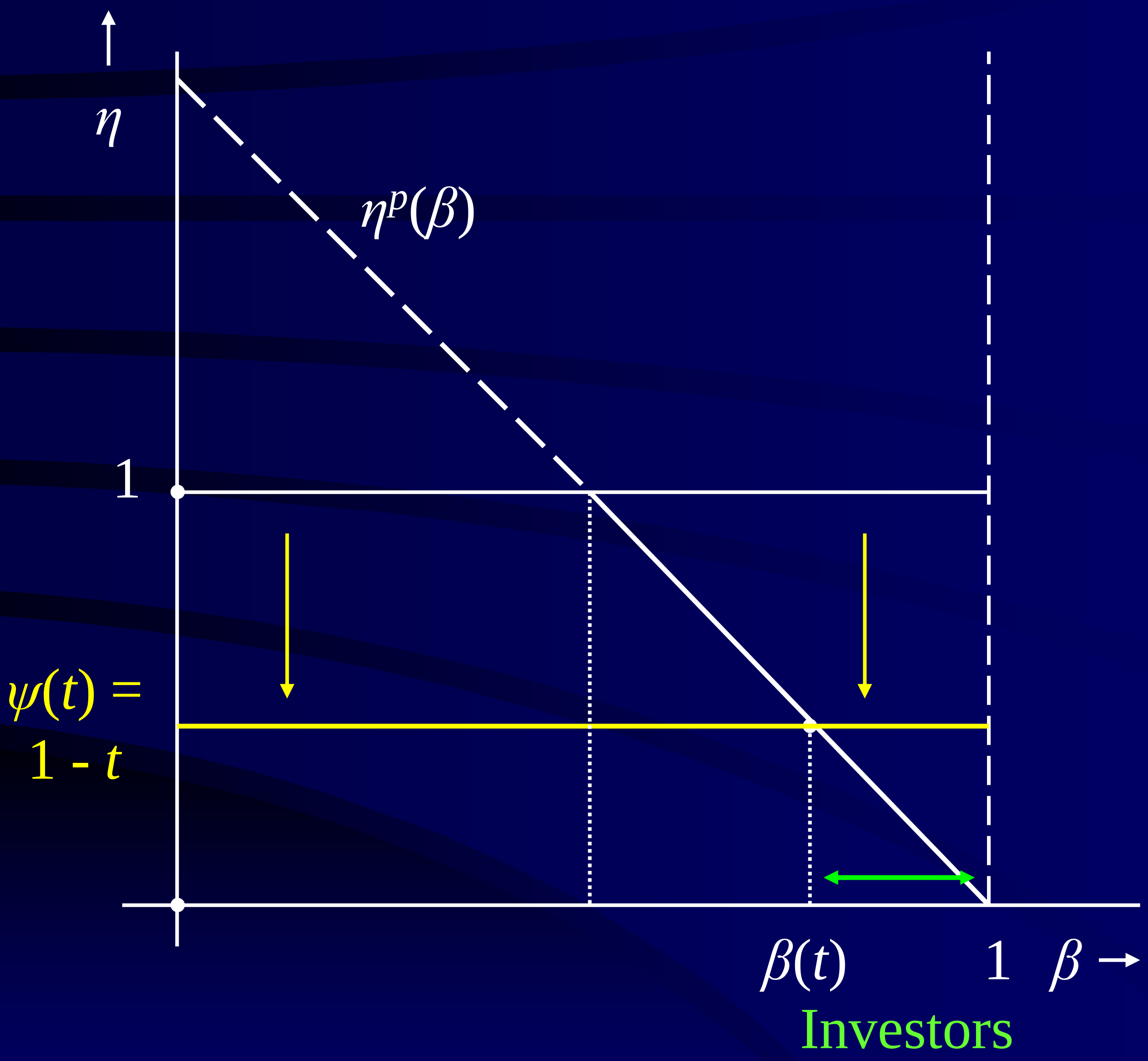
$$\eta^i(\beta) \begin{cases} = 0 & \text{if } 1/2 + 1/2\,\beta < 1/2 \\ \in \{0, 1\} & \text{if } 1/2 + 1/2\,\beta = 1/2 \\ = 1 & \text{if } 1/2 + 1/2\,\beta > 1/2. \end{cases}$$


η
$\eta^p(\beta)$
1
$\eta^i(\beta)$
1/2
Investors
1
β

- If time passes, investors become more familiar with the ambiuous asset and the maximal attainable level of ambiguity $\psi(t)$ falls.
- For $t \in [0,1]$ the attainable levels of ambiguity $\eta(t)$ are restricted to be in $[0, \psi(t)]$ with $\psi(t) := t - 1$.
- The marginal investor is $\beta(t) = 1/2 + 1/2\, t$.
- The endogenous level of ambiguity is

$$\eta^i(\beta) \quad \begin{cases} = 0 & \text{if } \beta < 1/2 + 1/2\, t \\ \in \{0, 1 - t\} & \text{if } \beta = 1/2 + 1/2\, t \\ = 1 - t & \text{if } \beta > 1/2 + 1/2\, t. \end{cases}$$

Under- and Overvaluation of Ambiguous Assets

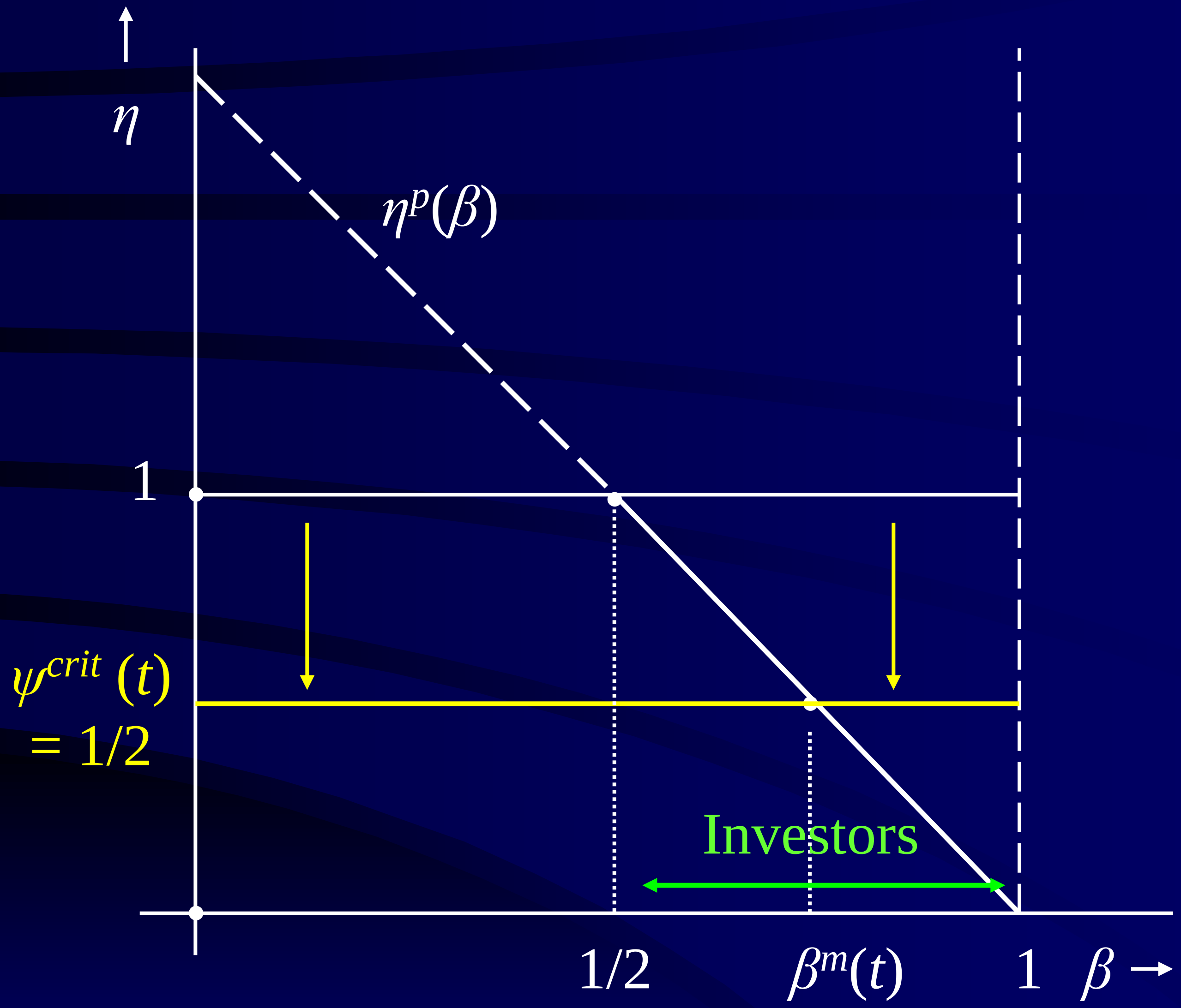


Under- and Overvaluation of Ambiguous Assets

Illiquid asset

- If the ambiguous asset is illiquid, investors who no longer want to hold the assets cannot dis-invest in.
- But they find no buyers for the ambiguous asset.
- So the price of the asset - the willingness to pay of the marginal investor β - falls below 1.
- For some t, the median investor no longer pefers the maximum ambiguity over no ambiguity.
- The level of ambiguity abruptly falls to zero and the price recovers.
- If there is inertia in the level of ambiguity, the shape of the utility function comes to bear: price falls (much) further before it recovers

Under- and Overvaluation of Ambiguous Assets



7. General Results

Theorem 1

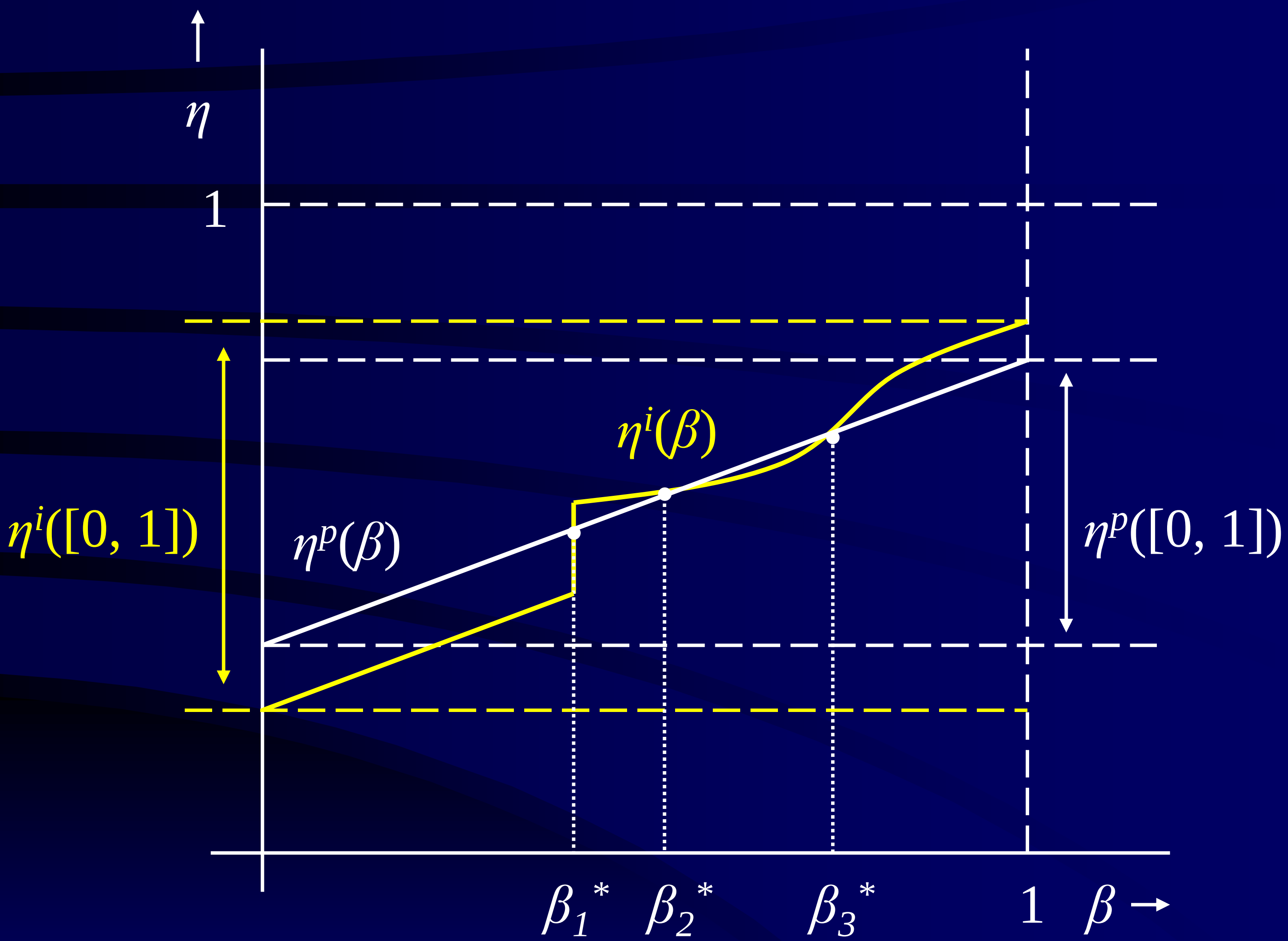
Let $\eta^i: [0, 1] \rightrightarrows [0, 1]$ be a non-empty and convex valued upper-hemi continuous correspondence and let $\eta^p: [0,1] \to \mathbb{R}$ be continuous function such that either

$\eta^i([0,1])$ is a subset of $\eta^p([0,1])$

or vice versa.

Then an equilibrium exists.

Under- and Overvaluation of Ambiguous Assets



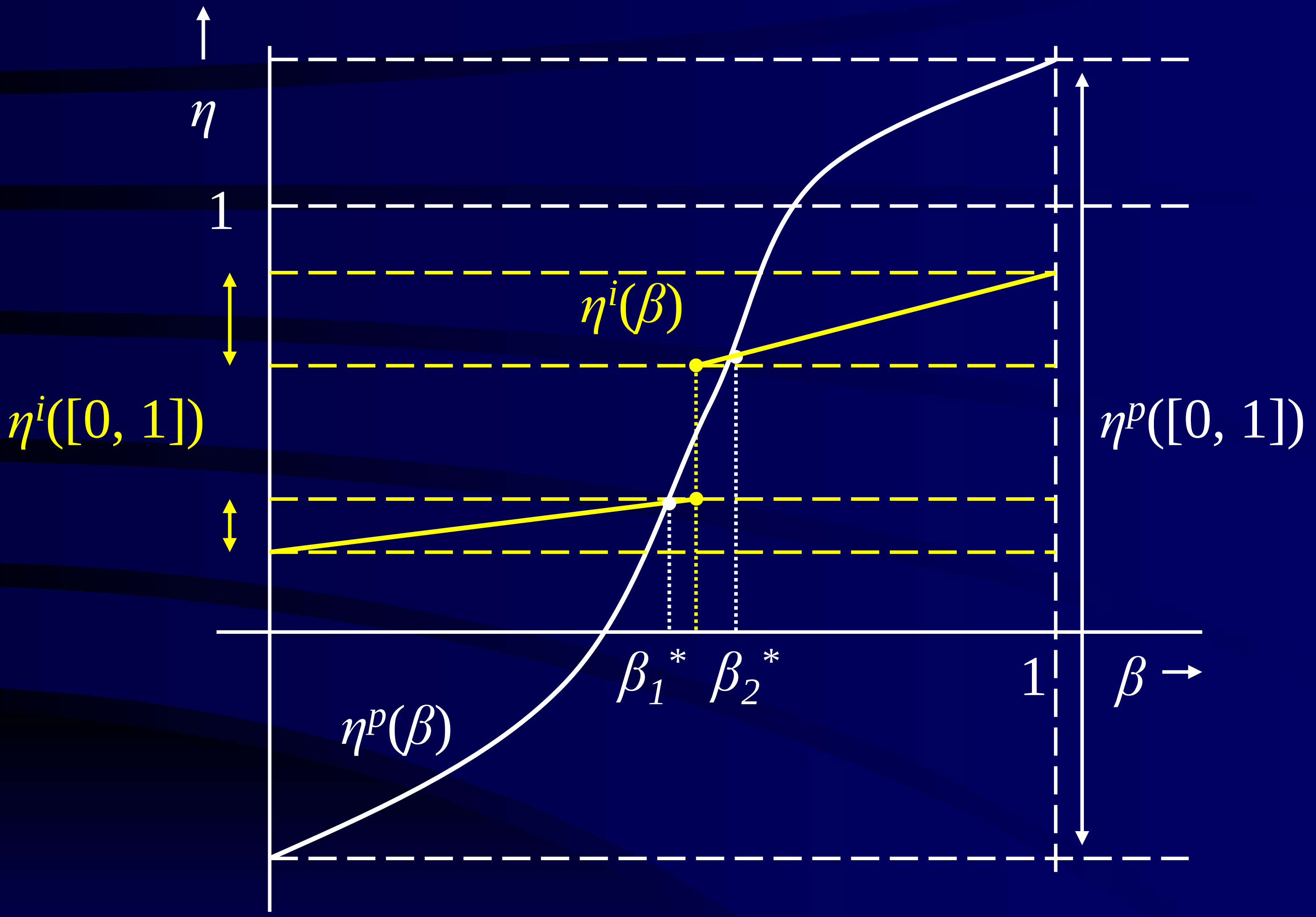
Theorem 2

Let η^i: [0, 1] $\rightrightarrows$ [0, 1] be a correspondence with a selection f: [0, 1] $\longrightarrow$ [0, 1] that is an increasing function and let η^p: [0,1] $\longrightarrow \mathbb{R}$ be continuous function such that

$$\eta^p([0,1]) \text{ is a subset of } \eta^i([0,1]).$$

Then an equilibrium exists.

Under- and Overvaluation of Ambiguous Assets



8. Concluding Remarks

- Ambiguity can be a potential cause for under- and overvaluation.
- In our examples:

  - if investors believe that increased ambiguity coincides with higher fundamental values, the asset is undervalued and no bubble occurs.

  - if investors believe that increased ambiguity coincides with lower fundamental values, the asset is overvalued and a bubble will occur.

  - if, in addition, the ambiguous asset is illiquid, there may be a sudden fall in the level of ambiguity after a period of declining prices.

Agenda for further work:

1. relate under- and overvaluation to general properties of $E(\eta)$.
2. endogenously derive η^i for an appropriate class of institutional environments.
3. develop a framework to in which multiple equilibria lead to coordination problems.
4. address the resulting equilibrium selection issue.
5. identify plausible properties for the decay process of ambiguity as described by ψ.